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Thermal Deflection in a Semi- Infinite Hollow Cylinder with Heat Source inside the Cylinder

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Abstract

This paper is concerned with the determination of temperature and the thermal deflection in a semi infinite hollow cylinder due to internal heat generation within it. The conduction equation has been solved by using March –Zgrablich and Fourier transform technique.

Keywords: *Non homogeneous heat condition, heat generation, semi-infinite hollow cylinder, integral Transform*

1. Introduction

Deshmukh and Wankhede [1]have discuss an inverse problem of thermoelasticity in a thin circular plate by determining the temperature on the curved surface of the plate so for find displacement and thermal stresses in the plate by using quasi-static approach by employing integral transform techniques. Deshmukh et al.[2]have determine inverse heat conduction problem in a semi-infinite circular plate and its thermal deflection by quasi-static approach. Deshmukh, K.C.et al.[3] have determined Inverse heat conduction problem in a semi-infinite cylinder and its thermal deflection by quasi-static approach Khobragade and Deshmukh[5] have discuss an inverse axially symmetric quasi-static problem of thermoelasticity for a thin clamped circular plate in which a heat flux is prescribed on an internal cylindrical surface of the plate and suitable heat exchange conditions are met on the upper and lower surfaces of the plate is solved with the help of a generalized integral transform technique. Recently Kulkarni and Deshmukh[6]have discus an inverse quasi-static steady state thermal stresses in a thick circular plate. Sabherwal [8]has discuss inverse problem of heat conduction in ceramic plate.

In this paper we consider heat conduction problem studied by Deshmukh et al. discuss the thermal stresses in a semi- infinite hollow cylinder. A hollow cylinder is subjected to arbitrary known temperature under unsteady state condition. Initially, the cylinder is at zero temperature and the lower surface is at zero temperature. The governing heat conduction equation has been solved by using integral transform method. The results are obtained in series form in terms of Bessel's functions. Mathematical model has been constructed of a semi-infinite hollow cylinder with the help of numerical illustration.

The problem is very important in view of its relevance to various industrial mechanics subjected to heating such as the main shaft of lathe, turbines, and the role of the rolling mill.

2. Statement of the problem

Consider semi infinite hollow cylinder occupying the space D: $a \le r \le b, 0 \le z < \infty$ for t > 0 heat generated within the semi infinite hollow cylinder at rate g(r, z, t)

The differential equation satisfying the deflection function is $\omega(r, t)$ given by

$$\nabla^4 \omega = \frac{\nabla^2 M_T}{D(1-\nu)} \tag{2.1}$$

Where M_T is the thermal moment of cylinder defined as

$$M_T = a_t E \int_0^h T(r\,,z\,,t) z dz \tag{2.2}$$

D is the flexural rigidity of the cylinder denoted as

$$D = \frac{Eh^3}{12(1-\nu^2)} \tag{2.3}$$

$a_t, E\ and\ \nu$ are the coefficient of the linear thermal expansion, Young's modulus and Poisson's ratio of the cylinder material respectively and

$$\nabla^2 = \frac{\partial^2}{\partial r^2} + \frac{1}{r}\frac{\partial}{\partial r} \tag{2.4}$$

since the edge of the semi-infinite cylinder is fixed,

$$\omega = \frac{\partial \omega}{\partial r} = 0 \quad at\ r = a\,, b \tag{2.5}$$

The temperature of the semi-infinite cylinder $T(r\,,z\,,t)$ at time t satisfies the differential equation

$$\frac{\partial^2 T}{\partial r^2}+\frac{1}{r}\frac{\partial T}{\partial r}+\frac{\partial^2 T}{\partial z^2}+\frac{g(r\,,z\,,t)}{k}=\frac{1}{\propto}\frac{\partial T}{\partial t} \tag{2.6}$$

With the boundary conditions

$$T+k_1\frac{\partial T}{\partial r}=F_1(\,z\,,t)\;\;at\;r=a\,,t>0 \tag{2.7}$$

$$T+k_2\frac{\partial T}{\partial r}=F_2(\,z\,,t)\;\;at\;r=b\,,t>0 \tag{2.8}$$

$$T\;=F_3(\,r\,,t)\;\;at\;z=0\,,t>0 \tag{2.9}$$

$$T\;=F(\,r\,,z)\;\;at\;t=0 \tag{2.10}$$

Where $T\;=T(\,r\,,z\,,t)\;and\;\;k_1\,,k_2$ are radiation constants and $\propto$ is thermal diffusivity of the material

Equation (2.1) to (2.10) constitutes mathematical formulation of the problem.

3. Solution of the problem

Applying March-Zgrablich and Fourier transform to the equation (2.6) one obtains

$$\frac{d\bar{T}^*}{dt}+\propto(\mu_n{}^2+\eta^2)\bar{T}^*=A(\mu_n\,,\eta\,,t) \tag{3.1}$$

$$A(\mu_n\,,\eta\,,t)=\frac{\alpha}{k}\bar{g}^*+\phi_1\overline{F_3}+\phi_2F_1{}^*+\phi_3F_2{}^* \tag{3.2}$$

$$\bar{T}^*(\mu_n\,,\eta\,,t)=\bar{F}^*(\mu_n\,,\eta)\;\;for\;t=0 \tag{3.3}$$

Equation (3.1) is a first order linear diff. equation, whose solution is given by

$$\bar{T}^*=e^{-\propto(\mu_n{}^2+\eta^2)t}\left[\bar{F}^*+\int_{t'=0}^{t}A(\mu_n\,,\eta\,,t)e^{\propto(\mu_n{}^2+\eta^2)t'}dt'\right] \tag{3.4}$$

Applying inverse March-Zgrablich and Fourier transform to the equation (3.4) one obtains

$$T=\sum_{n=1}^{\infty}\frac{S_0(\mathrm{k}_1,\mathrm{k}_2,\mu_\mathrm{n}r)}{C_n}\int_{\eta=0}^{\infty}\sqrt{\frac{2}{\pi}}sin\eta zd\eta e^{-\propto(\mu_n{}^2+\eta^2)t}\left[\bar{F}^*+\int_{t'=0}^{t}A(\mu_n\,,\eta\,,t)e^{\propto(\mu_n{}^2+\eta^2)t'}dt'\right] \tag{3.5}$$

Where

$$\bar{F}^*=\iint_{r'=a,z'=0}^{b,\infty}r'f(r',z')\,S_0(\mathrm{k}_1,\mathrm{k}_2,\mu_\mathrm{n}r)\sqrt{\frac{2}{\pi}}sin\;\eta z'dr'dz'$$

$$\bar{g}^*=\iint_{r'=a,z'=0}^{b,\infty}r'g(r',z',t)\,S_0(\mathrm{k}_1,\mathrm{k}_2,\mu_\mathrm{n}r)\sqrt{\frac{2}{\pi}}sin\;\eta z'dr'dz'$$

4. Determination of thermal deflection

Using equation (3.5) in equation (2.2) one obtains

$$M_T=a_tE\int_0^h\sum_{n=1}^{\infty}\frac{S_0(\mathrm{k}_1,\mathrm{k}_2,\mu_\mathrm{n}r)}{C_n}\int_{\eta=0}^{\infty}\sqrt{\frac{2}{\pi}}sin\eta zd\eta e^{-\propto(\mu_n{}^2+\eta^2)t}\left[\bar{F}^*+\int_{t'=0}^{t}A(\mu_n\,,\eta\,,t)e^{\propto(\mu_n{}^2+\eta^2)t'}dt'\right]zdz \tag{4.1}$$

We assume the solution of equation (2.1) satisfying condition (2.5) as

$$\omega(r,t)=\sum_{m=1}^{\infty}C_m\left[\,S_0(\mathrm{k}_1,\mathrm{k}_2,\mu_\mathrm{m}r)-\;S_0(\mathrm{k}_1,\mathrm{k}_2,\mu_\mathrm{m}\mathrm{b})\right] \tag{4.2}$$

Where μ_m are the positive roots of the transcendental equation

$$S_0(\mathrm{k}_1,\mathrm{k}_2,\mu_\mathrm{m}r)-\;S_0(\mathrm{k}_1,\mathrm{k}_2,\mu_\mathrm{m}\mathrm{b})=0 \tag{4.3}$$

It can be easily seen that

$$\omega=\frac{\partial\omega}{\partial r}=0\;\;at\;r=a\,,b$$

Hence the solution of (4.2)

Now

$$\nabla^4\omega=\left(\frac{\partial^2}{\partial r^2}+\frac{1}{r}\frac{\partial}{\partial r}\right)^2\sum_{m=1}^{\infty}C_m\left[\,S_0(\mathrm{k}_1,\mathrm{k}_2,\mu_\mathrm{m}r)-S_0(\mathrm{k}_1,\mathrm{k}_2,\mu_\mathrm{m}\mathrm{b})\right] \tag{4.4}$$

We use well result

$$\left(\frac{\partial^2}{\partial r^2}+\frac{1}{r}\frac{\partial}{\partial r}\right)S_0(\mathrm{k}_1,\mathrm{k}_2,\mu_\mathrm{m}r)=-\,\mu_\mathrm{m}{}^2\,S_0(\mathrm{k}_1,\mathrm{k}_2,\mu_\mathrm{m}r) \tag{4.5}$$

Substitute these values in (4.4)

$$\nabla^4\omega=\sum_{m=1}^{\infty}C_m\;\;\mu_\mathrm{m}{}^4\,S_0(\mathrm{k}_1,\mathrm{k}_2,\mu_\mathrm{m}r) \tag{4.6}$$

And

$$\nabla^2M_T=-a_tE\int_0^h\sum_{n=1}^{\infty}\frac{z\mu_n{}^2\,S_0(\mathrm{k}_1,\mathrm{k}_2,\mu_\mathrm{n}r)}{C_n}\int_{\eta=0}^{\infty}\sqrt{\frac{2}{\pi}}sin\eta zd\eta e^{-\propto(\mu_n{}^2+\eta^2)t}\left(\bar{F}^*+\int_{t'=0}^{t}A(\mu_n\,,\eta\,,t)e^{\propto(\mu_n{}^2+\eta^2)t'}dt'\right)dz \tag{4.7}$$

Using equation (4.6)and (4.7)in the equation (2.1)one obtains

$$\sum_{m=1}^{\infty}C_m\,\mu_\mathrm{m}{}^4\,S_0(\mathrm{k}_1,\mathrm{k}_2,\mu_\mathrm{m}r)=\frac{-a_tE}{D(1-v)}\int_0^h\sum_{n=1}^{\infty}\frac{z\mu_n{}^2\,S_0(\mathrm{k}_1,\mathrm{k}_2,\mu_\mathrm{n}r)}{C_n}\int_{\eta=0}^{\infty}\sqrt{\frac{2}{\pi}}sin\eta zd\eta e^{-\propto(\mu_n{}^2+\eta^2)t}\left(\bar{F}^*+\int_{t'=0}^{t}A(\mu_n\,,\eta\,,t)e^{\propto(\mu_n{}^2+\eta^2)t'}dt'\right)dz \tag{4.8}$$

On solving

$$C_m(t)=\frac{-a_tE}{D(1-v)}\int_0^h\sum_{n=1}^{\infty}\frac{z}{\mu_n{}^2C_n}\int_{\eta=0}^{\infty}\sqrt{\frac{2}{\pi}}sin\eta zd\eta e^{-\propto(\mu_n{}^2+\eta^2)t}\left(\bar{F}^*+\int_{t'=0}^{t}A(\mu_n\,,\eta\,,t)e^{\propto(\mu_n{}^2+\eta^2)t'}dt'\right)dz \tag{4.9}$$

Using equation (4.9) in equation (4.2) one obtains

$$\omega(r,t) = \sum_{m=1}^{\infty}\frac{-a_tE}{D(1-v)}\int_0^h\sum_{n=1}^{\infty}\frac{z}{\mu_n{}^2C_n}\int_{\eta=0}^{\infty}\sqrt{\frac{2}{\pi}}sin\eta zd\eta e^{-\propto(\mu_n{}^2+\eta^2)t}\left(\bar{F}^* + \int_{t'=0}^{t}A(\mu_n,\eta,t)e^{\propto(\mu_n{}^2+\eta^2)t'}dt'\right)dz\left[S_0(\mathrm{k}_1,\mathrm{k}_2,\mu_\mathrm{m}r) - S_0(\mathrm{k}_1,\mathrm{k}_2,\mu_\mathrm{m}\mathrm{b})\right] \quad (4.10)$$

5. Special case and numerical result

Setting,

$$F(r,z) = 0, \phi_2 = 0, \phi_3 = 0$$

$$g(r,z,t) = \frac{\delta(r-r_0)}{2\pi r}\delta(z-z_0)\delta(t-0) \quad (5.1)$$

$$T = \frac{\propto}{k\pi}\int_0^{\frac{z-z_0}{\sqrt{4\alpha t}}}e^{-\eta^2}d\eta\sum_{n=1}^{\infty}\frac{e^{-\propto(\mu_n{}^2)t}S_0(\mathrm{k}_1,\mathrm{k}_2,\mu_\mathrm{n}r)S_0(\mathrm{k}_1,\mathrm{k}_2,\mu_\mathrm{n}r_0)}{C_n} \quad (5.2)$$

$$\omega(r,t) = \frac{-a_tE}{D(1-v)}\frac{\propto}{k\pi}\int_0^{\frac{z-z_0}{\sqrt{4\alpha t}}}e^{-\eta^2}d\eta\sum_{n=1}^{\infty}\frac{e^{-\propto(\mu_n{}^2)t}S_0(\mathrm{k}_1,\mathrm{k}_2,\mu_\mathrm{n}r_0)}{C_n}\left[S_0(\mathrm{k}_1,\mathrm{k}_2,\mu_\mathrm{m}r) - S_0(\mathrm{k}_1,\mathrm{k}_2,\mu_\mathrm{m}\mathrm{b})\right] \quad (5.3)$$

Here a=1, b=3, $\mathrm{k}_1 = \mathrm{k}_2 = 0.25$,k=117 , $r_0 = 0.5$ t=1
$z_0 = 0.5$, $\propto = 3.33$, $a_t = 23 * 10^{-6}$,
$v = 0.35$, z=1.50, D=1, E=6.9*10^6

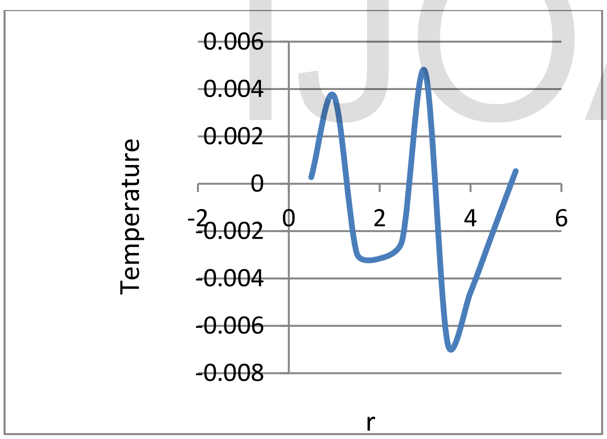

Fig.1 Temperature distribution along r

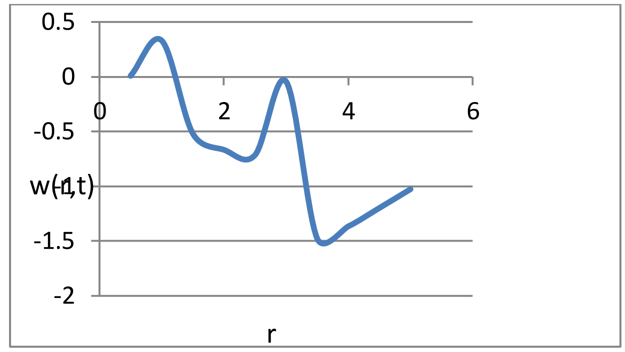

Fig.2 Thermal deflection along r

6. Conclusion

In this paper we extended the work of Deshmukh et al for two dimensional non homogeneous boundary values of problem of heat conduction and determination of temperature and the thermal deflection in a semi infinite hollow cylinder due to internal heat generation within it. The conduction equation has been solved by using March –Zgrablich and Fourier transform technique. The result presented here may be useful in solving engineering problem, particularly for aerospace engineering any particular case of special interest can be derived by assigning suitable value of the parameters and function in the expression.

7. References

[1].Deshmukh,K.C.and Wankhede,P.C., "An inverse quasi-static transient thermoelastic problem in a thin circular plate", *Bulletin of Pure and Applied Sciences, 17(1), pp. 209-207,(1998).*

[2].Deshmukh,K.C., Warbhe, S.D., Kedar, G.D. and Kulkarni, V.S., "Inverse heat conduction problem in a semi-infinite circular plate and its thermal deflection by quasi-static approach', *An International Journal (AAM), 5(1), pp. 120-127, (2010).*

[3].Deshmukh, K.C., Warbhe, S.D., Kedar, G.D. and Kulkarni, V.S., "Inverse heat conduction problem in a semi-infinite cylinder and its thermal deflection by quasi-static approach", *An International Journal (AAM), Vol. 6, issue 11 pp. 1883 – 1892(2011).*

[4].Grysa, K. and Cialkowski, M.J., 'On a certain inverse problem of temperature and thermal stress field", *Acta mechanica, 36, pp.169-185, (1980).*

[5].Khobragade, N.L. and Deshmukh, K.C., "An inverse quasi-static thermal deflection problem for a thin clamped

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circular plate", *J. Thermal Stresses, 28(4), pp. 353-361, (2005).*

[6].Kulkarni, V.S. and Deshmukh, K.C., " An inverse quasi-static thermal stresses in a thick circular plate", *J. franklin institute, 345, pp. 29-38, (2008).*

[7].Ozisik, M.N., "Boundary value problems of heat conduction", *International Text Book Company, Scranton, pennsylvania. (1968).*

[8].Sabherwal, K.C., *Indian Journal of Pure and Applied Physics, Vol.XL, No.8; pp. 448-450, (1966).*

[9].Khobragade,N.W.and Walde,R.T., "Thermal Deflection of a Clamped Annular Disc Due to Heat Generation", *Int. J latest Trend Math Vol-2 No. 1pp.37-43,(2012)*

Appendix

1. The Fourier integral transform of f(x), $0 \leq x < \infty$ is defined to be

$$\bar{F}(\beta) = \int_{x'=0}^{\infty} K(\beta, x')F(x')dx'$$

then inverse formula

$$F(x) = \int_{\beta=0}^{\infty} K(\beta, x)\bar{F}(\beta)\, d\beta$$

where $K(\beta, x) = \sqrt{\frac{2}{\pi}} sin(\beta)x$

2. The finite Marchi-Zgrablich integral transform of order p is defined as

$$\bar{f}_p(n) = \int_a^b x f(x) S_p(\mathrm{k}_1, \mathrm{k}_2, \mu_n x) dx$$

And inverse Marchi-Zgrablich integral transform as

$$f(x) = \sum_{n=1}^{\infty} \frac{\bar{f}_p(n) S_p(\mathrm{k}_1, \mathrm{k}_2, \mu_n x)}{C_n}$$

Where

$$S_p(\mathrm{k}_1, \mathrm{k}_2, \mu_n x) = J_p(\mu_n x)\{Y_p(\mathrm{k}_1, \mu_n a) + Y_p(\mathrm{k}_2, \mu_n b)\} - Y_p(\mu_n x)\{J_p(\mathrm{k}_1, \mu_n a) + J_p(\mathrm{k}_2, \mu_n b)\}$$

$$C_n = \frac{b^2}{2}\{S_p{}^2(\mathrm{k}_1, \mathrm{k}_2, \mu_n b) - S_{p-1}(\mathrm{k}_1, \mathrm{k}_2, \mu_n b). S_{p+1}(\mathrm{k}_1, \mathrm{k}_2, \mu_n b)\} - \frac{a^2}{2}\{S_p{}^2(\mathrm{k}_1, \mathrm{k}_2, \mu_n a) - S_{p-1}(\mathrm{k}_1, \mathrm{k}_2, \mu_n a). S_{p+1}(\mathrm{k}_1, \mathrm{k}_2, \mu_n a)\}$$

An operational property is given by

$$\int_a^b \left[\frac{\partial^2 f}{\partial x^2} + \frac{1}{x}\frac{\partial f}{\partial x} + \frac{p^2 f}{x^2}\right] S_p(\mathrm{k}_1, \mathrm{k}_2, \mu_n x) = \frac{b}{\mathrm{k}_2} S_p(\mathrm{k}_1, \mathrm{k}_2, \mu_n b) \left[f + \mathrm{k}_2 \frac{\partial f}{\partial x}\right]_{x=b} - \frac{a}{\mathrm{k}_1} S_p(\mathrm{k}_1, \mathrm{k}_2, \mu_n a) \left[f + \mathrm{k}_1 \frac{\partial f}{\partial x}\right]_{x=a} - \mu_n{}^2 \bar{f}_p(n)$$

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