The Effect of Heat-Loss Function on the Self- Gravitational Instability of Gaseous Plasma in the Presence of Fine-Dust Particles and Magnetic Field

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Abstract:-

The effect of heat-loss function on the self-gravitational instability of gaseous plasma in the presence of fine-dust-paticles and magnetic field has been studied, with the help of linearized MHD equations, a general dispersion relation is obtained using normal mode analysis. The conditions of modified Jeans instability and stability are discussed in the different cases of our interest. We found that the presence of arbitrary heat-loss functions and thermal conductivity modifies the fundamental Jeans condition of gravitational instability into a radiative instability condition. It is found that the radiative mode of instability arise in the medium depends on the arbitrary radiative heat-loss functions on the local density and temperature of the system. Applying Routh-Hurwitz, the stability of the medium is discussed and it is found that Jeans criterion determines the stability of the medium. Thermal conductivity modifies the Jeans criterion and the viscosity and permeability has damping effect. It is found that the effect of fine-dust-particles is to destabilize the system. It is also found that the condition of instability for propagation parallel to the magnetic field is independent of the magnetic field strength. For transverse propagation, however the condition for instability depends on the magnetic field strength.

Keywords- Jeans Instability, Magneto hydrodynamics (MHD), Arbitrary radiave heat-loss function, Fine-dust-particles, Thermal conductivity, Viscosity, Permeability, Interstellar Medium (ISM).

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1. Introduction:-

The gravitational instability is one of the fundamental concepts of modern astrophysical plasma and it is connected with the fragmentation of interstellar matter in regard to star formation. James Jeans(1) first investigated this instability problem and suggested that an infinite homogeneous self-gravitating fluid is unstable for all wave number which is less than critical Jeans wave number. In terms of the wavelength of a fluations, Jeans criterion say that λ should be greater than a critical value $\lambda_j = \sqrt{\frac{\pi c^2}{G\rho}}$, which is named the Jeans length. In this formula G is the gravitational constant, ρ is the unperturbed matter density and $C = \sqrt{\frac{k_B T}{m}}$, is the sound speed for adiabatic perturbation, where k_B is the Boltzmann's constant, T is the physical temperature and m is the mass of the particle. Jeans(2) discussed the condition under which a fluid becomes gravitational unstable under the action of its own gravity.

Now a days, for any relevant length scales in the universe [stars, galaxies, planetary rings, asteroid, clusters, etc] gravitational instability has been recognized as the key mechanism to explain the gravitational formations of structures and there evolution in the linear regime. The conclusion of Jeans work is that perturbations with mass greater a critical mass M_j (named as Jeans –mass) may grow there by producing gravitational bounded structures, where as perturbation a mass smaller then M_j do not grow and behave like acoustic waves. Such as instability has been studied by many researchers(3-14) and they found that this gravitational instability criterion remains basically valid and plays a fundamental role even in the expanding universe.

The Jeans problem has been extensively investigated under varying assumption. A comprehensive account of the investigations has been given by Chandrasekhar(15) in his monograph on hydrodynamic and hydro magnetic stability. Ebert(16) studied the problem of an isothermal gas sphere subjected to external pressure and he found that disturbances of length

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scale approximately equal to the Jeans length based on the central density were unstable to gravitational collapse.

Since $\lambda_j \propto \rho^{-\frac{1}{2}}$ and $M_j \propto \rho\lambda_j^3 \propto \rho^{-\frac{1}{2}}$; this considerably reduced the minimum unstable mass and demonstrated that an 'O' star could from in the centre of an interstellar cloud. Hunter(17) investigated the growth of perturbations with initial scale of the order of or less than the Jeans length grew less rapidly relative to back ground density than did perturbations of substantially layer dimension.

The virial theorem condition for the collapse of an isolated cloud is

$$2E_T + E_G < 0$$

Where E_T is the total internal kinetic energy of the cloud and E_G its gravitational energy. For the case of isothermal gas sphere and for given the density and temperature of the gas we can estimate a lower bound to the mass [and radius] of the sphere which is gravitationally unstable. The radius derived from condition is found to correspond closely to the Jeans length i.e. a sphere of mass above would be unstable against its own gravitational forces. These results have confirmed by(16-18) considering more accurate configuration of the gas. Furthermore, so long as the gas remains isothermal, pressure forces will be unable to halt the collapse and as Hunter(17) has pointed fragmentation into smaller sub condensations will probably occur. It should be noted that "instability" in this sense is dependent on the consideration of isothermally [or at least ratio of specific heats $\gamma < \frac{4}{3}$] where the Jeans result, which is applicable only in crucial order, does not depend on γ in any crucial way [the value of c^2 is slightly changed].

In the recent ISM observations, the importance of the radiative heat-loss mechanism in interstellar gas dynamics has been recognized parameter. The radiative heat-loss mechanism plays an important role in the star formation and molecular cloud condensations process in connection with thermal instability. The ISM structure shows that the heat-loss procession is the major cause of condensation of large astrophysical compact objects. Basically these radiative heat-loss functions show the decay of heat in an embedded system with respect to local temperature and density. These radiative heat-loss functions are similar to those of cooling functions considered by(19-20) for the H_{11} region. It leads to several important phenomena of astrophysics and space plasma physics.

In addition to this partially-ionized plasma represent a state which often exists in the universe. The interaction between the neutral and the ionized gas components becomes important in cosmic sphere. The nature of the coupling of the magnetic field to the neutrals through ion-neutral collisions has been studied by(21-22). In this connection many investigators(23-24) investigated the gravitational instability in the presence of fine-dust-particles with various parameters.

From the above studies, we find that viscosity, permeability, thermal conductivity, radiative heat-loss functions and fine-dust-particles are the important parameters to discuss Jeans instability of gaseous plasma. Thus in the present problem, we investigate the effects of arbitrary radiative heat–loss functions and thermal conductivity on the Jeans instability of viscous with permeability, magnetized gaseous plasma in the presence of fine-dust-particles.

2.Linearized Perturbation Equation:-

We consider an infinite homogeneous self-gravitating gas particle medium of thermal conductivity having radiative heat-loss effects in the presence of fine-dust-particles and it is acted by a uniform vertical magnetic field $H(0,0,H)$ Let $u(u,v,\omega),v,\rho$ and N be the gas velocity, the particle velocity, the density of gas and the number of density of particles. If we assume uniform particle size, spherical shape and small relative velocities between the two phases, then the net effect of particles on the gas is equivalent to an extra body force term per unit volume $K_s N(v-u)$ and is added to the momentum transfer equation for gas, where the constant K_s is given by Stoke's drag formula $K_s = 6\pi\rho\upsilon r$, r being the particle radius and υ is the kinetic viscosity of clean gas. Self-gravitational attraction U is added with kinetic viscosity term in equation of motion for gas, k_1 is the permeability of the medium.

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In writing the equation of motion for particles, we neglect the bouyancy force as its stabilizing effect for the case of two free boundaries is extremely small. Interparticles reactions are also ignored by assuming the distance between particles to be too large compared with their diameters.

The stability of the system is investigated by writing the solutions to the full equations as initial state plus a perturbation. The initial state of the system is taken to be quiescent layer with a uniform particle distribution. The equations thus obtained are linearized by neglecting the product of two perturbed quantities.

Hence the linearized perturbation equations for motion of such medium are:

$$\frac{\partial u}{\partial t} = -\frac{1}{\rho}\nabla\delta p + \nabla\delta U + \frac{K_S N}{\rho}(v-u) + \upsilon\left[\nabla^2\vec{u} - \frac{u}{k_1}\right] + \frac{1}{4\pi\rho}(\nabla\times h)\times H \tag{1}$$

$$\frac{\partial}{\partial t}\delta\rho = -\rho\nabla.u \tag{2}$$

$$\delta p = c^2\delta\rho \tag{3}$$

$$\nabla.h = 0 \tag{4}$$

$$\left(\tau\frac{\partial}{\partial t} + 1\right)v = u \tag{5}$$

$$\frac{\partial}{\partial t} = \nabla \times (u \times H) \tag{6}$$

$$\nabla^2 \delta U = -4\pi G \delta\rho \tag{7}$$

$$\frac{1}{(\gamma-1)}\frac{\partial}{\partial t}\delta p - \left(\frac{\gamma}{\gamma-1}\right)\frac{p}{\rho}\frac{\partial}{\partial t}\delta\rho + \rho\left[\angle_\rho \delta\rho + \angle_T \delta T\right] - \lambda\Delta^2 \delta T = 0 \tag{8}$$

$$\frac{\delta p}{p} = \frac{\delta T}{T} + \frac{\delta\rho}{\rho} \tag{9}$$

Where $\delta\rho, \delta p, \delta U, \delta T$ and $h(h_X, h_y, h_z)$ denote respectively the perturbation in density ρ, pressure p, gravitational potential U, temperature T and magnetic field H, G is gravitational constant, c is the velocity of sound, $\tau = m/K_s$ and mN is the mass of particles per unit volume. Here $\angle_\rho and \angle_T$ respectively denote partial derivatives $\left(\frac{\partial\angle}{\partial\rho}\right)_T and \left(\frac{\partial\angle}{\partial T}\right)_\rho$ of heat-loss function evaluated for the initial state.

3. Dispersion Relation:-

Let us assume the perturbation of all the quantities vary as

$$\text{Exp. } \left[i\left(K_x x + K_z z + \omega t\right)\right] \tag{10}$$

Where ω is the growth rate of the perturbation and K_x, K_z are the wave numbers of the perturbation along the x- and z- directions respectively such that

$$K_x^2 + K_z^2 = K^2 \tag{11}$$

Combining equation (8) and (9), we get

$$\delta p = \left(\frac{\alpha + \sigma C^2}{\sigma + \beta} \right) \delta \rho \tag{12}$$

Where $\sigma = i\omega$

$C = \left(\frac{\gamma p}{\rho} \right)^{1/2}$ Is the adiabatic velocity of sound in the medium

$$\left.\begin{aligned} \alpha &= (\gamma - 1)\left[\angle_T T - \angle_\rho \rho + \frac{\lambda K_s^2 T}{\rho} \right] \\ \beta &= (\gamma - 1)\left[\frac{\angle_T T \rho}{p} + \frac{\lambda K_s^2 T}{p} \right] \end{aligned}\right] \tag{13}$$

We get the following equations from -(1), using (2) - (7), (12) and variations

$$\begin{aligned} &\tau\sigma^3 + \sigma^2 \left(1 + \frac{KsN\tau}{\rho} + \Omega_v \right) + \sigma\Omega_v + (1 + \sigma\tau) K_s^2 V^2 \quad]u \\ &= -\frac{iK_x}{K_s^2} \Omega_T^2 (1 + \tau\sigma) \sigma s \end{aligned} \tag{14}$$

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$$\tau\sigma^3 + \sigma^2\left(1 + \frac{K_s N\tau}{\rho} + \Omega_v \tau\right) + \sigma\left(\frac{4}{3}\Omega_v\right) + (1+\sigma\tau)K_z^2 V^2 \Big] v = 0 \tag{15}$$

$$\left[\tau\sigma^2 + \sigma\left(1 + \frac{K_s N\tau}{\rho} + \frac{4}{3}\upsilon K_s^2 \tau\right) + \frac{4}{3}\upsilon K^2\right]\omega = \frac{-iK_z}{K_s^2}\Omega_T^2(1+\tau\sigma)s \tag{16}$$

On taking the divergence of equation (1) using (2)- (7), (12) and performing the above said variation we get

$$iK_x K_s^2 V^2(1+\tau\sigma)u + \begin{bmatrix} \tau\sigma^4 + \sigma^3\left(1 + \frac{K_s N\tau}{\rho} + \Omega_v\tau\right) + \frac{4}{3}\upsilon K_s^2 \tau \\ + \frac{4}{3}\upsilon K^2 \sigma^2 + \sigma(1+\sigma\tau)\Omega_T^2 \end{bmatrix} = 0 \tag{17}$$

(14) - (17) can be written in the form

$$\begin{vmatrix} \sigma\xi_3 + \xi_2 K_s^2 V^2 & 0 & 0 & \frac{iK_x}{K_s^2}\Omega_T^2\xi_2\sigma \\ 0 & \sigma\xi_3 + \xi_2 K_z^2 V^2 & 0 & 0 \\ 0 & 0 & \xi_3 & \frac{iK_z}{K_s^2}\Omega_T^2\xi_2 \\ -iK_x K_s^2 V^2 \xi_2 & 0 & 0 & \sigma(\sigma\xi_3 + \xi_2\Omega_T^2) \end{vmatrix} \begin{vmatrix} u \\ v \\ \omega \\ s \end{vmatrix} = 0$$

(18)

Where $s = \frac{\delta\rho}{\rho}$ is the condensation of the medium

$$\left.\begin{aligned}
&1+\frac{K_s N\tau}{\rho}+\Omega_v\tau=\xi_1\\
&(1+\sigma\tau)=\xi_2\\
&\tau\sigma^2+\sigma\left(1+\frac{K_s N\tau}{\rho}+\Omega_v\right)+\Omega_v=\xi_3\\
&\Omega_{Tt}^2=\frac{\sigma\Omega_j^2+\Omega_I^2}{\sigma+\beta}\\
&\Omega_I^2=K_s^2\alpha-4\pi G\rho\beta\\
&\Omega_j^2=K_S^2C^2-4\pi G\rho\\
&\Omega_v=\upsilon\left(K_S^2+\frac{1}{k_1}\right)\\
&V=\frac{H}{(4\pi\rho)^{1/2}}
\end{aligned}\right\} \tag{19}$$

is the Alfven velocity V

The determinant of the matrix of (19) gives the dispersion relation

$$\sigma\left[\sigma\xi_3+\xi_2K_Z^2V^2\right][\xi_3]\begin{bmatrix}\left\{\sigma\xi_3+\xi_2K_S^2V^2\right\}\left\{\sigma\xi_3+\xi_2\Omega_T^2\right\}\\ -\frac{\Omega_T^2}{K_S^2}\xi_2^2K_x^2V^2K_S^2\end{bmatrix}=0$$

Equation (20) represents the dispersion relation for combined influence of an infinite homogeneous, thermally conducting, radiating and self-gravitating gas particle medium in the presence of fine-dust-particles, having uniform magnetic field in z-direction. If we neglect the effect of thermal conductivity and radiative term in Equation (20), we get the result as obtained by Sharma (1977) and that of Chhajalani et.al. (1978) neglecting the contribution of electrical conductivity.

4.Discussion:-

It is convenient to discuss this dispersion relation for longitudinal and transverse propagation separately.

Case 4.1: Longitudinal propagation:-

We assume the perturbation in parallel direction to magnetic field $\left(K_x = 0, K_z = K_s\right)$. The dispersion relation (20) is reduced to

$$\sigma\left[\sigma\xi_3 + \xi_2 K_Z^2 V^2\right]^2 \left[\xi_3\right]\left[\sigma\xi_3 + \xi_2 \Omega_T^2\right] = 0 \qquad (21)$$

Re substituting the values of ξ_2, ξ_3 and Ω_T^2 we get of twelfth degree equation in the terms of σ. This equation splits into four factors representing different modes due to various effects discussed below;

The first factor of Equation (21) gives us

$$\sigma = 0 \qquad (22)$$

Which is neutrally stable mode.

This second factor of Equation (21) gives the cubic equation

$$\tau\sigma^3 + \left(1 + \frac{K_s N \tau}{\rho} + \Omega_V \tau\right)\sigma^2 + \left(\Omega_V + \tau K_s^2 V^2\right)\sigma + K_s^2 V^2 = 0 \qquad (23)$$

This is the dispersion relation for a magnetized, viscous gas particle medium in the presence of the fine-dust-particles. This mode does not depend on self-gravitation thermal conductivity and heat-loss function. But this is a stable mode as the term is constant with positive sign, and Equation (23) can never have a negative real root or a complex root whose real part is not positive values. The sufficient condition is that the Routh-Hurwitz criterion must be satisfied, according to which all the principal diagonal minors of the Hurwitz matrix must be positive for a stable system and we get

$$\Delta_1 = \tau\left(1 + \frac{K_s N\tau}{\rho} + \Omega_V \tau\right) \rangle 0$$

$$\Delta_2 = \Omega_V + \left(\frac{K_s N\tau}{\rho} + \Omega_V \tau\right)\left(\Omega_V + \tau K_s^2 V^2\right) \rangle 0$$

$$\Delta_3 = K_s^2 V^2 \Delta_2 \rangle 0$$

We find that all Δ's are positive so we find that a magnetized, viscous gas particle medium is stable even in the presence of fine-dust-particles.

The third factor of Equation (21) gives us a quadratic equation

$$\tau\sigma^2 + \left(1 + \frac{K_s N\tau}{\rho} + \Omega_V \tau\right)\sigma + \Omega_V = 0 \qquad (24)$$

Which is identical to Equation (17) of Sharma (1977) and (16) of Chhajlani et.at.(1978). This mode is independent of thermal conductivity, heat-loss function, self-gravitation and magnetic field. According to the necessary and the sufficient condition, Equation (24) has all the roots with negative real parts, giving stable mode.

The last factor of Equation (21) gives

$$\tau\sigma^4 + \left[\xi_1 + \tau\beta\right]\sigma^3 + \left[\xi_1\beta + \Omega_V + \tau\Omega_j^2\right]\sigma^2 + \left[\Omega_j^2 + \tau\Omega_I^2 + \beta.\Omega_V\right]\sigma + \Omega_I^2 = 0 \qquad (25)$$

This is the dispersion relation for a viscous, self-gravitating gas particle medium in the presence of fine-dust-particles and incorporating thermal conductivity and heat-loss function. This mode is independent of magnetic field. Equation (25) may be reduced to particular cases so that the effect of each factor may be discussed separately.

For only self-gravitating, viscous gas particle medium in the presence of fine-dust-particles, we have $\Omega_I^2 = 0, \beta = 0$ and Equation (25) is reduced to

$$\tau\sigma^3 + \left[\xi_1\right]\sigma^2 + \left[\Omega_V + \tau\Omega_j^2\right]\sigma + \Omega_j^2 = 0 \qquad (26)$$

This is a third degree equation and it is identical to Equation (16) of Sharma (1975) and the condition of instability obtained from Equation (26) from constant term is $K_s < K_j$ where K_j is the critical Jeans wave number given as

$$K_j = \sqrt{\frac{4\pi G\rho}{C^2}} \qquad (27)$$

And corresponding critical Jeans wave length is

$$\lambda_j = \sqrt{\frac{\pi}{G\rho}} \qquad (28)$$

The system is unstable for all Jeans length $\lambda > \lambda_j$ or wave numbers $K_s < K_j$ thus we find that Jeans criterion of instability holds good in the presence of fine-dust-particles on the gravitational instability of an infinite homogeneous gas-particle medium.

For viscous, non-radiating, but thermally conducting self-gravitating gas particle medium in the presence of fine-dust-particles, Equation (25) becomes

$$\tau\sigma^4 + \left[\xi_1 + \tau\Omega_K\right]\sigma^3 + \left[\xi_1\Omega_K + \Omega_V + \tau\Omega_j^2\right]\sigma^2 + \begin{bmatrix}\Omega_j^2 + \tau\Omega_K + \Omega_{jI}^2 \\ + \Omega_K.\Omega_V K_s^2\end{bmatrix}\sigma + \Omega_K.\Omega_{jI}^2 = 0 \qquad (29)$$

Where

$$\Omega_{K}. = \frac{\gamma\lambda K_s^2}{\rho C_p}$$

$$\Omega_{jI}^2 = C`^2 - 4\pi G\rho$$

$C` = \left(\frac{p}{\rho}\right)^{1/2}$ Is the isothermal velocity of sound

Equation (29) represents the dispersion relation showing the effect of viscosity and thermal conductivity on the gravitational instability of an infinite homogeneous gas particle medium in the presence of fine-dust-particles. This is fourth degree equation and the condition of instability obtained from Equation(29) from constant term is $K_s < K_{jI}$,where K_{jI} is the modified critical Jeans wave number, given as

$$K_{jI} = \sqrt{\frac{4\pi G\rho}{C`^2}} \qquad (30)$$

And corresponding critical Jeans wave length is

$$\lambda_{jI} = C`\sqrt{\frac{\pi}{G\rho}} \qquad (31)$$

On comparing Equations (27) and (30), it is obvious that due to thermal conduction, the sonic velocity is altered form adiabatic to isothermal one in Jeans expression. Also comparing (28) and (31) we have

$$\lambda_{jI} = \lambda_j \sqrt{\frac{1}{\gamma}} \tag{32}$$

Since $\gamma > 1, C > C`$ therefore owing to thermal conduction. Jeans length is reduced. Thus the thermal conduction destabilizes the system.

If we consider viscous, self-gravitating radiating and thermally non conducting gas particle medium in the presence of fine-dust-particles, then Equation (25) will reduce to

$$\tau\sigma^4 + \left[\xi_1 + \tau\beta_0\right]\sigma^3 + \left[\xi_1\beta_0 + \Omega_V + \tau\Omega_j^2\right]\sigma^2 + \left[\Omega_j^2 + \Omega_{I0}^2 + \beta_0\Omega_V K_s^2\right]\sigma + .\Omega_{I0}^2 = 0 \tag{33}$$

Where

$$\alpha_0 = (\gamma - 1)(\angle_T T - \rho\angle_\rho)$$

$$\beta_o = \frac{(\gamma - 1)\angle_T T\rho}{p} \qquad \text{and} \qquad \Omega_{I0}^2 = K^2\alpha_0 - 4\pi G\rho\beta_0$$

Evidently, if $\Omega_{I0}^2 < 0$ the above equation will possess at least one real positive root implying there-by instability of the system. Thus we find that the system is unstable for all the wave number K_s such $K_s < K_{j2}$

Where $$K_{j2} = K_j\left[\sqrt{\frac{\gamma\angle_T}{\angle_T - \frac{\rho}{T}\angle_T}}\right]^{1/2} \tag{34}$$

It is clear from Equation (34) that in this case the critical Jeans wave number depends on the derivatives of the heat-loss function with respect to local temperature and density in the configuration. The critical Jeans wave number vanishes if the heat-loss function is independent of temperature $(\angle_T = 0) and \sqrt{\gamma}$ times of original critical Jeans wave number if the heat-loss function is purely temperature-dependent $(\angle_T = 0)$ it may be remarked that the critical wave number decreases or increases as the heat-loss function respectively increases or decreases with increases in density. Owing to simultaneous effect of all the parameters represented by the original dispersion relation (25), the condition of instability obtained from(25) form constant term is $\Omega_I^2 < 0$, therefore the system represented by Equation(25) will remain unstable for all the wave number $K_s < K_{j3}$

Where

$$K_{j3}^2 = \frac{1}{2}\left[a_1 \pm \sqrt{\left(a_1^2 + b_1\right)}\right] \tag{35}$$

and

$$a_1 = \frac{4\pi G\rho}{C'^2} + \frac{\rho^2 \angle_\rho}{\lambda T} - \frac{\rho \angle_T}{\lambda}$$

$$b_1 = \frac{16\pi G\rho^2}{\lambda C'^2}\angle_T$$

It is observed, from Equation (35) that the critical wave number K_{j3} is very much different from the classical Jeans value K_j and depends upon the thermal conductivity and the derivatives of the heat-loss function with respect to local temperature and density in the configuration. It can be easily worked out that for a purely density-dependent heat-loss function $[\angle_t = 0]$,the critical wave number is increased or decreased depending on whether the heat-loss function is an increasing or decreasing function of the density. Furthermore, it can be seen that for a purely temperature-dependent heat-loss function $(\angle_\rho = 0)$ which increase with temperature $(\angle_T = 0)$ the condition (35) suggests a monotonic instability if

$$K_s < K_{j1}$$

However, if instead, the heat-loss function decreases with temperature $(\angle_T < 0)$ the modified Jeans critical wave number lies between the values of $\frac{|\angle_T|}{\lambda}$ and $\frac{4\pi G\rho}{C^{`2}}$

We now discuss the dynamical stability of the system by applying the Routh-Hurwitz criterion to Equation (25). If $\Omega_I^2 > 0$, then all the coefficients in Equation (25) are positive and satisfying the necessary condition for the stability. To achieve the sufficient condition, the Routh-Hurwitz criterion must be satisfied, according to which all the principal diagonal minors of the Hurwitz matrix must be positive for a stable system.

For Equation (25) the four principal diagonal minors of Hurwitz matrix are all positive as shown here under

$$\Delta_1 = \tau(\xi_1 + \tau\beta) > 0$$

$$\Delta_2 = \left[\xi_1(\xi_1\beta + \Omega_V) + \tau\left\{\tau\Omega_j^2\left(\frac{K_s N}{\rho} + \Omega_V\right) + \xi_1\beta^2\right\} + \tau^2 K_s^2(\gamma-1)\angle_\rho \rho\right] > 0,$$

$$\Delta_3 = \begin{bmatrix} K_s^2(\gamma-1)\angle_\rho.\rho\left\{\xi_1^2 + \tau^3\Omega_1^2 + \tau^2\beta\Omega_V + \tau\xi_1\beta + \tau^2\Omega_j^2\right\} \\ + \left(\frac{K_s N\tau}{\rho} + \Omega_V\tau\right).\tau.\left\{\beta\Omega_V\Omega_j^2 + \tau\Omega_j^2\Omega_I^2\right\} \\ + \xi_I\Omega_V\left\{\xi_I\beta^2 + \Omega_j^2\right\} + \xi_1.\beta\Omega_V\left\{\Omega_V\tau\beta^2\right\} + \xi_1.\Omega_V\tau\Omega_I^2 \\ + \tau^2\beta^2\Omega_j^2(\Omega_V) > 0 \end{bmatrix}$$

Since $\gamma > 0$

$$\Delta_4 = \Omega_I^2.\Delta_3 > 0$$

Therefore the system represented by (25) will remain stable if

$\Omega_I^2. = \left(K_s^2\alpha - 4\pi G\rho\beta\right) > 0$ Thus we find that for longitudinal wave propagation the gravitating plasma is stable if the condition $K_s^2\alpha > 4\pi G\rho\beta$ is satisfied.

Case 4.2: Transverse propagation:-

We assume the perturbations in perpendicular direction to magnetic field [$i.e. K_x = K_s$ and $K_z = 0$] the dispersion relation (20) is reduced to

$$\sigma^3.\xi_3^2.\left\{\sigma\xi_3^2 + \sigma\xi_2\xi_3\Omega_T^2 + \xi_2\xi_3K_s^2V^2\right\} = 0 \tag{36}$$

Re-substituting the values of ξ_2, ξ_3 and Ω_T^2 we get twelfth degree equation in term of σ

This equation splits into three factors representing different modes due to various effects as discussed below.

The first factor of Equation (36) gives us

$$\sigma = 0 \tag{37}$$

This is same as discussed in Equation (22) and representing stable mode.

The second factor of Equation (36) gives us a quadratic equation as

$$\sigma = 0$$

This is same as discussed in Equation (22) and representing again stable mode.

The second factor of Equation (36) gives us a quadratic equation as

$$\tau\sigma^2 + \left(1 + \Omega_V \tau + \frac{K_s N}{\rho}\right)\sigma + \Omega_V = 0 \tag{38}$$

This is same as discussed in Equation (24) and representing again stable mode.

The third factor of Equation (36) gives us

$$\begin{aligned}
&\tau^2\sigma^6 + \left[\tau^2\beta + 2\tau\xi_1\right]\sigma^5 + \left[2\tau\xi_1\beta + \xi_1^2 + \tau\left(\tau\Omega_j^2 + \tau V^2 K_s^2 + 2\Omega_V\right)\right]\sigma^4 \\
&+ \left[\xi_1\left(2\Omega_V + \tau\Omega_j^2 + \tau V^2 K_s^2\right) + \tau\left(\Omega_j^2 + V^2 K_s^2\right) + \xi^2\left(\Omega_I^2 + V^2 K_s^2 \beta\right) + \beta\left(\xi_1^2 + 2\Omega_V\right)\right]\sigma^3 \\
&+ \begin{bmatrix} \Omega_V\left(\tau\Omega_j^2 + \Omega_V\right) + \Omega_V K_s^2 V^2 \tau + \xi_1\left(V^2 K_s^2 + \Omega_j^2\right) \\ + \beta\left(2\Omega_V \xi_1 + \tau\xi_I V^2 K_s^2 + \tau V^2 K_s^2\right) + \Omega_I^2\left(\tau + \tau\xi_I\right) \end{bmatrix}\sigma^2 \\
&+ \begin{bmatrix} \Omega_V\left(\Omega_j^2 + K_s^2 V^2\right) + \beta\left\{\begin{pmatrix} \Omega_V + \\ K_s^2 V^2 \tau + \xi_1 V^2 \\ K_s^2 \Omega_V \end{pmatrix}^2\right\} \\ + \Omega_I^2\left(\Omega_v \tau + \xi_1\right) \end{bmatrix}\sigma + \\
&\Omega_V\left[\Omega_I^2 + \beta K_s^2 V^2\right] = 0
\end{aligned} \tag{39}$$

This dispersion relation represents the effect of simultaneous inclusion of the viscosity, thermal conductivity and heat-loss function on the magneto gravitational instability of a infinite homogeneous self-gravitating gas particle medium in the presence of the fine-dust-particles, when disturbances are propagating perpendicular to the direction of the magnetic field. The condition for the constant term of Equation (39) is

$$\Omega_I^2 + \beta K_s^2 V^2 < 0 \tag{40}$$

Therefore the system represented by Equation (39) will remain unstable for all the wave number

$K_s = K_{j4}$

Where

$$K_{j4}^2 = \frac{1}{2}\left[a_2 \pm \sqrt{a_2^2 + b_2}\right]$$

$$a_2 = \left\{\frac{4\pi G\rho^2}{C^{`2}} \frac{\rho^2 \angle_\rho}{\lambda T}\right\}\left\{1 + \frac{V^2}{C^{`2}}\right\}^{-1} - \frac{\angle_T \rho}{\lambda}$$

$$b_2 = \frac{16\pi G\rho^2}{C^{`2}} \frac{\angle_T}{\lambda}\left\{1 + \frac{V^2}{C^{`2}}\right\}^{-1} \tag{41}$$

It may be remarked here that the critical wave number depends upon the strength of magnetic field, thermal conductivity and derivative of general heat-loss function as temperature dependent and density-dependent configuration. It is obvious that the magnetized field decreases the Jeans wave number. Thus the magnetized field stabilizes the medium for traverse propagation.

When $\left(\Omega_I^2 + \beta K_s^2 V^2\right) > 0$, Equation (39) has all the coefficient positive. Hence according to Hurwitz's necessary condition the system is stable; but this is not a sufficient condition for stability. The sufficient condition for above stability is that all principal diagonal minor of the Hurwitz matrix must be positive. The principal diagonal minors are as

$$\Delta_I = [\beta + \Omega_V] > 0$$
$$\Delta_2 = \Omega_V\left[\Omega_V \beta + \Omega_j^2 + \beta^2\right] + K_s^2 \angle_\rho \rho(\gamma - 1)$$
$$\Delta_3 = \Omega_I^2 \Delta_2$$

All the Δ's are positive, hence the system represented by Equation (34) is a stable system if $\Omega_1^2 > 0 and \Omega_j^2 > 0$

5. Conclusion:-

In the present paper, we have investigated the problem of a self-gravitational instability of infinite homogeneous thermally conducting viscous gaseous plasma in the presence of fine-dust-particles under the effective of arbitrary radiative heat-loss functions. The general dispersion relation is obtained using normal mode analysis, which is modified due to the presence of these parameters. We find that the Jeans criterion of instability remains valid but the expression of the critical Jeans wave number is modified. Owing to the inclusion of thermal conductivity the isothermal sound velocity is replaced by the adiabatic velocity of the sound. The effect of the viscosity and permeability parameters are found to stabilize the system. It is also found that the effect of fine-dust-particles is to destabilize the system. The thermal conductivity has a destabilize influence. The density-dependent heat-loss function also has destabilizing influence of the Jeans instability and the system becomes more and more unstable for higher values of the density-dependent heat-loss function.

In the transverse mode of propagation, it is observed that for an infinitely conducting medium the condition for radiative instability is modified due to presence of magnetic field and it is independent of viscosity, permeability and fine-dust-particles of the medium.

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