TO FIND A NON-SPLIT RESTRAINED DOMINATING SETS OF INTERVAL GRAPH AND COMPARING RAINBOW CONNECTION NUMBER, DIAMETER, CARDINALITY OF RDS AND RADIUS OF THE INTERVAL GRAPHS

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ABSTRACT:

Among the various applications of the theory of restrained domination, the most often discussed is communication network. There has been persistent in the Algorithmic aspects of interval graphs in past decades spurred much by their numerous applications of an interval graphs corresponding to an interval family I. A set $D \subseteq V(G)$ is a Restrained dominating set of a graph G, if every vertex not in D is adjacent to a vertex in D and to a vertex in $V-D$. In graph theory, a connected component of an undirected graph is a subgraph in which any two vertices are connected to each other by paths. For a graph G, if the induced subgraph of G itself is a connected component then the graph G is called connected. A Restrained dominating set RDS of a graph $G(V,E)$ is a Non-split restrained dominating set, if the induced subgraph $< V-RDS >$ is connected. In this paper we find a non-split restrained dominating set of an interval graph and compare the rainbow connection number, diameter, cardinality of RDS and radius of the interval graphs.

KEY WORDS: Interval family, interval graph, connected graph, restrained dominating set, Non-split restrained dominating set, rainbow connection number, diameter, cardinality of RDS and radius.

1. INTRODUCTION

THE research of the domination in graphs has been an evergreen of the graph theory. Its basic concept is the dominating set and the domination number. The theory of domination in graphs was introduced by Ore [1] and Berge [2]. A survey on results and applications of dominating sets was presented by E.J.Cockayane and S.T.Hedetniemi [3]. In 1997 Kulli et.al introduced the concept of Non-split domination [4] and studied these parameters for various standard graphs and obtained the bounds for these parameters.

In general an undirected graph $G=(V,E)$ is an interval graph (IG), if the vertex set V can be put into one-to-one correspondence with a set of intervals I on the real line R, such that two vertices are adjacent in G, if and only if their corresponding intervals have non-empty intersection. The set I is called an interval representation of G and G is referred to as the intersection graph I. Let $I=\{I_1, I_2, I_3, I_4, ...I_n\}$ be any interval family where, each I_i is an interval on the real line and $I_i=[a_i, b_i]$ for $i=1,2,3,4,..........n$. Here a_i is called

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the left end point labeling and b_i is the right end point labeling of I_i. Without loss of generality we assume that all end points of the intervals in I are distinct numbers between 1 and $2n$. Two intervals i and j are said to be intersect each other if they have non empty intersection. Also we say that the intervals contain both its end points and that no two intervals share a common end point. The intervals and vertices of an interval graph are one and the same thing. The graph G is connected, and the list of sorted end point is given and the intervals in I are indexed by increasing right end points, that is $b_1 < b_2 < b_3 < .... < b_n$.

Let $G = (V, E)$ be a graph. A set $D \subseteq V(G)$ is a dominating set of G if every vertex in V / D is adjacent to some vertex in D. A set $S \subseteq V$ is a restrained dominating set (RDS) if every vertex not in S is adjacent to a vertex in S and to a vertex in $V - S$. Every graph has a RDS, since $S = V$ is such a set. The restrained domination number of G, denoted by $\gamma_r(G)$, is the minimum cardinality of a RDS of G. A RDS S is called a $\gamma_r(G)$-set of G if $|S| = \gamma_r(G)$.

The concept of restrained domination was introduced by Telle and Proskurowski [5], albeit indirectly, as a vertex partitioning problem. One application of domination is that of prisoners and guards. For security, each prisoner must be seen by some guard; the concept is that of domination. However, in order to protect the rights of prisoners, we may also require that each prisoner is seen by another prisoner; the concept is that of restrained domination.

A restrained dominating set RDS of G is connected restrained dominating set, if the induced subgraph $< V - RDS >$ is connected. i.e., A restrained dominating set RDS of a graph $G(V, E)$ is a non-split restrained dominating set, if the induced subgraph $< V - RDS >$ is connected.

2. MAIN THEOREMS

2.1 Theorem:

Let $I = \{i_1, i_2, ..., i_n\}$ be an n interval family and G is an interval graph corresponding to I. If i and j are any two intervals in I such that $i \in RDS$, where RDS is a restrained dominating Set, $j \neq 1$ and j is contained in i, if there is at least one interval to the left of j that intersect j and there is at least one interval $k \neq i$ to the right of j that intersect j. Then the restrained domination occurs in G and the non-split restrained dominating set $< V - RDS >$ is connected as $|RDS| = 3$. Also we can prove rc(G)>diam(G)>$|RDS|$ $\geq$rad(G).

Proof: Let $I = \{i_1, i_2, ..., i_n\}$ be the given n interval family an G is an interval graph corresponding to I. First we will find the restrained dominating set corresponding to G. Suppose there is at least one interval $k \neq i$ to the right of j that intersect j. Then it is obvious that j is adjacent to k in $< V - RDS >$, so that there will not be any disconnection in $< V - RDS >$. Since, there is at least one interval to the left of j that intersect j, there will not be any disconnection in $< V - RDS >$, to its left. Thus we get non-split restrained domination in G.

Now we will find the restrained dominating set of an interval graph with an illustration as follows,

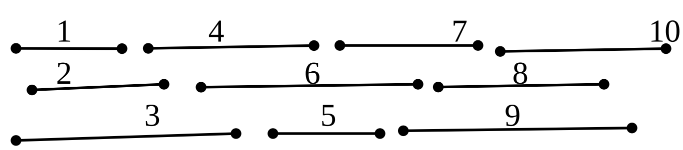

Fig.1: Interval family I

nbd [1] = {1,2,3}, nbd [2] = {1,2,3,4},
nbd[3]={1,2,3,4,6},nbd[4]={2,3,4,5,6},
nbd [5] = {4,5,6,7}, nbd [6] = {3,4,5,6,7,9},

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nbd[7] = {5,6,7,8,9}, nbd [8] = {7,8,9,10},
nbd [9] = {6,7,8,9,10}, nbd [10] = {8,9,10}.

The corresponding Interval graph is as follows,

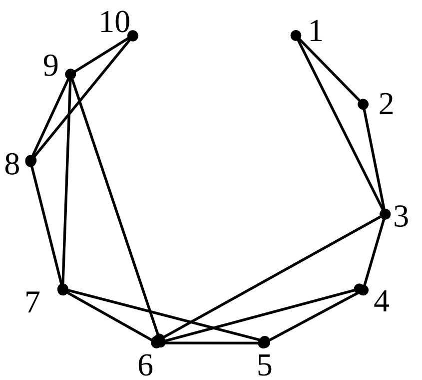

Fig.2: Interval Graph G

The restrained dominating set in the above interval graph is clearly{3, 7, 10}

$$\therefore |RDS| = 3$$

Thus we get the Non-spilt Restrained dominating set $< V - RDS >$ as follows,

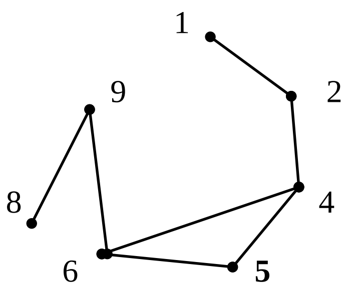

Fig.3: Vertex induced subgraph

$< V - RDS >$- Connected graph from G

Now we will prove the inequality rc(G)>diam(G)>$|RDS|$ ≥rad(G).

Let $I = \{i_1, i_2, ..., i_n\}$ be an n interval family and G is an interval graph corresponding to I.

If i and j are any two intervals in I such that i∈RDS, where RDS is a restrained dominating set, j≠1 and j is contained in i, if there is atleast one interval to the left of j that intersects j and there is atleast one interval k≠i to the right of j that intersects j. Then the non-split restrained domination occurs in G and

rc(G)>diam(G) $> |RDS|$ ≥rad(G).

If there is no interval k≠i to the right of j that intersects j, then we get a contradiction as RDS gives the split restrained domination in G and there is no change in

rc(G)>diam(G) $> |RDS|$ ≥rad(G),

it is shown in the following way:

If there is no interval to the left of j that intersects j, then also we get a contradiction as the cardinality of RDS increases. So we must have there is atleast one interval to the left of j that intersects j and there is atleast one interval k≠i to the right of j that intersects j and j is contained i∈RDS, it is shown in the following illustration clearly,

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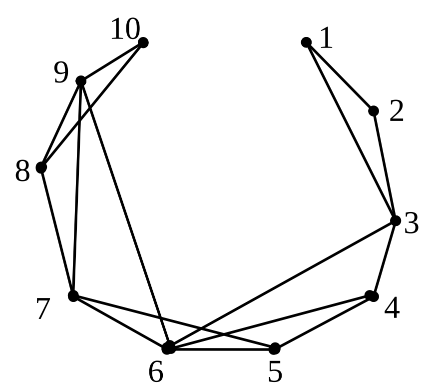

Fig.4: Interval graph G

For this we will find the distances of the path lengths from Interval graph G as follows,

$d(v_1,v_1)=0$	$d(v_2,v_1)=1$	$d(v_3,v_1)=1$	$d(v_4,v_1)=2$	$d(v_5,v_1)=3$
$d(v_1,v_2)=1$	$d(v_2,v_2)=0$	$d(v_3,v_2)=1$	$d(v_4,v_2)=2$	$d(v_5,v_2)=3$
$d(v_1,v_3)=1$	$d(v_2,v_3)=1$	$d(v_3,v_3)=0$	$d(v_4,v_3)=1$	$d(v_5,v_3)=2$
$d(v_1,v_4)=2$	$d(v_2,v_4)=2$	$d(v_3,v_4)=1$	$d(v_4,v_4)=0$	$d(v_5,v_4)=1$
$d(v_1,v_5)=3$	$d(v_2,v_5)=3$	$d(v_3,v_5)=2$	$d(v_4,v_5)=1$	$d(v_5,v_5)=0$
$d(v_1,v_6)=2$	$d(v_2,v_6)=2$	$d(v_3,v_6)=1$	$d(v_4,v_6)=1$	$d(v_5,v_6)=1$
$d(v_1,v_7)=3$	$d(v_2,v_7)=3$	$d(v_3,v_7)=2$	$d(v_4,v_7)=2$	$d(v_5,v_7)=1$
$d(v_1,v_8)=4$	$d(v_2,v_8)=4$	$d(v_3,v_8)=3$	$d(v_4,v_8)=3$	$d(v_5,v_8)=2$
$d(v_1,v_9)=3$	$d(v_2,v_9)=3$	$d(v_3,v_9)=2$	$d(v_4,v_9)=2$	$d(v_5,v_9)=2$
$d(v_1,v_{10})=4$	$d(v_2,v_{10})=4$	$d(v_3,v_{10})=3$	$d(v_4,v_{10})=3$	$d(v_5,v_{10})=3$

$d(v_6,v_1)=2$	$d(v_7,v_1)=3$	$d(v_8,v_1)=4$	$d(v_9,v_1)=3$	$d(v_{10},v_1)=4$
$d(v_6,v_2)=2$	$d(v_7,v_2)=3$	$d(v_8,v_2)=4$	$d(v_9,v_2)=3$	$d(v_{10},v_2)=4$
$d(v_6,v_3)=1$	$d(v_7,v_3)=2$	$d(v_8,v_3)=3$	$d(v_9,v_3)=2$	$d(v_{10},v_3)=3$
$d(v_6,v_4)=1$	$d(v_7,v_4)=2$	$d(v_8,v_4)=3$	$d(v_9,v_4)=2$	$d(v_{10},v_4)=3$
$d(v_6,v_5)=1$	$d(v_7,v_5)=1$	$d(v_8,v_5)=2$	$d(v_9,v_5)=2$	$d(v_{10},v_5)=3$
$d(v_6,v_6)=0$	$d(v_7,v_6)=1$	$d(v_8,v_6)=2$	$d(v_9,v_6)=1$	$d(v_{10},v_6)=2$
$d(v_6,v_7)=1$	$d(v_7,v_7)=0$	$d(v_8,v_7)=1$	$d(v_9,v_7)=1$	$d(v_{10},v_7)=2$
$d(v_6,v_8)=2$	$d(v_7,v_8)=1$	$d(v_8,v_8)=0$	$d(v_9,v_8)=1$	$d(v_{10},v_8)=1$
$d(v_6,v_9)=1$	$d(v_7,v_9)=1$	$d(v_8,v_9)=1$	$d(v_9,v_9)=0$	$d(v_{10},v_9)=1$
$d(v_6,v_{10})=2$	$d(v_7,v_{10})=2$	$d(v_8,v_{10})=1$	$d(v_9,v_{10})=1$	$d(v_{10},v_{10})=0$

The eccentricity of the vertices is as follows, $ecc(v) = \max_{x \in V(G)} d(v,x)$

$ecc(v_1) = \max \{d(v_1,v_1), d(v_1,v_2), d(v_1,v_3), d(v_1,v_4), d(v_1,v_5), d(v_1,v_6), d(v_1,v_7), d(v_1,v_8), d(v_1,v_9), d(v_1,v_{10})\}$

$= \max \{0, 1, 1, 2, 3, 2, 3, 4, 3, 4\} = 4$

$ecc(v_2) = \max \{d(v_2,v_1), d(v_2,v_2), d(v_2,v_3), d(v_2,v_4), d(v_2,v_5), d(v_2,v_6), d(v_2,v_7), d(v_2,v_8), d(v_2,v_9), d(v_2,v_{10})\}$

$= \max \{1, 0, 1, 2, 3, 2, 3, 4, 3, 4\} = 4$

$ecc(v_3) = \max \{d(v_3,v_1), d(v_3,v_2), d(v_3,v_3), d(v_3,v_4), d(v_3,v_5), d(v_3,v_6), d(v_3,v_7), d(v_3,v_8), d(v_3,v_9), d(v_3,v_{10})\}$

$= \max \{1, 1, 0, 1, 2, 1, 2, 3, 2, 3\} = 3$

$ecc(v_4) = \max \{d(v_4,v_1), d(v_4,v_2), d(v_4,v_3), d(v_4,v_4), d(v_4,v_5), d(v_4,v_6), d(v_4,v_7), d(v_4,v_8), d(v_4,v_9), d(v_4,v_{10})\}$

$= \max \{2, 2, 1, 0, 1, 1, 2, 3, 2, 3\} = 3$

$ecc(v_5) = \max \{d(v_5,v_1), d(v_5,v_2), d(v_5,v_3), d(v_5,v_4), d(v_5,v_5), d(v_5,v_6), d(v_5,v_7), d(v_5,v_8), d(v_5,v_9), d(v_5,v_{10})\}$

$= \max \{3, 3, 2, 1, 0, 1, 1, 2, 2, 3\} = 3$

$ecc(v_6) = \max \{d(v_6,v_1), d(v_6,v_2), d(v_6,v_3), d(v_6,v_4), d(v_6,v_5), d(v_6,v_6), d(v_6,v_7), d(v_6,v_8), d(v_6,v_9), d(v_6,v_{10})\}$

$= \max \{2, 2, 1, 1, 1, 0, 1, 2, 1, 2\} = 2$

$ecc(v_7) = \max \{d(v_7,v_1), d(v_7,v_2), d(v_7,v_3), d(v_7,v_4), d(v_7,v_5), d(v_7,v_6), d(v_7,v_7), d(v_7,v_8), d(v_7,v_9), d(v_7,v_{10})\}$

$= \max \{3, 3, 2, 2, 1, 1, 0, 1, 1, 2\} = 3$

$ecc(v_8) = \max \{d(v_8,v_1), d(v_8,v_2), d(v_8,v_3), d(v_8,v_4), d(v_8,v_5), d(v_8,v_6), d(v_8,v_7), d(v_8,v_8), d(v_8,v_9), d(v_8,v_{10})\}$

$= \max \{4, 4, 3, 3, 2, 2, 1, 0, 1, 1\} = 4$

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$ecc(v_9) = \max \{d(v_9,v_1), d(v_9,v_2), d(v_9,v_3), d(v_9,v_4), d(v_9,v_5), d(v_9,v_6), d(v_9,v_7), d(v_9,v_8), d(v_9,v_9), d(v_9,v_{10})\}$

$= \max \{3, 3, 2, 2, 2, 1, 1, 1, 0, 1\} = 3$

$ecc(v_{10}) = \max\{d(v_{10},v_1), d(v_{10},v_2), d(v_{10},v_3), d(v_{10},v_4), d(v_{10},v_5), d(v_{10},v_6), d(v_{10},v_7), d(v_{10},v_8), d(v_{10},v_9), d(v_{10},v_{10})\}$

$= \max \{4, 4, 3, 3, 3, 2, 2, 1, 1, 0\} = 4$

The diameter of a graph G is the maximum of eccentricity of all its vertices and is denoted by $diam(G)$

That is $diam(G) = \max\{e(v) : v \in V(G)\}$

diam(G) = max{4, 4, 3, 3, 3, 2, 3, 4, 3, 4} = 4

The radius of a graph G is the minimum of eccentricity of all its vertices and is denoted by rad(G).

That is rad(G) = min{ecc(v):v∈V(G)}

rad(G)=min{4, 4, 3, 3, 3, 2, 3, 4, 3, 4} =2

The rainbow connection number rc (G) can be determined as follows,

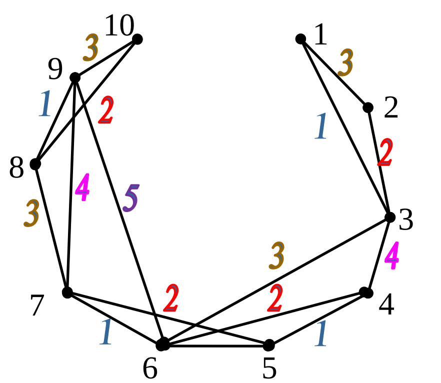


Fig.5: Interval graph with the edge colors

In the above interval graph with the edge colors clearly we can see that the rainbow connection number rc(G)=5.

Hence we can conclude that rc(G)>diam(G)> $|RDS| \geq$ rad(G).

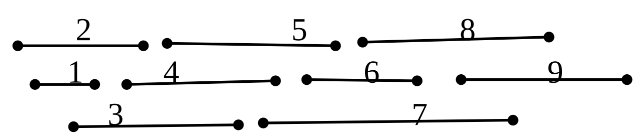
2.3 THEOREM:

Let G be an Interval graph corresponding to an n interval family $I = \{i_1, i_2, \ldots\ldots, i_n\}$. If i and j are any two intervals in I such that $i \in RDS$, $j = 1$, j intersects i and if there is one more interval that intersects j or contains j. Then restrained domination occurs in G and the non-split restrained dominating set $< V - RDS >$ is connected as $|RDS| = 2$. Also we can prove rc(G)>diam(G)> $|RDS| \geq$ rad(G).

Proof:

Let $I = \{i_1, i_2, \ldots\ldots, i_n\}$ be the given n interval family and G is an interval graph corresponding to I. First we will find the restrained dominating set corresponding to G. Now let $j = 1$ be an interval contained in an interval $k \neq i$ or intersects k which is not in the restrained dominating set. Suppose j intersects i, since $i \in RDS, < V - RDS >$ does not contain i. Further in $< V - RDS >$, the vertex j is adjacent to the vertex k, since j is contained in k or j intersects k and hence there will not be any disconnection in $< V - RDS >$. Therefore we get non-split dominating in G.

Next we will find the restrained dominating set as follows from an interval family,



Interval Family Fig.6: I

nbd [1] = {1,2,3}, nbd [2] = {1,2,3,4}, nbd [3] = {1,2,3,4,5}, nbd [4] = {2,3,4,5,7}, nbd [5] = {3,4,5,6,7}, nbd [6] = {5,6,7,8}, nbd [7] = {4,5,6,7,8,9},nbd [8] = {6,7,8,9}, nbd [9] = {7,8,9}

The corresponding interval graph is as follows,

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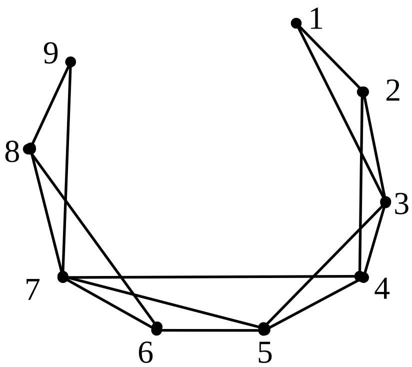

Fig.7: Interval graph G

The restrained dominating set in the above interval graph is clearly {3, 8}

$\therefore$ $|RDS|$ = The cardinality of RDS = 2.

Thus we get the Non-spilt restrained dominating set $< V - RDS >$ as follows,

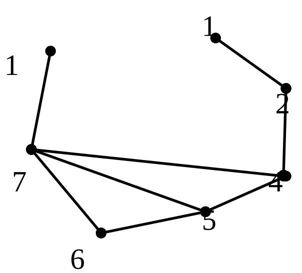

Fig.8: Vertex induced subgraph $< V - RDS >$- Connected graph from G

Now we will prove the inequality rc(G)>diam(G)>$|RDS|$ $\geq$rad(G).

If the first interval I in RDS does not intersects i+2, then we get a contradiction as rc(G) must be equal to diam(G) and the cardinality of RDS increases, which leads to $|RDS| \neq 2$. Hence the first interval I in RDS must intersect the interval i+2, then rc(G)>diam(G)>$|RDS|$ $\geq$rad(G). This can be shown in the following illustration,

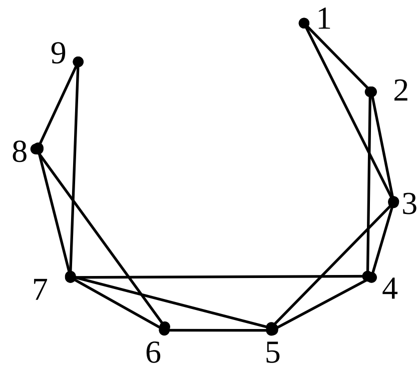

Fig.9: Interval graph G

For this we will find the distances of the path lengths from Interval graph G as follows,

$d(v_1,v_1)$=0	$d(v_2,v_1)$= 1	$d(v_3,v_1)$= 1	$d(v_4,v_1)$= 2	$d(v_5,v_1)$= 2
$d(v_1,v_2)$=1	$d(v_2,v_2)$= 0	$d(v_3,v_2)$= 1	$d(v_4,v_2)$= 1	$d(v_5,v_2)$= 2
$d(v_1,v_3)$=1	$d(v_2,v_3)$= 1	$d(v_3,v_3)$= 0	$d(v_4,v_3)$= 1	$d(v_5,v_3)$= 1
$d(v_1,v_4)$= 2	$d(v_2,v_4)$= 1	$d(v_3,v_4)$= 1	$d(v_4,v_4)$= 0	$d(v_5,v_4)$= 1
$d(v_1,v_5)$= 2	$d(v_2,v_5)$= 2	$d(v_3,v_5)$= 1	$d(v_4,v_5)$= 1	$d(v_5,v_5)$= 0
$d(v_1,v_6)$= 3	$d(v_2,v_6)$= 3	$d(v_3,v_6)$= 2	$d(v_4,v_6)$= 2	$d(v_5,v_6)$= 1
$d(v_1,v_7)$= 3	$d(v_2,v_7)$= 2	$d(v_3,v_7)$= 2	$d(v_4,v_7)$= 1	$d(v_5,v_7)$= 1
$d(v_1,v_8)$= 4	$d(v_2,v_8)$= 3	$d(v_3,v_8)$= 3	$d(v_4,v_8)$= 2	$d(v_5,v_8)$= 2
$d(v_1,v_9)$= 4	$d(v_2,v_9)$= 3	$d(v_3,v_9)$= 3	$d(v_4,v_9)$= 2	$d(v_5,v_9)$= 2

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$d(v_6,v_1)=3$	$d(v_7,v_1)=3$	$d(v_8,v_1)=4$	$d(v_9,v_1)=4$
$d(v_6,v_2)=3$	$d(v_7,v_2)=2$	$d(v_8,v_2)=3$	$d(v_9,v_2)=3$
$d(v_6,v_3)=2$	$d(v_7,v_3)=2$	$d(v_8,v_3)=3$	$d(v_9,v_3)=3$
$d(v_6,v_4)=2$	$d(v_7,v_4)=1$	$d(v_8,v_4)=2$	$d(v_9,v_4)=2$
$d(v_6,v_5)=1$	$d(v_7,v_5)=1$	$d(v_8,v_5)=2$	$d(v_9,v_5)=2$
$d(v_6,v_6)=0$	$d(v_7,v_6)=1$	$d(v_8,v_6)=1$	$d(v_9,v_6)=2$
$d(v_6,v_7)=1$	$d(v_7,v_7)=0$	$d(v_8,v_7)=1$	$d(v_9,v_7)=1$
$d(v_6,v_8)=1$	$d(v_7,v_8)=1$	$d(v_8,v_8)=0$	$d(v_9,v_8)=1$
$d(v_6,v_9)=2$	$d(v_7,v_9)=1$	$d(v_8,v_9)=1$	$d(v_9,v_9)=0$

The eccentricity of the vertices is as follows, $ecc(v) = \max_{x \in V(G)} d(v,x)$

$ecc(v_1) = \max\{d(v_1,v_1), d(v_1,v_2), d(v_1,v_3), d(v_1,v_4), d(v_1,v_5), d(v_1,v_6), d(v_1,v_7), d(v_1,v_8), d(v_1,v_9)\}$

$= \max\{0, 1, 1, 2, 2, 3, 3, 4, 4\} = 4$

$ecc(v_2) = \max\{d(v_2,v_1), d(v_2,v_2), d(v_2,v_3), d(v_2,v_4), d(v_2,v_5), d(v_2,v_6), d(v_2,v_7), d(v_2,v_8), d(v_2,v_9)\}$

$= \max\{1, 0, 1, 1, 2, 3, 2, 3, 3\} = 3$

$ecc(v_3) = \max\{d(v_3,v_1), d(v_3,v_2), d(v_3,v_3), d(v_3,v_4), d(v_3,v_5), d(v_3,v_6), d(v_3,v_7), d(v_3,v_8), d(v_3,v_9)\}$

$= \max\{1, 1, 0, 1, 1, 2, 2, 3, 3\} = 3$

$ecc(v_4) = \max\{d(v_4,v_1), d(v_4,v_2), d(v_4,v_3), d(v_4,v_4), d(v_4,v_5), d(v_4,v_6), d(v_4,v_7), d(v_4,v_8), d(v_4,v_9)\}$

$= \max\{2, 1, 1, 0, 1, 2, 1, 2, 2\} = 2$

$ecc(v_5) = \max\{d(v_5,v_1), d(v_5,v_2), d(v_5,v_3), d(v_5,v_4), d(v_5,v_5), d(v_5,v_6), d(v_5,v_7), d(v_5,v_8), d(v_5,v_9)\}$

$= \max\{2, 2, 1, 1, 0, 1, 1, 2, 2\} = 2$

$ecc(v_6) = \max\{d(v_6,v_1), d(v_6,v_2), d(v_6,v_3), d(v_6,v_4), d(v_6,v_5), d(v_6,v_6), d(v_6,v_7), d(v_6,v_8), d(v_6,v_9)\}$

$= \max\{3, 3, 2, 2, 1, 0, 1, 1, 2\} = 3$

$ecc(v_7) = \max\{d(v_7,v_1), d(v_7,v_2), d(v_7,v_3), d(v_7,v_4), d(v_7,v_5), d(v_7,v_6), d(v_7,v_7), d(v_7,v_8), d(v_7,v_9)\}$

$= \max\{3, 2, 2, 1, 1, 1, 0, 1, 1\} = 3$

$ecc(v_8) = \max\{d(v_8,v_1), d(v_8,v_2), d(v_8,v_3), d(v_8,v_4), d(v_8,v_5), d(v_8,v_6), d(v_8,v_7), d(v_8,v_8), d(v_8,v_9)\}$

$= \max\{4, 3, 3, 2, 2, 1, 1, 0, 1\} = 4$

$ecc(v_9) = \max\{d(v_9,v_1), d(v_9,v_2), d(v_9,v_3), d(v_9,v_4), d(v_9,v_5), d(v_9,v_6), d(v_9,v_7), d(v_9,v_8), d(v_9,v_9)\}$

$= \max\{4, 3, 3, 2, 2, 2, 1, 1, 0\} = 4$

The diameter of a graph G is the maximum of eccentricity of all its vertices and is denoted by $diam(G)$

That is $diam(G) = \max\{e(v) : v \in V(G)\}$

diam(G) = max{4, 3, 3, 2, 2, 3, 3, 4,4} = 4

The radius of a graph G is the minimum of eccentricity of all its vertices and is denoted by rad(G).

That is $rad(G) = \min\{ecc(v) : v \in V(G)\}$

rad(G)=min{4, 3, 3, 2, 2, 3, 3, 4,4} =2

The rainbow connection number rc (G) can be determined as follows,

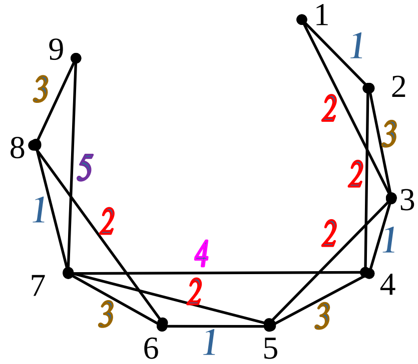


Fig.10: Interval graph with the edge colors

In the above interval graph with the edge colors we can seen that the rainbow connection number rc(G)=5.

Hence we can say that, rc(G)>diam(G)>$|RDS| \geq$ rad(G).

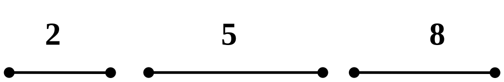
2.4 THEOREM:

Let us consider an n interval family $I = \{i_1, i_2, ..., i_n\}$ and G be an interval graph of I. If i, j, k are three consecutive intervals such that $i < j < k$ and $j \in RDS$, , i intersects j, j intersect k and i intersect k. Also the first interval i_1 in the interval family must intersects i_1+3. Then restrained domination occurs in G and the non-split restrained dominating set $< V - RDS >$ is connected as $|RDS| = 2$. Also we can prove rc(G)>diam(G)>$|RDS| \geq$rad(G).

Proof:

Let $I = \{i_1, i_2, ..., i_n\}$ be an n interval family and G be an interval graph of I. Let i, j, k be three consecutive intervals satisfy the hypothesis. Now i and k intersect implies that i and k are adjacent in $< V - RDS >$. So that there will not be any disconnection in $< V - RDS >$.

Now we will find Restrained dominating set using the following interval family,



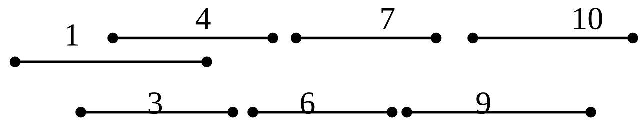


Fig.11: Interval Family I

nbd [1] = {1,2,3,4}, nbd [2] = {1,2,3,4},
nbd [3] = {1,2,3,4,5}, nbd [4] = {1,2,3,4,5,6},
nbd [5] = {3,4,5,6,7}, nbd [6] = {4,5,6,7,8},
nbd [7] = {5,6,7,8,9}, nbd [8] = {6,7,8,9,10},
nbd [9] = {7,8,9,10}, nbd [10] = {8,9,10}.

The corresponding interval graph is as follows,

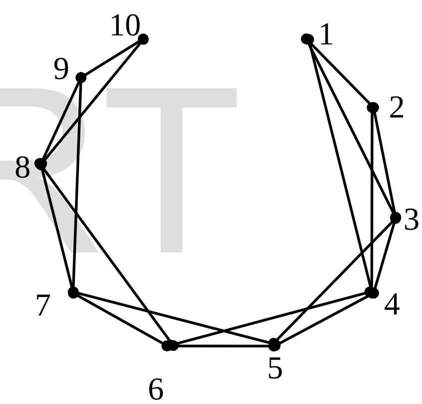
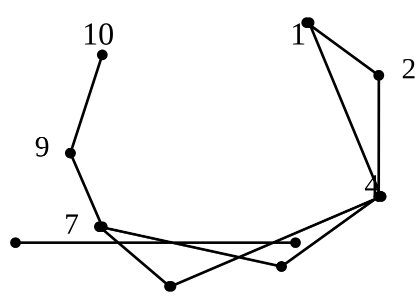


Fig.12: Interval graph G

The restrained dominating set in the above interval graph is clearly {3, 8}

$\therefore$ $|RDS|$ = The cardinality of RDS = 2.

Thus we get the Non-spilt restrained dominating set $< V - RDS >$ as follows,



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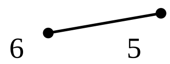

Fig.13: Vertex induced subgraph $< V - RDS >$- Connected graph from G

Now we will prove the inequality rc(G)>diam(G)> $|RDS| \geq$rad(G).

If the first interval i_1 in the interval family $I = \{i_1, i_2, ..., i_n\}$ does not intersects the interval i_1+3, then we get a contradiction as

rc(G) does not greater than diam(G) $> |RDS|$ does not greater than or equal to rad(G)

ie., rc(G) decreases and diam(G) increases and rad(G) increases from $|RDS|$. So i_1 must intersect the interval i_1+3. This can clearly seen in the following illustration.

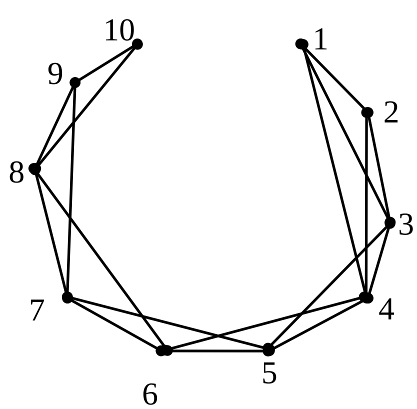

Fig.14: Interval graph G

For this we will find the distances of the path lengths from interval graph G as follows,

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$d(v_1,v_1)=0$	$d(v_2,v_1)=1$	$d(v_3,v_1)=1$	$d(v_4,v_1)=2$	$d(v_5,v_1)=2$
$d(v_1,v_2)=1$	$d(v_2,v_2)=0$	$d(v_3,v_2)=1$	$d(v_4,v_2)=1$	$d(v_5,v_2)=2$
$d(v_1,v_3)=1$	$d(v_2,v_3)=1$	$d(v_3,v_3)=0$	$d(v_4,v_3)=1$	$d(v_5,v_3)=1$
$d(v_1,v_4)=2$	$d(v_2,v_4)=1$	$d(v_3,v_4)=1$	$d(v_4,v_4)=0$	$d(v_5,v_4)=1$
$d(v_1,v_5)=2$	$d(v_2,v_5)=2$	$d(v_3,v_5)=1$	$d(v_4,v_5)=1$	$d(v_5,v_5)=0$
$d(v_1,v_6)=2$	$d(v_2,v_6)=2$	$d(v_3,v_6)=2$	$d(v_4,v_6)=1$	$d(v_5,v_6)=1$
$d(v_1,v_7)=3$	$d(v_2,v_7)=3$	$d(v_3,v_7)=2$	$d(v_4,v_7)=2$	$d(v_5,v_7)=1$
$d(v_1,v_8)=4$	$d(v_2,v_8)=3$	$d(v_3,v_8)=3$	$d(v_4,v_8)=2$	$d(v_5,v_8)=2$
$d(v_1,v_9)=4$	$d(v_2,v_9)=4$	$d(v_3,v_9)=3$	$d(v_4,v_9)=3$	$d(v_5,v_9)=2$
$d(v_1,v_{10})=4$	$d(v_2,v_{10})=4$	$d(v_3,v_{10})=4$	$d(v_4,v_{10})=3$	$d(v_5,v_{10})=3$

$d(v_6,v_1)=2$	$d(v_7,v_1)=3$	$d(v_8,v_1)=4$	$d(v_9,v_1)=4$	$d(v_{10},v_1)=4$
$d(v_6,v_2)=2$	$d(v_7,v_2)=3$	$d(v_8,v_2)=3$	$d(v_9,v_2)=4$	$d(v_{10},v_2)=4$
$d(v_6,v_3)=2$	$d(v_7,v_3)=2$	$d(v_8,v_3)=3$	$d(v_9,v_3)=3$	$d(v_{10},v_3)=4$
$d(v_6,v_4)=1$	$d(v_7,v_4)=2$	$d(v_8,v_4)=2$	$d(v_9,v_4)=3$	$d(v_{10},v_4)=3$
$d(v_6,v_5)=1$	$d(v_7,v_5)=1$	$d(v_8,v_5)=2$	$d(v_9,v_5)=2$	$d(v_{10},v_5)=3$
$d(v_6,v_6)=0$	$d(v_7,v_6)=1$	$d(v_8,v_6)=1$	$d(v_9,v_6)=2$	$d(v_{10},v_6)=2$
$d(v_6,v_7)=1$	$d(v_7,v_7)=0$	$d(v_8,v_7)=1$	$d(v_9,v_7)=1$	$d(v_{10},v_7)=2$
$d(v_6,v_8)=1$	$d(v_7,v_8)=1$	$d(v_8,v_8)=0$	$d(v_9,v_8)=1$	$d(v_{10},v_8)=1$
$d(v_6,v_9)=2$	$d(v_7,v_9)=1$	$d(v_8,v_9)=1$	$d(v_9,v_9)=0$	$d(v_{10},v_9)=1$
$d(v_6,v_{10})=2$	$d(v_7,v_{10})=2$	$d(v_8,v_{10})=1$	$d(v_9,v_{10})=1$	$d(v_{10},v_{10})=0$

The eccentricity of the vertices is as follows, $ecc(v) = \max_{x\in V(G)} d(v,x)$

$ecc(v_1) = \max \{d(v_1,v_1), d(v_1,v_2), d(v_1,v_3), d(v_1,v_4), d(v_1,v_5), d(v_1,v_6), d(v_1,v_7), d(v_1,v_8), d(v_1,v_9), d(v_1,v_{10})\}$

$= \max \{0, 1, 1, 2, 2, 2, 3, 4, 4, 4\} = 4$

$ecc(v_2) = \max \{d(v_2,v_1), d(v_2,v_2), d(v_2,v_3), d(v_2,v_4), d(v_2,v_5), d(v_2,v_6), d(v_2,v_7), d(v_2,v_8), d(v_2,v_9), d(v_2,v_{10})\}$

$= \max \{1, 0, 1, 1, 2, 2, 3, 3, 4, 4\} = 4$

$ecc(v_3) = \max \{d(v_3,v_1), d(v_3,v_2), d(v_3,v_3), d(v_3,v_4), d(v_3,v_5), d(v_3,v_6), d(v_3,v_7), d(v_3,v_8), d(v_3,v_9), d(v_3,v_{10})\}$

$= \max \{1, 1, 0, 1, 1, 2, 2, 3, 3, 4\} = 4$

$ecc(v_4) = \max \{d(v_4,v_1), d(v_4,v_2), d(v_4,v_3), d(v_4,v_4), d(v_4,v_5), d(v_4,v_6), d(v_4,v_7), d(v_4,v_8), d(v_4,v_9), d(v_4,v_{10})\}$

$= \max \{2, 1, 1, 0, 1, 1, 2, 2, 3, 3\} = 3$

$ecc(v_5) = \max \{d(v_5,v_1), d(v_5,v_2), d(v_5,v_3), d(v_5,v_4), d(v_5,v_5), d(v_5,v_6), d(v_5,v_7), d(v_5,v_8), d(v_5,v_9), d(v_5,v_{10})\}$

$= \max \{2, 2, 1, 1, 0, 1, 1, 2, 2, 3\} = 3$

$ecc(v_6) = \max \{d(v_6,v_1), d(v_6,v_2), d(v_6,v_3), d(v_6,v_4), d(v_6,v_5), d(v_6,v_6), d(v_6,v_7), d(v_6,v_8), d(v_6,v_9), d(v_6,v_{10})\}$

$= \max \{2, 2, 2, 1, 1, 0, 1, 1, 2, 2\} = 2$

$ecc(v_7) = \max \{d(v_7,v_1), d(v_7,v_2), d(v_7,v_3), d(v_7,v_4), d(v_7,v_5), d(v_7,v_6), d(v_7,v_7), d(v_7,v_8), d(v_7,v_9), d(v_7,v_{10})\}$

$= \max \{3, 3, 2, 2, 1, 1, 0, 1, 1, 2\} = 3$

$ecc(v_8) = \max \{d(v_8,v_1), d(v_8,v_2), d(v_8,v_3), d(v_8,v_4), d(v_8,v_5), d(v_8,v_6), d(v_8,v_7), d(v_8,v_8), d(v_8,v_9), d(v_8,v_{10})\}$

$= \max \{4, 3, 3, 2, 2, 1, 1, 0, 1, 1\} = 4$

$ecc(v_9) = \max \{d(v_9,v_1), d(v_9,v_2), d(v_9,v_3), d(v_9,v_4), d(v_9,v_5), d(v_9,v_6), d(v_9,v_7), d(v_9,v_8), d(v_9,v_9), d(v_9,v_{10})\}$

$= \max \{4, 4, 3, 3, 2, 2, 1, 1, 0, 1\} = 4$

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$ecc(v_{10})= \max\{d(v_{10},v_1), d(v_{10},v_2), d(v_{10},v_3), d(v_{10},v_4), d(v_{10},v_5), d(v_{10},v_6), d(v_{10},v_7), d(v_{10},v_8), d(v_{10},v_9), d(v_{10},v_{10})\}$

$= \max \{4, 4, 4, 3, 3, 2, 2, 1, 1, 0\} = 4$

The diameter of a graph G is the maximum of eccentricity of all its vertices and is denoted by $diam(G)$

That is $diam(G) = \max\{e(v) : v \in V(G)\}$

diam(G) = max{4, 4, 4, 3, 3, 2, 3, 4, 4, 4} = 4

The radius of a graph G is the minimum of eccentricity of all its vertices and is denoted by rad(G).

That is rad(G) = min{ecc(v):v∊V(G)}

rad(G)=min{4, 4, 4, 3, 3, 2, 3, 4, 4, 4} =2

The rainbow connection number rc (G) can be determined as follows,

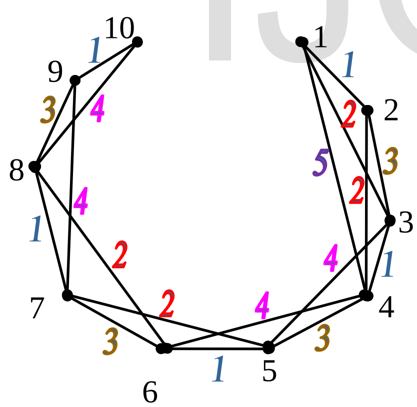

Fig.15: Interval graph G with the edge colors

In the above interval graph with the edge colors clearly we can see that the rainbow connection number rc(G)=5.

Hence we can conclude that rc(G)>diam(G)>$|RDS|$ ≥rad(G).

II . CONCLUSION

In this paper we study the **non-split restrained dominating set and comparing rainbow connection number, diameter, and cardinality of rds and radius of** an Interval graph corresponding to an Interval family I. In future, efforts in the paper eventually open up many an avenue in the field of research on interval graph.

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