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Steady flows in pipes of equilateral triangular cross–section through porous medium with magnetic field

Dr. Anand Swrup Sharma
Associate Professor, Dept. of Applied Sciences, Ideal Institute of Technology, Ghaziabad (U.P) India.
Email: sharma.as09@gmail.com

ABSTRACT: In this paper we have investigated the Steady flow in pipes of equilateral triangular cross–section through porous medium with magnetic field. We have obtained the velocity, volumetric flow and vortex lines.

KEY WORDS: Steady flow, Equilateral triangular cross section, incompressible fluid, pipes, porous medium and magnetic field.

NOMENCLATURE

u =velocity component along x – axis
v = velocity component along y – axis
w (x , y) = velocity in x-y plane
t = the time
ρ = the density of fluid
P = the fluid pressure
K= the thermal conductivity of the fluid
μ = Coefficient of viscosity
υ = Kinematic viscosity
Q = the volumetric flow
Ω_x = Vorticity component in x - direction
Ω_y = Vorticity component in y - direction
Ω_z = Vorticity component in z - direction

INTRODUCTION

We have investigated the Steady flow in pipes of equilateral triangular cross–section through porous medium with magnetic field. Attempts have been made by several researchers i.e. P. Eguia, J. Zueco, E. Granada & D. Patio [1] NSM solution for unsteady MHD Couette flow of a dusty conducting fluid with variable viscosity and electric conductivity. A. Elcrat, B. Fornberg, M. Horn & K. Miller [2] some steady vortex flows past a circular cylinder. J. W. Elder [3] Transient convection in a porous medium. S. M. M. EL-Kabeir, A. M. Rashad & S. R. G. Rama [4] unsteady MHD combined convection over a moving vertical sheet in a fluid saturated porous medium with uniform surface heat flux. K. Ellgene & H. O. Abdul [5] the effect of surface tensions the wave growth and transition to slug flow. E. Erturk & C. Gokcol [6] fourth order compact formulation of Navier-stokes equations and driven cavity flow at high Reynolds numbers. E. Erturk, T. C. Corke & C. Gokcol [7] numerical solutions of 2-D steady incompressible driven cavity flow at high Reynolds numbers. E. Erturk, T. C. Corke [8] boundary layer leading-edge receptivity to sound at incidence angles. E. Erturk, O. M. Haddad & T. C. Corke [9] numerical solutions of laminar incompressible flow past parabolic bodies at angles of attack. A. T. Esware & G. Nath [10] unsteady non-similar two-dimensional and axisymmetric water boundary layer with variable viscosity and Prandti number. M. A. Ezzat & A. A. EI-Bary [11] space approach to the hydro-magnetic flow of dusty fluid through a porous medium. M. A. Ezzat [12] magneto hydrodynamic unsteady flow of a non-newtonian fluid past an infinite porous plate. H. Faisst & B. Eckhardt [13] Transition from the Couette–Taylor system to the plane Couette system. D. Florea [14] non associated elastics viscoplastic model for bituminous concrete. In this paper we have investigated the velocity, volumetric flow and vortex lines.

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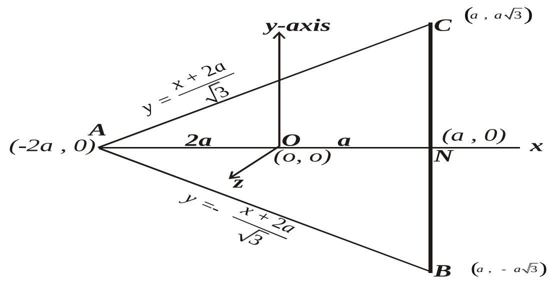
FORMULATION OF THE PROBLEM

Let z - axis be taken the direction of flow along the axis of the pipe. Then u = 0, v = 0 for steady and incompressible fluid the velocity component is independent of z . The equation of continuity.



$$AB = BC = CA = 2a\sqrt{3}\ ,\ AN = 3a$$

Figure-1

$$\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}+\frac{\partial w}{\partial z}=0 \quad ....(1)$$

$$But \quad u=0,\ \ v=0 \quad \frac{\partial w}{\partial z}=0......(2)$$

$$\Rightarrow \quad w = w\ (x, y)........(3)$$

i.e. w is independent of **z**

The Navier-Stokes equations of in the absence of body forces .

$$-\frac{\partial P}{\partial y}=0 \quad ....(4)$$

$$\frac{\partial P}{\partial z}+\mu\left(\frac{\partial^2 w}{\partial x^2}+\frac{\partial^2 w}{\partial y^2}\right)-\left(\frac{1}{\rho K}+\frac{\sigma B_0^2}{\rho\,\mu}\right)\mu\, w=0...........(5)$$

It is clear from (3) & (4) P is independent of **x** & **y** i.e. **p** is the Function of **z**

SOLUTION OF THE PROBLEM: p = p (z) $\frac{\partial p}{\partial z} = \frac{d\,p}{d\,z}$ = Constant = –P

$$let \quad \left(\frac{1}{\rho K}+\frac{\sigma B_0^2}{\rho\,\mu}\right)=B^2 \ \Rightarrow\ \mu\left[\frac{\partial^2 w}{\partial x^2}+\frac{\partial^2 w}{\partial y^2}-B^2 w\right]=\frac{dp}{dz} \quad \Rightarrow \quad \frac{\partial^2 w}{\partial x^2}+\frac{\partial^2 w}{\partial y^2}-B^2 w=-\frac{P}{\mu}.............(6)$$

$\left(D^2+D'^2-B^2\right)w=-\frac{P}{\mu}$, $\therefore$ $C.F. = \sum a_n\, e^{h_n x+h'_n y}$ where h_n & h'_n are related by $h_n{}^2+h'^2_n-B^2=0$

and $P.I. = \frac{1}{D^2+D'^2-B^2}\left(-\frac{P}{\mu}\right) = \frac{P}{B^2\mu}$ $\Rightarrow$ w (x, y) = $\sum_{n=1}^{\infty} a_n\, e^{h_n x+h'_n y}+\frac{1}{B^2\mu}P$ Where $h_n^2+h_n'^2=B^2$

Case -I: using boundary conditions at $y = \frac{x+2a}{\sqrt{3}}$

$$-\frac{P}{B^2\mu}=\sum_{n=1}^{\infty} a_n\, e^{h_n x+h'_n\left(\frac{x+2a}{\sqrt{3}}\right)} \quad ....(7)$$

$$\&\ \text{at}\ \ y=-\frac{x+2a}{\sqrt{3}} \qquad -\frac{P}{B^2\mu}=\sum_{n=1}^{\infty} a_n\, e^{h_n x-h'_n\left(\frac{x+2a}{\sqrt{3}}\right)} \quad ....(8)$$

From (7) & (8) $\sum_{n=1}^{\infty} a_n\left[e^{h_n x+h'_n\left(\frac{x+2a}{\sqrt{3}}\right)}-e^{h_n x+h'_n\left(\frac{x+2a}{\sqrt{3}}\right)}\right]=0 \Rightarrow e^{h_n x+h'_n\left(\frac{x+2a}{\sqrt{3}}\right)}=e^{h_n x-h'_n\left(\frac{x+2a}{\sqrt{3}}\right)}$

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$$at\ x = a \quad e^{h_n a + \frac{h'_n a}{\sqrt{3}} + \frac{2ah'_n}{\sqrt{3}}} = e^{h_n a - \frac{ah'_n}{\sqrt{3}} - \frac{2ah'_n}{\sqrt{3}}} \Rightarrow h'_n = 0 \Rightarrow h_n^2 + h'^2_n = B^2 \quad \therefore h_n = B \ \& \ h'_n = 0$$

$$\sum_{n=1}^{\infty} a_n\, e^{Bx} + \frac{P}{B^2\mu} = 0 \Rightarrow \frac{P}{B^2\mu} = e^{Ba} \sum_{n=1}^{\infty} a_n \Rightarrow \sum_{n=1}^{\infty} a_n = -\frac{P}{B^2\mu} P\, e^{-Ba} \Rightarrow w_1(x,y) = \frac{P}{B^2\mu}\left[1 - e^{B(x-a)}\right]$$

Case - II: $w\ (x,\ y)\ =\ 0\ \ at\ (-2a,\ 0)\ \ \&\ w(x,\ y)\ = 0\ at\ \ (a,\ a\sqrt{3})$

$$-\frac{P}{B^2\mu} = \sum_{n=1}^{\infty} a_n\, e^{-2\,ah_n} \quad ....(9)$$

$$-\frac{P}{B^2\mu} = \sum_{n=1}^{\infty} a_n\, e^{ah_n + a\sqrt{3}\,h'_n} \quad ....(10)$$

$$ah_n + a\sqrt{3}\,h'_n = -2ah_n \Rightarrow a\sqrt{3}\,h'_n = -3ah_n \Rightarrow h'_n = -\sqrt{3}\,h_n$$

$$h'^2_n + h_n^2 = B^2 \Rightarrow 4h_n^2 = B^2 \Rightarrow h_n = \pm\frac{B}{2} \quad \& \quad h'_n = \mp\frac{\sqrt{3}\,B}{2}$$

$$\sum_{n=1}^{\infty} a_n = -\frac{P}{B^2\mu}\, e^{Ba} \Rightarrow w_2(x,y) = \frac{P}{B^2\mu}\left[1 - e^{\frac{B\left(x-\sqrt{3}\,y+2a\right)}{2}}\right]$$

Case – III: $w\ (x,\ y)\ =\ 0\ \ at\ (-2a,\ 0)\ \ \&\ w(x,y)\ =\ 0\,at\ \ (a,\ -a\sqrt{3})$

$$-\frac{P}{B^2\mu} = \sum_{n=1}^{\infty} a_n\, e^{-2ah_n} \quad ....(11)$$

$$-\frac{P}{B^2\mu} = \sum_{n=1}^{\infty} a_n\, e^{ah_n - a\sqrt{3}\,h'_n} \quad ....(12)$$

On solving: $h_n = \frac{B}{2},\ \ h'_n = \frac{\sqrt{3}\,B}{2}\ \ \&\ \sum_{n=1}^{\infty} a_n = -\frac{P}{B^2\mu}\, e^{Ba} \Rightarrow w_3(x,y) = \frac{P}{B^2\mu}\left[1 - e^{\frac{B\left(x+\sqrt{3}\,y+2a\right)}{2}}\right]$

$$w(x,y) = \frac{P}{B^2\mu}\left[1 - \left\{e^{\frac{B(x+2a)}{2}} \,.\, e^{-\frac{\sqrt{3}By}{2}} + e^{\frac{B(x+2a)}{2}}\, e^{\frac{\sqrt{3}By}{2}}\right\} - e^{B(x-a)}\right]$$

$$= \frac{P}{B^2\mu}\left[1 - e^{\frac{B(x+2a)}{2}}\left\{e^{\frac{\sqrt{3}By}{2}} + e^{-\frac{\sqrt{3}By}{2}}\right\} - e^{B(x-a)}\right]$$

$$w(x,y) = \frac{P}{B^2\mu}\left[1 - 2\ Cosh\ \frac{\sqrt{3}\,B\,y}{2}\ e^{\frac{B(x+2a)}{2}} - e^{B(x-a)}\right] \quad ....(13)$$

The volumetric Flow:

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$$Q=\iint w\,(x,y)\,dx\,dy==\int_{x=-2a}^{a}\int_{y=-\left(\frac{x+2a}{\sqrt{3}}\right)}^{y=\left(\frac{x+2a}{\sqrt{3}}\right)}\frac{P}{B^2\mu}\left\{1-2\,Cosh\frac{\sqrt{3}\,By}{2}e^{\frac{B(x+2a)}{2}}-e^{B(x-a)}\right\}dx\,dy$$

$$=\frac{2P}{B^2\mu}\int_{-2a}^{a}\int_{0}^{\frac{x+2a}{\sqrt{3}}}\left\{1-2\,Cosh\frac{\sqrt{3}\,By}{2}\;e^{\frac{B(x+2a)}{2}}-e^{B(x-a)}\right\}dy\,dx$$

$$=\;\frac{2P}{B^2\mu}\int_{-2a}^{a}\left\{\frac{x+2a}{\sqrt{3}}-\frac{4}{\sqrt{3B}}\;Sinh\frac{B\left(x+2a\right)}{2}e^{\frac{B(x+2a)}{2}}-\left(\frac{x+2a}{\sqrt{3}}\right)e^{B(x-a)}\right\}dx$$

$$Let\;\;I_1=\int_{-2a}^{a}e^{\frac{B(x+2a)}{2}}\;Sinh\frac{B\left(x+2a\right)}{2}\;.\,dx=\int_{-2a}^{a}e^{\frac{B(x+2a)}{2}}\left\{\frac{e^{\frac{B(x+2a)}{2}}-e^{-\frac{B(x+2a)}{2}}}{2}\right\}dx$$

$$=\frac{1}{2}\int_{-2a}^{a}\left\{e^{B(x+2a)}-1\right\}dx\;=\frac{1}{2}\left\{\frac{e^{B(x+2a)}}{B}\;-x\right\}_{-2a}^{a}=\frac{1}{2}\left\{\frac{e^{3aB}}{B}\;-a-\;\frac{1}{B}-2a\right\}$$

$$=\frac{1}{2}\left\{\frac{e^{3aB}}{B}\;-\frac{1}{B}-3a\right\}=\frac{1}{2B}\left\{e^{3aB}-1-3aB\right\}$$

$$Let\quad I_2=\int_{-2a}^{a}\frac{\left(x+2a\right)}{\sqrt{3}}\;e^{B(x-a)}\;dx==\frac{1}{\sqrt{3}}\left[\left\{\frac{\left(x+2a\right)}{B}\;e^{B(x-a)}\right\}_{-2a}^{a}-\int_{-2a}^{a}\frac{1}{B}\;\;e^{B(x-a)}\;.\,dx\right]$$

$$=\frac{1}{\sqrt{3}}\left\{\frac{3a}{B}-\frac{1}{B^2}\left\{e^{B(x-a)}\right\}_{-2a}^{a}\right\}\;=\frac{1}{\sqrt{3}}\left[\frac{3a}{B}-\frac{1}{B^2}\left\{1-e^{-3aB}\right\}\right]=\frac{1}{\sqrt{3}}\left[\frac{3a}{B}-\frac{1}{B^2}+\frac{e^{-3aB}}{B^2}\right]=\frac{1}{\sqrt{3}\,B^2}\left[3aB-1+e^{-3aB}\right]$$

$$Let\quad I_3=\int_{-2a}^{a}\frac{\left(x+2a\right)}{\sqrt{3}}\;dx=\left[\frac{\left(x+2a\right)}{2\sqrt{3}}\right]_{-2a}^{a}=\frac{\left(3a\right)^2}{2\sqrt{3}}=\frac{9a^2}{2\sqrt{3}}=\frac{3\sqrt{3}\;a^2}{2}$$

$$\therefore\;\;Q=\frac{2P}{\mu B^2}\left[I_3-\frac{4}{\sqrt{3}B}I_1-I_2\right]=\frac{2P}{\mu B^2}\left[\frac{3\sqrt{3}\;a^2}{2}-\frac{4}{\sqrt{3}\,B}.\frac{1}{2B}\left\{e^{3aB}-1-3aB\right\}-\frac{1}{\sqrt{3}\,B^2}\left\{3aB-1+e^{-3aB}\right\}\right]$$

$$Q=\frac{2P}{\mu B^2}\left\{\frac{9a^2}{2\sqrt{3}}-\frac{2}{\sqrt{3}\,B^2}\;e^{3aB}+\frac{2}{\sqrt{3}\,B^2}+\frac{6a}{\sqrt{3}\,B}-\frac{3a}{\sqrt{3}\,B}+\frac{1}{\sqrt{3}\,B^2}-\frac{1}{\sqrt{3}\,B^2}\;e^{-3aB}\right\}$$

$$=\frac{2P}{\mu B^2}\left\{\frac{9a^2}{2\sqrt{3}}+\frac{3}{\sqrt{3}\,B^2}+\frac{3a}{\sqrt{3}\,B}-\frac{2}{\sqrt{3}\,B^2}\;e^{3aB}-\frac{1}{\sqrt{3}\,B^2}\;e^{-3aB}\right\}$$

$$Q=\frac{2P}{\sqrt{3}\mu B^2}\left\{\frac{9a^2}{2}+\frac{3}{B}(a+\frac{1}{B})-\frac{1}{B^2}\left(2e^{3aB}-e^{-3aB}\right)\right\}\;.............(14)$$

The equation of vortex line: $\frac{dx}{\Omega_x}=\frac{dy}{\Omega_y}=\frac{dz}{\Omega_z}$ where $\Omega_x,\,\Omega_y\;\&\;\Omega_z$ are vorticity components

Let $\bar{q} = ui + vj = wk = \frac{P}{\mu B^2}\left[1 - 2\, Cosh\frac{\sqrt{3}\,B\,y}{2} e^{\frac{B(x+2a)}{2}} - e^{B(x-a)}\right]k$

$$\Omega_x = \frac{\partial w}{\partial y} - \frac{\partial v}{\partial z} = \frac{P}{\mu B^2}\left[-\sqrt{3}\,B\, Sinh\frac{\sqrt{3}\,B\,y}{2} e^{\frac{B(x+2a)}{2}}\right] = -\frac{\sqrt{3}\,P}{\mu B}\, e^{\frac{B(x+2a)}{2}}\, Sinh\frac{\sqrt{3}\,B\,y}{2}$$

$$\Omega_y = \frac{\partial u}{\partial z} - \frac{\partial w}{\partial x} = -\frac{P}{\mu B^2}\left[-B\, Cosh\frac{\sqrt{3}\,B\,y}{2} e^{\frac{B(x+2a)}{2}} - Be^{B(x-a)}\right]$$

$$= \frac{P}{\mu B}\left[Cosh\frac{\sqrt{3}\,B\,y}{2} e^{\frac{B(x+2a)}{2}} + e^{B(x-a)}\right] \quad \& \quad \Omega_z = 0$$

$$\frac{dx}{-\frac{\sqrt{3}\,P}{\mu B}\, e^{\frac{B(x+2a)}{2}}\, Sinh\frac{\sqrt{3}\,B\,y}{2}} = \frac{dy}{\frac{P}{\mu B}\left[Cosh\frac{\sqrt{3}\,B\,y}{2}\, e^{\frac{B(x+2a)}{2}} + e^{B(x-a)}\right]} = \frac{dz}{0}$$

Taking I^{st} Two $\frac{dx}{-\sqrt{3}\, e^{\frac{B(x+2a)}{2}}\, Sinh\frac{\sqrt{3}\,B\,y}{2}} = \frac{dy}{\left[Cosh\frac{\sqrt{3}\;By}{2}\, e^{\frac{B(x+2a)}{2}} + e^{B(x-a)}\right]}$

$$\int \frac{\left(Cosh\frac{\sqrt{3}\,B\,y}{2} e^{\frac{B(x+2a)}{2}} + e^{B(x-a)}\right)}{e^{\frac{B(x+2a)}{2}}}\, dx + \sqrt{3}\int Sinh\frac{\sqrt{3}\,B\,y}{2}\, dy = C_1$$

$$\int Cosh\frac{\sqrt{3}\,B\,y}{2}\, dx + \int e^{\frac{B(2x-2a-x-2a)}{2}}\, dx + \sqrt{3}\,.\,\frac{2}{\sqrt{3}\,B}\, Cosh\frac{\sqrt{3}\,B\,y}{2} = C_1$$

$$\int Cosh\frac{B(x+2a)}{2}\, dx + \int e^{\frac{B(x-4a)}{2}}\, dx + \frac{2}{B}\, Cosh\frac{\sqrt{3}\,B\,y}{2} = C_1$$

$$\frac{2}{B}\, Sinh\frac{B(x+2a)}{2} + \frac{2}{B}\, e^{\frac{B(x-4a)}{2}} + \frac{2}{B}\, Cosh\frac{\sqrt{3}\,B\,y}{2} = C_1 \text{ or } Sinh\frac{B(x+2a)}{2} + e^{\frac{B(x-4a)}{2}} + Cosh\frac{\sqrt{3}\,B\,y}{2} = \frac{C_1 B}{2} = A$$

the first vortex line is $e^{\frac{B(x-4a)}{2}} + Sinh\frac{\sqrt{3}\,B\,y}{2} + Cosh\frac{\sqrt{3}\,B\,y}{2} = A$ .... (15)

taking last two $dz = 0$ the second vortex line $z = B$ .... (16)

Tables for velocity: Case- I

$$Let \quad P=2, \quad \mu=.5, \quad a=3, \; are\, same, \; \frac{1}{\sqrt{\sigma K}}=\sqrt{\frac{\sigma B_0^2}{\rho\mu}}=\frac{1}{2} \Rightarrow B=\sqrt{\frac{1}{\sigma K}+\frac{\sigma B_0^2}{\rho\mu}}=\frac{1}{\sqrt{2}}$$

Table -1 (for velocity)

	(x,y)	$\left(-9,\frac{1}{6\sqrt{3}}\right)$	$\left(-12,\frac{1}{2\sqrt{3}}\right)$	$\left(-15,\frac{1}{\sqrt{3}}\right)$	$\left(-18,\frac{2}{\sqrt{3}}\right)$	$\left(-21,\frac{3}{\sqrt{3}}\right)$	$\left(-24,\frac{4}{\sqrt{3}}\right)$	$\left(-27,\frac{5}{\sqrt{3}}\right)$
$\frac{1}{\sqrt{\rho K}}=\frac{1}{2}$	$w(x,y)$	**.8315**	**8.795**	**12.519**	**14.203**	**15.025**	**15.45**	**15.683**
$\sqrt{\frac{\sigma B_0^2}{\rho\mu}}=\frac{1}{2}$	$w(x,y)$	**.8315**	**8.795**	**12.519**	**14.203**	**15.025**	**15.45**	**15.683**
$\sqrt{\frac{1}{\sigma K}+\frac{\sigma B_0^2}{\rho\mu}}=\frac{1}{\sqrt{2}}$	$w(x,y)$	**2.449**	**6.052**	**7.294**	**7.71**	**7.871**	**7.939**	**7.971**

Case-II:

$$Let \quad P=2, \quad \mu=.5, \quad a=3, \; are\, same, \; \frac{1}{\sqrt{\sigma K}}<\sqrt{\frac{\sigma B_0^2}{\rho\mu}} \; Let \; \frac{1}{\sqrt{\sigma K}}=\frac{1}{2} \; and \; \sqrt{\frac{\sigma B_0^2}{\rho\mu}}=1 \Rightarrow B=\sqrt{\frac{1}{\sigma K}+\frac{\sigma B_0^2}{\rho\mu}}=\frac{\sqrt{5}}{2}$$

Table -2 (for velocity)

	(x,y)	$\left(-9,\frac{1}{6\sqrt{3}}\right)$	$\left(-12,\frac{1}{2\sqrt{3}}\right)$	$\left(-15,\frac{1}{\sqrt{3}}\right)$	$\left(-18,\frac{2}{\sqrt{3}}\right)$	$\left(-21,\frac{3}{\sqrt{3}}\right)$	$\left(-24,\frac{4}{\sqrt{3}}\right)$	$\left(-27,\frac{5}{\sqrt{3}}\right)$
$\frac{1}{\sqrt{\rho K}}=\frac{1}{2}$	$w(x,y)$	**.8315**	**8.795**	**12.519**	**14.203**	**15.025**	**15.45**	**15.683**
$\sqrt{\frac{\sigma B_0^2}{\rho\mu}}=1$	$w(x,y)$	**2.209**	**3.589**	**3.8997**	**3.969**	**3.9896**	**3.996**	**3.999**
$\sqrt{\frac{1}{\sigma K}+\frac{\sigma B_0^2}{\rho\mu}}=\frac{\sqrt{5}}{2}$	$w(x,y)$	**1.998**	**2.968**	**3.151**	**3.187**	**3.196**	**3.199**	**3.2**

Case- III:

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$Let\quad P=2,\quad \mu=.5,\quad a=3,\ are\,same,\ \frac{1}{\sqrt{\sigma K}}>\sqrt{\frac{\sigma B_0^2}{\rho\mu}}\ \ Let\ \ \frac{1}{\sqrt{\sigma K}}=1\ and\ \sqrt{\frac{\sigma B_0^2}{\rho\mu}}=\frac{1}{2}\ \Rightarrow\ B=\sqrt{\frac{1}{\sigma K}+\frac{\sigma B_0^2}{\rho\mu}}=\frac{\sqrt{5}}{2}$

Table -3 (for velocity)

	(x,y)	$\left(-9,\frac{1}{6\sqrt{3}}\right)$	$\left(-12,\frac{1}{2\sqrt{3}}\right)$	$\left(-15,\frac{1}{\sqrt{3}}\right)$	$\left(-18,\frac{2}{\sqrt{3}}\right)$	$\left(-21,\frac{3}{\sqrt{3}}\right)$	$\left(-24,\frac{4}{\sqrt{3}}\right)$	$\left(-27,\frac{5}{\sqrt{3}}\right)$
$\sqrt{\frac{\sigma B_0^2}{\rho\mu}}=\frac{1}{2}$	$w(x,y)$	**.8315**	**8.795**	**12.519**	**14.203**	**15.025**	**15.45**	**15.683**
$\frac{1}{\sqrt{\sigma K}}=1$	$w(x,y)$	**2.209**	**3.589**	**3.8997**	**3.969**	**3.9896**	**3.996**	**3.999**
$\sqrt{\frac{1}{\sigma K}+\frac{\sigma B_0^2}{\rho\mu}}=\frac{\sqrt{5}}{2}$	$w(x,y)$	**1.998**	**2.968**	**3.151**	**3.187**	**3.196**	**3.199**	**3.2**

CONCLUSION AND DISCUSSION

In this paper, we have investigated the velocity by the **table-1** of equation (13) between velocity and point. The velocity in porous medium and magnetic field at $\frac{1}{\sqrt{\rho K}}=\sqrt{\frac{\sigma B_0^2}{\rho\mu}}=\frac{1}{2}$ is less than the value of velocity in porous with magnetic field at $\sqrt{\frac{1}{\rho K}+\frac{\sigma B_0^2}{\rho\mu}}=\frac{1}{\sqrt{2}}$ at point $\left(-9,\frac{1}{6\sqrt{3}}\right)$ but the value of velocity in porous medium and magnetic field at $\frac{1}{\sqrt{\rho K}}=\sqrt{\frac{\sigma B_0^2}{\rho\mu}}=\frac{1}{2}$ is greater than the corresponding value of velocity in porous with magnetic field at $\sqrt{\frac{1}{\rho K}+\frac{\sigma B_0^2}{\rho\mu}}=\frac{1}{\sqrt{2}}$ in the interval $\left(-12,\frac{1}{2\sqrt{3}}\right)\le(x,y)\le\left(-27,\frac{5}{\sqrt{3}}\right)$.

Again by the **table-2** the velocity in porous medium at $\frac{1}{\sqrt{\rho K}}=\frac{1}{2}$ is less than the corresponding value of velocity in magnetic field at $\sqrt{\frac{\sigma B_0^2}{\rho\mu}}=1$ and is also less than the corresponding value of velocity in porous with magnetic field at $\sqrt{\frac{1}{\rho K}+\frac{\sigma B_0^2}{\rho\mu}}=\frac{\sqrt{5}}{2}$ at point $\left(-9,\frac{1}{6\sqrt{3}}\right)$,but the value of velocity in porous medium at $\frac{1}{\sqrt{\rho K}}=\frac{1}{2}$ is greater than the corresponding value of velocity in magnetic field at $\sqrt{\frac{\sigma B_0^2}{\rho\mu}}=1$ and at porous with magnetic field at $\sqrt{\frac{1}{\rho K}+\frac{\sigma B_0^2}{\rho\mu}}=\frac{\sqrt{5}}{2}$ in the interval $\left(-12,\frac{1}{2\sqrt{3}}\right)\le(x,y)\le\left(-27,\frac{5}{\sqrt{3}}\right)$.

Again by the **table-3** the velocity in magnetic field at $\sqrt{\frac{\sigma B_0^2}{\rho\mu}}=\frac{1}{2}$ is less than the corresponding value of velocity in porous medium at $\frac{1}{\sqrt{\rho K}}=1$ and is also less than the corresponding value of velocity in porous with magnetic field at $\sqrt{\frac{1}{\rho K}+\frac{\sigma B_0^2}{\rho\mu}}=\frac{\sqrt{5}}{2}$ at point $\left(-9,\frac{1}{6\sqrt{3}}\right)$,but the value of velocity in magnetic field at $\sqrt{\frac{\sigma B_0^2}{\rho\mu}}=\frac{1}{2}$ is greater than the corresponding value of velocity in porous medium at $\frac{1}{\sqrt{\rho K}}=1$ and at porous with magnetic field at $\sqrt{\frac{1}{\rho K}+\frac{\sigma B_0^2}{\rho\mu}}=\frac{\sqrt{5}}{2}$ in the interval $\left(-12,\frac{1}{2\sqrt{3}}\right)\le(x,y)\le\left(-27,\frac{5}{\sqrt{3}}\right)$. Also we have investigated the volumetric flow and vortex lines by the equations (14), (15) and (16) respectively.

REFERENCES

[1] Eguia P., Zueco J., Granada E. & Patio D., NSM solution for unsteady MHD Couette flow of a dusty conducting fluid with variable viscosity and electric conductivity. Applied Mathematical Modelling, 35, pp 303-316, (2011).

[2] Elcrat A., Fornberg B., Horn M. & Miller K., some steady vortex flows past a circular cylinder. J. Fluid Mech. 409 pp 13–27, (2000).

[3] Elder J. W., Transient convection in a porous medium. the J. of Fluid Mechanics, vol. 47, pp. 609–623,(1967).

[4] EL-Kabeir S. M. M., Rashad A. M. & Rama S. R. G., unsteady MHD combined convection over a moving vertical sheet in a fluid saturated porous medium with uniform surface heat flux. Mathematical and Computer Modeling, Vol. 46, Issues 3-4, pp 384-397, (2007).

[5] Ellgene K. & Abdul H. O., the effect of surface tensions the wave growth and transition to slug flow. J. of Fluids Engng. Vol. 117, No. 3, pp 389-393, (1995).

[6] Erturk E. & Gokcol C., fourth order compact formulation of Navier-stokes equations and driven cavity flow at high Reynolds numbers. Int. J. for numerical methods in fluids 50, pp 421-436, (2006).

[7] Erturk E., Corke T. C. & Gokcol C., numerical solutions of 2-d steady incompressible driven cavity flow at high Reynolds numbers. Int. J. for numerical methods in fluids 48, pp 747-774, (2005).

[8] Erturk E., Corke T. C., boundary layer leading-edge receptivity to sound at incidence angles. J. Fluid Mech.; 444, pp 383–407, (2001).

[9] Erturk E., Haddad O. M. & Corke T. C., numerical solutions of laminar incompressible flow past parabolic bodies at angles of attack. AIAA journal 42, pp 2254-2265, (2004).

[10] Esware A. T & Nath G., unsteady non-similar two-dimensional and axisymmetric water boundary layer with variable viscosity and Prandti number. Int. J. Engng. Sci. Yo. 32, No. 2, pp 287-279, (1994).

[11] Ezzat M. A. & EI-Bary A. A., space approach to the hydro-magnetic flow of dusty fluid through a porous medium. J. of computer and mathematics with application. Vol.-59, pp 2868-2879, (2010).

[12] Ezzat M. A., magneto hydrodynamic unsteady flow of a non-newtonian fluid past an infinite porous plate. Indian J. Pure Appl. Maths, Yo. 25 No. 6, pp 655-664, (1994).

[13] Faisst H. & Eckhardt B., Transition from the Couette–Taylor system to the plane Couette system. Phys. Rev. E, 61, pp 7227–7230, (2000).

[14] Florea D., non associated elastics viscoplastic model for bituminous concrete. Int. J. Engng. Sci. Vol. 32, No.1. pp 81-93, (1994).