

Soret and Dufour Effects on Steady free Convection in MHD Micropolar Fluid Flow, Mass and Heat Transfer with Hall Current

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ABSTRACT

In the present paper is an investigation of heat and mass transfer of a steady free convection in MHD micropolar electrically conducting fluid flow on a vertical plate in the presence of Soret and Dufour effects under the influence of an applied uniform magnetic field and the effects of Hall current are taken into account. A magnetic field of uniform strength is assumed to be applied transversely to the direction of the main flow. Using suitable similarity transformations the governing equations of the problem are reduced to couple nonlinear ordinary differential equations and are solved numerically by Runge- Kutta fourth-fifth order method using symbolic software MATLAB. The numerical results concerned with the velocity, secondary velocity, concentration, micro rotation and temperature profiles effects of various parameters on the flow fields are investigated and presented graphically.

Keywords : MHD free convection; Micropolar fluid; Magnetic parameter; Hall parameter; Soret and Dufour effect.;

Nomenclature			
MHD	Magnetohydrodynamics	j	Micro-inertia density
c_p	Specific heat of with constant pressure	q_w	Heat transfer
g^*	Gravitational acceleration	S_c	Schmidt number
g	Secondary Velocity	q_m	Mass Transfer
M	Magnetic parameter	B_0	Constant magnetic field intensity
m	Hall parameter	T_w	Temperature at the Plate
v	Kinematic viscosity	T_∞	Temperature of the fluid outside the boundary layer
α	Thermal diffusivity	ψ	Stream function
β_T	Thermal Expansion Coefficient	γ^*	Spin-gradient viscosity
β_c^*	Coefficient of expansion with concentration	η	Similarity variable
ρ	Density	ξ	Micro-inertia density parameter
σ	Micro rotation Component	κ	Vortex viscosity
θ	Dimensionless temperature	K_T	Thermal diffusion ratio
u	Velocity component in x-direction	T_m	Mean fluid temperature
v	Velocity component in y-direction	C_s	Concentration susceptibility
w	Secondary Velocity	δ	Vortex viscosity parameter
T	Temperature	L, γ	Buoyancy parameter
D	Thermal molecular diffusivity	A	Reference velocity
C	Concentration	D_f	Dufour number
C_∞	Concentration of the fluid outside the boundary layer	σ^*	Dimensionless micro rotation
P_r	Prandlt number	Subscripts	
S_r	Soret number	W	Quantities at wall
		∞	Quantities at the free stream

1 INTRODUCTION

In recent years MHD flow problems have become in view of its significant applications in industrial manufacturing processes such as plasma studies, petroleum industries Magneto-hydrodynamics power generator cooling of clear reactors, boundary layer control in aerodynamics. Many authors have studied the effects of magnetic field on mixed, natural and force convection heat and mass transfer problems. The study of flow and heat transfer of an electrically conducting micropolar fluid past a porous plate under the influence of a magnetic field has attracted the interest of numerous researchers in view of its applications in many engineering problems, such as magnetohydrodynamic (MHD) generators, nuclear reactors, geothermal energy extractions and the boundary layer control in the field of aerodynamics. Keeping in mind some specific industrial applications such as polymer processing technology, numerous attempts have been made to analyze the effect of transverse magnetic field on boundary layer flow characteristics. However, the effect of thermal radiation on the flow and heat transfer has not been provided in most investigations. The effect of radiation on MHD flow and heat transfer problem has become more important industrially. At high operating temperature, radiation effect can be quite significant. Many processes in engineering areas occur at high temperature and knowledge on radiation heat transfer becomes very important for design of reliable equipment, nuclear plants, gas turbines and various propulsion devices or aircraft, missiles, satellites and space vehicles. On the basis of these applications. Cogley et al. [13] shows that in the optically thin limit, the fluid does not absorb its own emitted radiation but the fluid does not absorb radiation emitted by the boundaries. Makinde [14] examined the transient free convection interaction with thermal radiation with thermal radiation of an absorbing emitting fluid along moving vertical permeable plate. Ibrahim et al. [15] discussed the case of mixed convection flow of a micropolar fluid past a semi-infinite, steady moving porous plate with varying suction velocity normal to the plate in the presence of thermal radiation and viscous dissipation. The concept of micropolar fluid deals with a class of fluids that exhibit certain micropolar effects arising from the local structure and micromoments of the fluid elements. These fluids contain dilute suspensions or rigid macromolecules with individual motion that stress and body moments and are influenced by spin inertia. The dynamics of micropolar fluid has been a popular area of research because of their application in a number of processes that occur in industry. Such application includes the flow of exotic lubricants colloidal suspensions, solidification of liquid crystals, extrusion of polymer fluids, cooling of metallic plate in bath, animal blood, body fluids and many other situations. The dynamics of micropolar fluid. Originated from the work of Eringen [1],[2]. An excellent review about micropolar fluid mechanics and its application was given by Ariman et al. [3],[4], Qukaszewicz [5] and Eringen [6]. A significant amount of research on micropolar fluid flow and heat transfer caused by continuously stretched or moving surfaces [7]-[12] under different conditions and in the presence of

various physical effects has been reported. From the point of applications, model studied on the effect of magnetic field on free convection flows have been made by several investigators. Some of them are Georgantopoulos [16], Nanousis et al. [17] and Raptis and Singh [18]. Along with the effects of magnetic field, the effect of transpiration parameter, being an effective method of controlling the boundary layer has been considered by Kafousias [19]. On the other hand, along with the free convection currents, caused by temperature difference, the flow is also affected by the difference in concentrations on material constitution. Gebhart and Pera [21] made extensive studies of such a combined heat and mass transfer flow to highlight the insight of the flow. In the above mentioned works, the level of concentration of foreign mass is assumed very low so that the Soret and Dufour effects can be neglected. However, exceptions are observed therein. The Soret effect, for instance, has been utilized for isotope separation and in mixture between gases with very light molecular weight (H_2, H_e) and of medium molecular weight (N_2, air). The Dufour effect was found to be of order of considerable magnitude such that it cannot be ignored Eckert and Drake, [20]. In view of the importance of above mentioned effects, Kafoussias and Williams [22] studied the Soret and Dufour effects on mixed free-forced convective and mass transfer boundary layer flow with temperature dependent viscosity. Anghel et al. [23] investigated the Dufour and Soret effects on free convection boundary layer flow over a vertical surface embedded in porous medium. Quite recently, Alam and Rahman [24] investigated the Dufour and Soret effects on mixed convection flow past a vertical porous flat plate with variable suction. Since the study of heat and mass transfer is important in most cases, in the present paper our object is to investigate the Soret and Dufour effects on steady free convection in MHD micropolar fluid flow, mass and heat transfer with Hall current.

2 FORMULATION OF THE PROBLEM AND SIMILARITY ANALYSIS

Let us consider steady two dimensional MHD free convection heat and mass transfer in an incompressible micropolar electrically conducting fluid flow with Soret and Dufour effects. The flow is subjected to a transverse magnetic field of strength B_0 which is assumed to be applied in the positive y -direction, normal to the surface. The viscosity and thermal conductivity of the fluid are assumed to be function of temperature. The pressure gradient and body force viscous dissipation and joule heating effects are neglected compared with effects of with internal heat source/sink.

Under the above assumptions and usual boundary layer approximation, the dimensional governing equations of continuity, momentum, angular momentum, concentration and energy under the influence of externally imposed magnetic field with the presence of Hall current are:

Equation of continuity:

$$u \frac{\partial u}{\partial x} + v \frac{\partial u}{\partial y} = 0 \quad (1)$$

Momentum equation:

$$u \frac{\partial u}{\partial x} + v \frac{\partial u}{\partial y} = \left(v + \frac{\kappa}{\rho} \right) \frac{\partial^2 u}{\partial y^2} + \frac{\kappa}{\rho} \frac{\partial \sigma}{\partial y} + g^* \beta_T (T - T_\infty) + g^* \beta_c (C - C_\infty) - \frac{\sigma_0 B_0^2}{\rho(1+m^2)} (u + mw) \quad (2)$$

$$u \frac{\partial w}{\partial x} + v \frac{\partial w}{\partial y} = \left(v + \frac{\kappa}{\rho} \right) \frac{\partial^2 w}{\partial y^2} + \frac{\kappa}{\rho} \frac{\partial \sigma}{\partial y} + \frac{\sigma_0 B_0^2}{\rho(1+m^2)} (mu - w) \quad (3)$$

Angular momentum Equation:

$$u \frac{\partial \sigma}{\partial x} + v \frac{\partial \sigma}{\partial y} = \frac{\gamma^*}{\rho j} \frac{\partial^2 \sigma}{\partial y^2} - \frac{\kappa}{\rho j} \left(2\sigma + \frac{\partial u}{\partial y} \right); \quad \gamma^* = \left(\mu + \frac{\kappa}{2} \right) j \quad (4)$$

Energy Equation:

$$u \frac{\partial T}{\partial x} + v \frac{\partial T}{\partial y} = \alpha \frac{\partial^2 T}{\partial y^2} + \frac{DK_T}{c_s c_p} \frac{\partial^2 C}{\partial y^2}, \quad \alpha = \frac{k}{\rho c_p} \quad (5)$$

Concentration Equation:

$$u \frac{\partial C}{\partial x} + v \frac{\partial C}{\partial y} = D \frac{\partial^2 C}{\partial y^2} + \frac{DK_T}{T_m} \frac{\partial^2 T}{\partial y^2} \quad (6)$$

Boundary conditions are :

$$u = 0, v = 0, w = 0, \frac{\partial T}{\partial y} = -\frac{q_w}{K}, \frac{\partial C}{\partial y} = -\frac{q_m}{D}, \text{ at } y = 0$$

$$u = 0, w = 0, \sigma = 0, T = T_\infty, C = C_\infty \text{ as } y \rightarrow \infty \quad (7)$$

To convert the governing equations into a set of similarity equations, we introduce the following similarity transformation:

$$\eta = y \sqrt{\frac{A}{2\nu x}}, \psi = \sqrt{2\nu Ax} f(\eta), \sigma = \sigma^* \sqrt{\frac{A^3}{2\nu x}}, w = Ag(\eta)$$

$$T - T_\infty = \theta(\eta) T^*, T^* = \sqrt{\frac{2\nu x}{A}} \frac{q_w}{K}, T_w - T_\infty = T^*$$

$$C - C_\infty = \varphi(\eta) C^*, C^* = \sqrt{\frac{2\nu x}{A}} \frac{q_m}{K}, C_w - C_\infty = C^* \quad (8)$$

Since $u = \frac{\partial \psi}{\partial y}$, and $v = -\frac{\partial \psi}{\partial x}$, then from Eq.(8) we have

$$u = Af', v = \sqrt{\frac{\nu A}{2x}} (\eta f' - f)$$

The dimensionless numbers are as follows:

$$M = \frac{2\sigma_0 B_0^2 x}{\rho A}, \delta = \frac{\kappa}{\mu}, \gamma = \frac{2g^* \beta_T T^* x}{A^2}, L = \frac{2g^* \beta_c C^* x^2}{A^2},$$

$$\xi = \frac{jA}{\nu x}, D_f = \frac{DK_T}{\nu c_p c_s}, S_c = \frac{\nu}{D}, S_r = \frac{DK_T q_w}{\nu q_m T_m}, P_r = \frac{\nu}{\alpha}$$

From the above transformations, the non-dimensional, nonlinear and coupled ordinary differential equations are obtained

$$\text{as } (1+\delta)f''' + ff'' + \delta(\sigma^*)' + \gamma\theta + L\varphi - \frac{M}{1+m^2} f' - \frac{Mm}{1+m^2} g = 0 \quad (9)$$

$$(1+\delta)g'' + fg' + \delta(\sigma^*)' + \frac{Mm}{1+m^2} f' - \frac{M}{1+m^2} g = 0 \quad (10)$$

$$\left(1 + \frac{1}{2}\delta\right)\xi(\sigma^*)'' - 2\delta(2\sigma^* + f'') + \xi\left[(\sigma^*)' f + \sigma^* f'\right] = 0 \quad (11)$$

$$\theta'' + P_r D_f \varphi'' + P_r (\theta' f - \theta f') = 0 \quad (12)$$

$$\varphi'' + S_c (\varphi' f - \varphi f') + S_r S_c \theta'' = 0 \quad (13)$$

The transform boundary conditions:

$$f = f' = 0, g = 0, \theta = 1, \theta' = -1, \varphi = 1, \varphi' = -1,$$

$$\sigma^* = 0 \text{ at } \eta = 0$$

$$\sigma^* = f = g = \theta = \varphi = 0 \text{ as } \eta \rightarrow \infty$$

3 RESULTS AND DISCUSSION

In this paper, the effect of different parameters entering into a steady free convection in MHD micropolar electrically conducting fluid flow on a vertical plate in the presence of Soret and Dufour effects under the influence of an applied uniform magnetic field and the effects of Hall current are taken into account has been investigated by Runge- Kutta fourth-fifth order method using symbolic software MATLAB. The numerical results concerned with the velocity, secondary velocity, concentration, micro rotation and temperature profiles effects of various parameters on the flow fields are investigated and presented graphically. Figs.1-26 to illustrate the influence of physical parameters viz., magnetic parameter M, Hall parameter m, Schmidt number S_c, Soret number S_r, Prandtl Number P_r, buoyancy parameters L & γ, dufour number D_f, micro-inertia density parameter ξ and vortex viscosity parameter δ on the velocity, secondary velocity, temperature, micro rotation and concentration profiles. For various values of the magnetic parameter M, the velocity profiles are plotted in fig. 1 & fig. 2. It can be seen that as M increases, the velocity decreases. This result qualitatively agrees with the expectations,

since the magnetic field exerts a retarding force on the free convection flow. The effect of Hall parameter m on the velocity profile is presented in fig. 3. It can be easily seen from that the velocity decreases as m . The effect of the micro inertia parameter and vortex viscosity parameter on the velocity profile is presented in fig. 4 & fig. 5 and fig.6 respectively. It can be seen that as ξ increases, the velocity decreases but velocity increases for increasing values of δ . The effect of buoyancy parameters L & γ are shown in figs. 7-9. It can be seen that in a certain interval of typical value of η the velocity decreases for increasing values of L & γ and then velocity increases. From fig. 10 & fig. 11 it is seen that the secondary velocity decreases as m increases and in fig. 12 and fig. 13 the secondary velocity increases as M increases. From fig. 14 & fig. 15 it is observed that in a certain interval of typical value of η the velocity decreases for increasing values of δ and then velocity increases. From fig. 16 & fig. 17 it is observed that the micro rotation increases as ξ increases. From fig. 18 & fig. 19 it is observed that in a certain interval of typical value of η the micro rotation increases for increasing values of δ and then micro rotation decreases. From fig. 20 -fig. 23 it is observed that the temperature profile increases as P_r and D_f increases. Also from fig. 24-fig.26 it is seen that concentration increases for increasing values of S_c , & S_r .

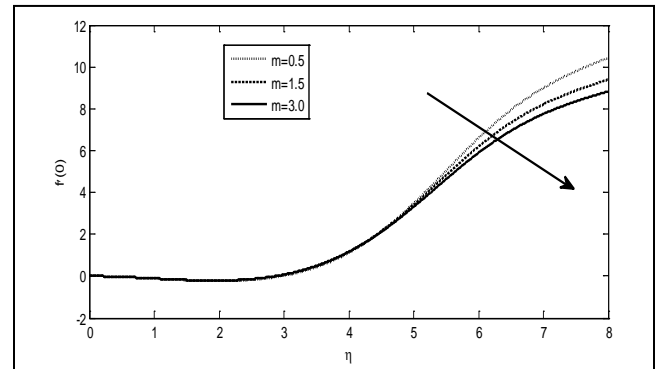


Fig. 3. Velocity profile for various values of m and $M=1.0, L=1.0, \gamma=1.0, \xi=5.0, \delta=5.0, P_r=0.7, D_f=0.1, S_c=0.1, S_r=1.$

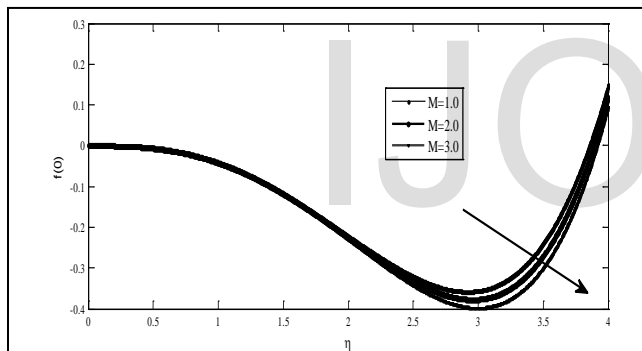


Fig. 1. Velocity profile for various values of M and $m=1.0, L=1.0, \gamma=1.0, \xi=5.0, \delta=5.0, P_r=0.7, D_f=0.1, S_c=0.1, S_r=1.$

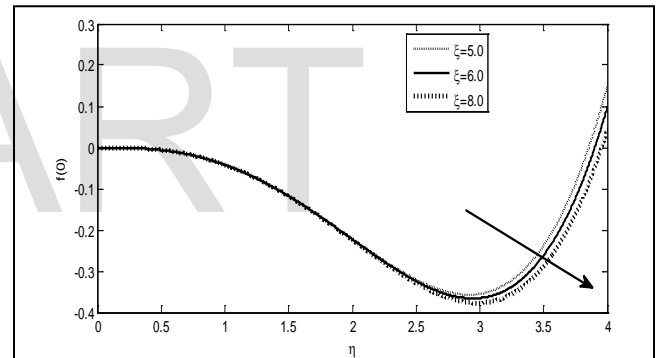


Fig. 4. Velocity profile for various values of ξ and $M=1.0, L=1.0, \gamma=1.0, m=1.0, \delta=5.0, P_r=0.7, D_f=0.1, S_c=0.1, S_r=1.$

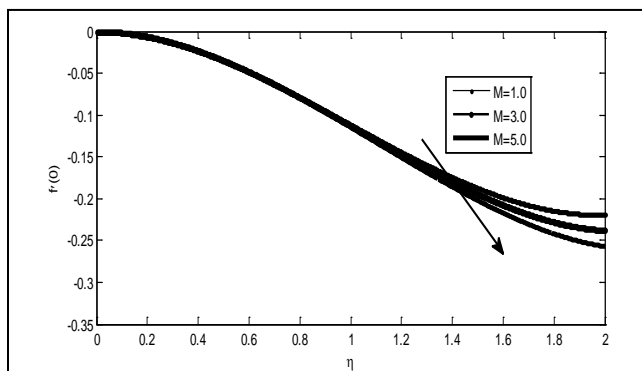


Fig. 2. Velocity profile for various values of M and $m=1.0, L=1.0, \gamma=1.0, \xi=5.0, \delta=5.0, P_r=0.7, D_f=0.1, S_c=0.1, S_r=1.$

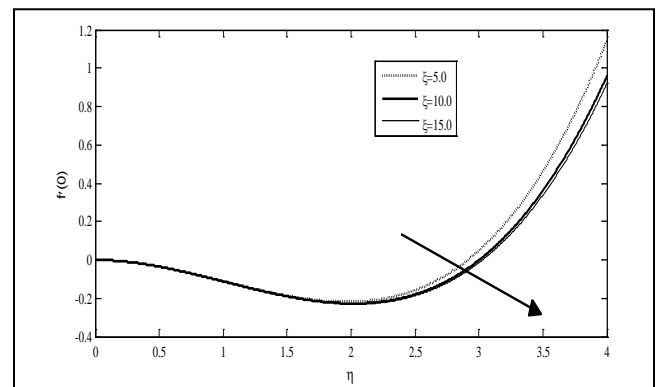


Fig. 5. Velocity profile for various values of ξ and $M=1.0, L=1.0, \gamma=1.0, m=1.0, \delta=5.0, P_r=0.7, D_f=0.1, S_c=0.1, S_r=1.$

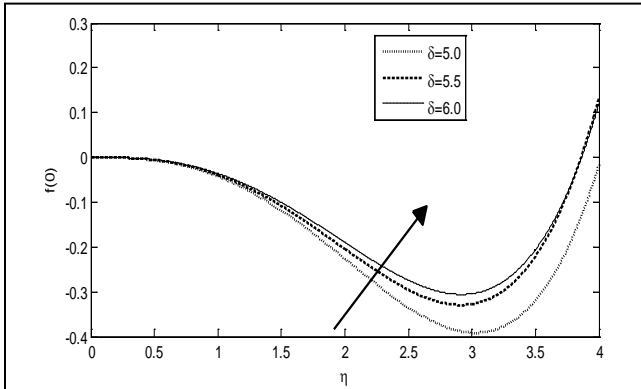


Fig. 6. Velocity profile for various values of δ and $M=1.0, L=1.0, \gamma=1.0, m=1.0, \xi=5.0, P_r=0.7, D_f=0.1, S_c=0.1, S_r=1.0$

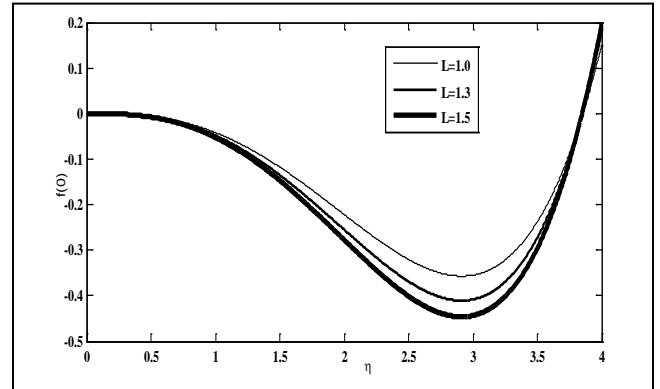


Fig. 9. Velocity profile for various values of L and $M=1.0, \gamma=1.0, \delta=5.0, m=1.0, \xi=5.0, P_r=0.7, D_f=0.1, S_c=0.1, S_r=1.0$

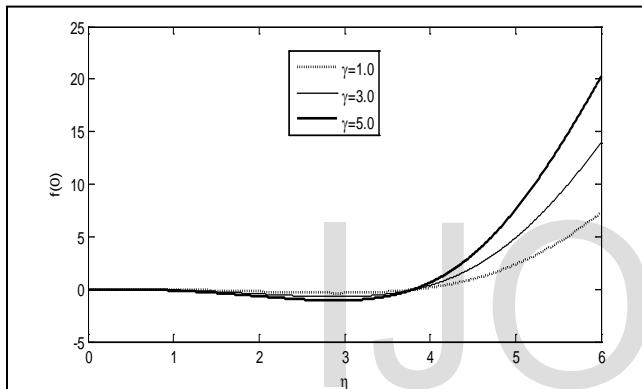


Fig. 7. Velocity profile for various values of γ and $M=1.0, L=1.0, \delta=5.0, m=1.0, \xi=5.0, P_r=0.7, D_f=0.1, S_c=0.1, S_r=1.0$

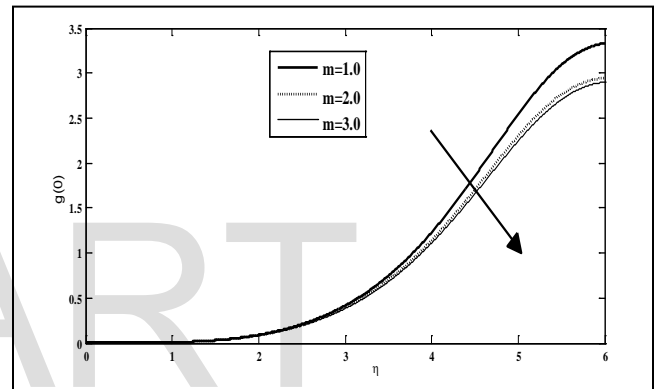


Fig. 10. Secondary velocity profile for various values of m and $M=1.0, \gamma=1.0, \delta=5.0, L=1.0, \xi=5.0, P_r=0.7, D_f=0.1, S_c=0.1, S_r=1.0$

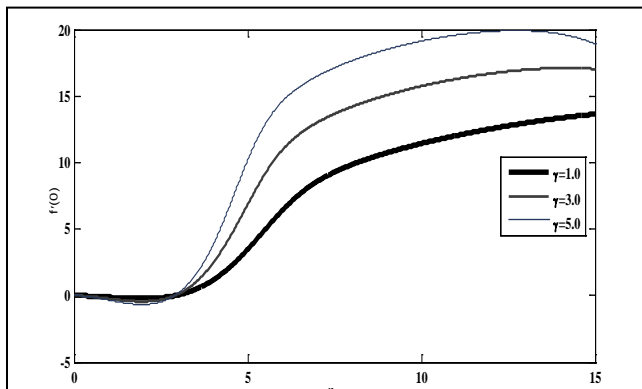


Fig. 8. Velocity profile for various values of γ and $M=1.0, L=1.0, \delta=5.0, m=1.0, \xi=5.0, P_r=0.7, D_f=0.1, S_c=0.1, S_r=1.0$

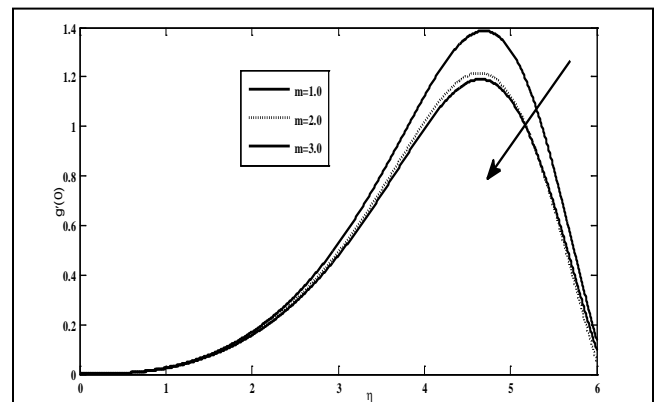


Fig. 11. Secondary velocity profile for various values of m and $M=1.0, \gamma=1.0, \delta=5.0, L=1.0, \xi=5.0, P_r=0.7, D_f=0.1, S_c=0.1, S_r=1.0$

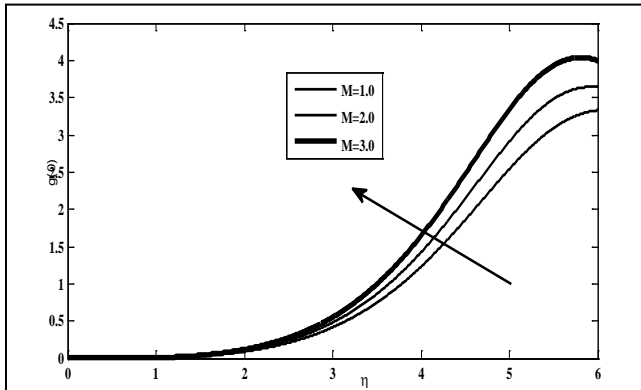


Fig. 12. Secondary velocity profile for various values of M and $L=1.0, \gamma=1.0, \delta=5.0, m=1.0, \xi=5.0, P_r=0.7, D_f=0.1, S_c=0.1, S_r=1.0$

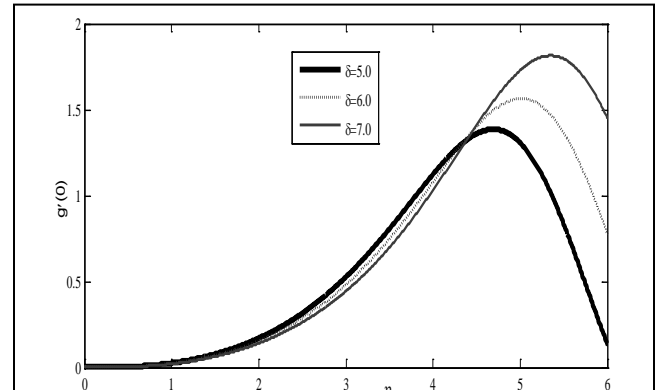


Fig. 15. Secondary velocity profile for various values of δ and $M=1.0, \gamma=1.0, L=1.0, m=1.0, \xi=5.0, P_r=0.7, D_f=0.1, S_c=0.1, S_r=1.0$

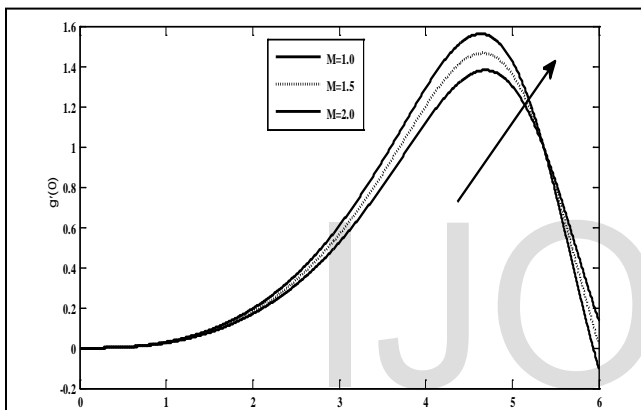


Fig. 13. Secondary velocity profile for various values of M and $L=1.0, \gamma=1.0, \delta=5.0, m=1.0, \xi=5.0, P_r=0.7, D_f=0.1, S_c=0.1, S_r=1.0$

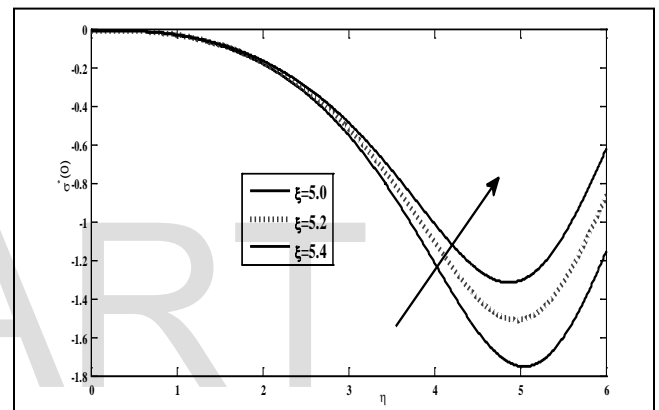


Fig. 16. Micro rotation profile for various values of ξ and $M=1.0, \gamma=1.0, \delta=5.0, m=1.0, L=1.0, P_r=0.7, D_f=0.1, S_c=0.1, S_r=1.0$

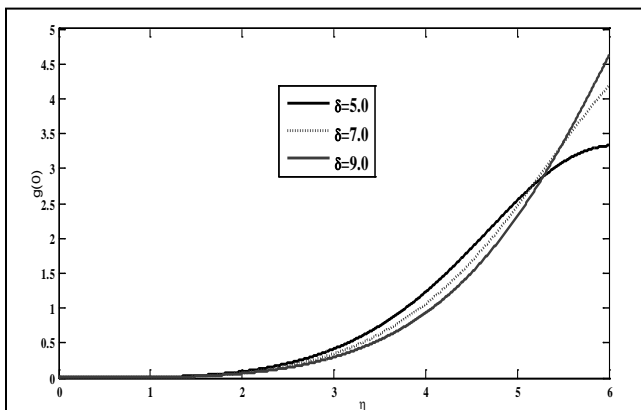


Fig. 14. Secondary velocity profile for various values of δ and $M=1.0, \gamma=1.0, L=1.0, m=1.0, \xi=5.0, P_r=0.7, D_f=0.1, S_c=0.1, S_r=1.0$

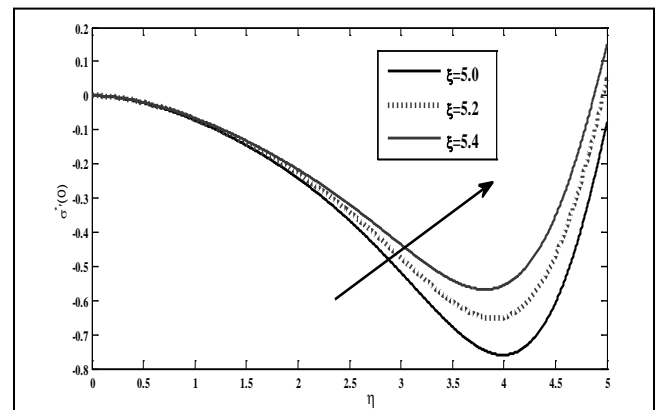


Fig. 17. Micro rotation profile for various values of ξ and $M=1.0, \gamma=1.0, \delta=5.0, m=1.0, L=1.0, P_r=0.7, D_f=0.1, S_c=0.1, S_r=1.0$

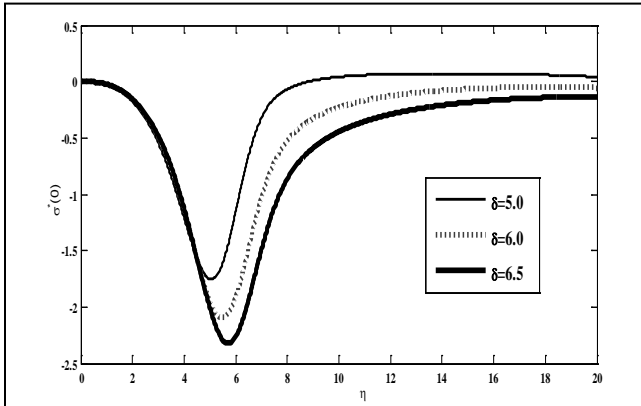


Fig. 18. Micro rotation profile for various values of δ and $M=1.0, \gamma=1.0, L=1.0, m=1.0, \xi=5.0, P_r=0.7, D_f=0.1, S_c=0.1, S_v=1.0$

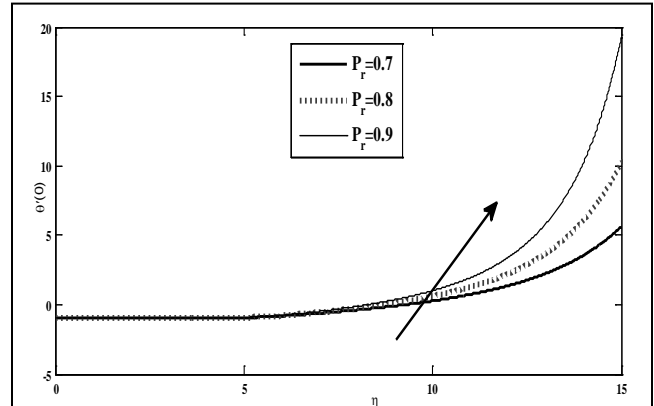


Fig. 21. Temperature profile for various values of P_r and $M=1.0, \gamma=1.0, \delta=5.0, m=1.0, \xi=5.0, L=1.0, D_f=0.1, S_c=0.1, S_v=1.0$

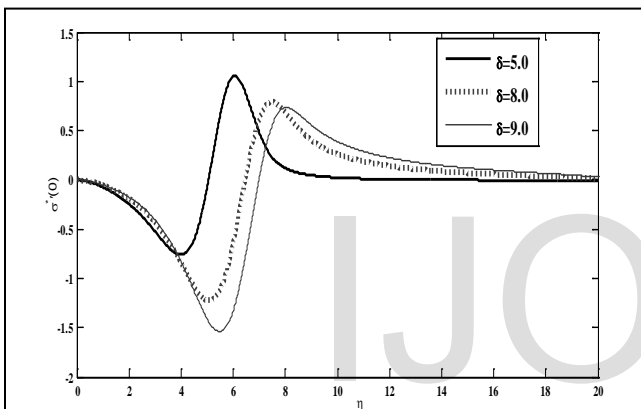


Fig. 19. Micro rotation profile for various values of δ and $M=1.0, \gamma=1.0, L=1.0, m=1.0, \xi=5.0, P_r=0.7, D_f=0.1, S_c=0.1, S_v=1.0$

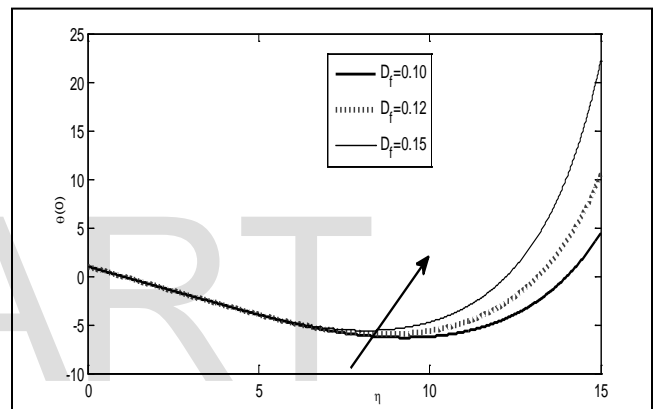


Fig. 22. Temperature profile for various values of D_f and $M=1.0, \gamma=1.0, \delta=5.0, m=1.0, \xi=5.0, P_r=0.7, L=1.0, S_c=0.1, S_v=1.0$

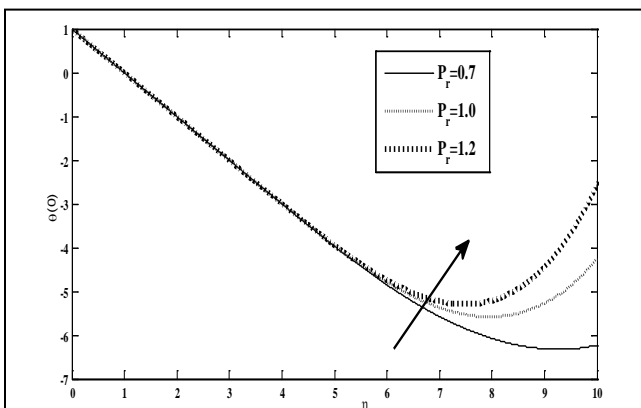


Fig. 20. Temperature profile for various values of P_r and $M=1.0, \gamma=1.0, \delta=5.0, m=1.0, \xi=5.0, L=1.0, D_f=0.1, S_c=0.1, S_v=1.0$

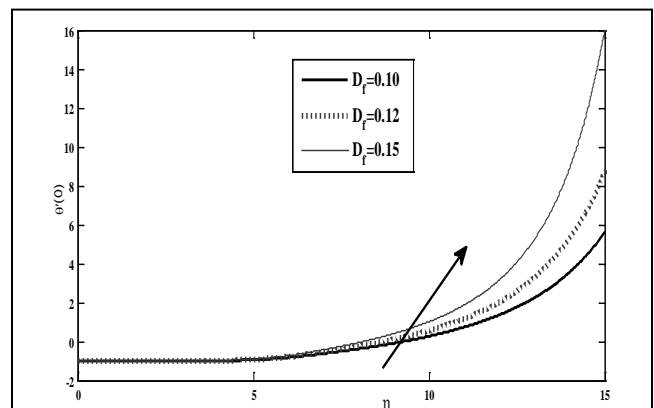
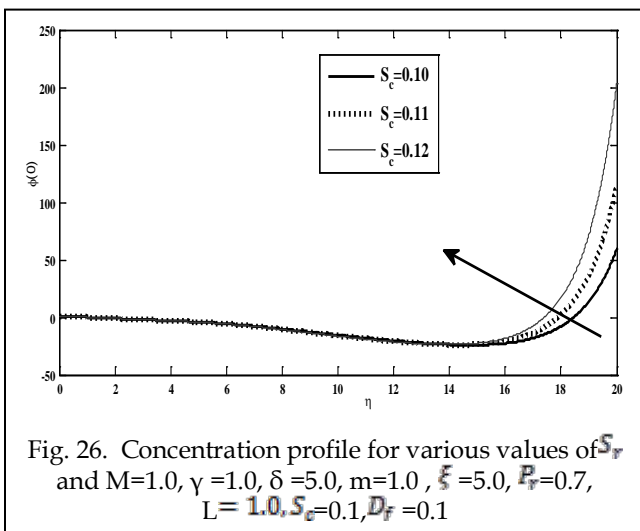
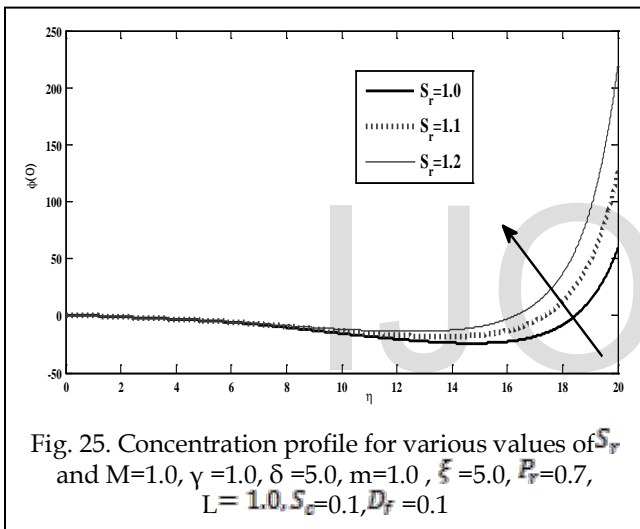
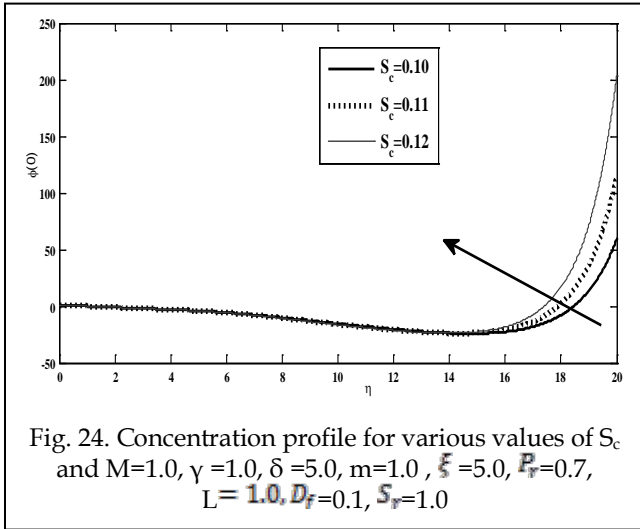


Fig. 23. Temperature profile for various values of D_f and $M=1.0, \gamma=1.0, \delta=5.0, m=1.0, \xi=5.0, P_r=0.7, L=1.0, S_c=0.1, S_v=1.0$



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