Solving an RBC model: Technological and Government Expenditure Shocks*

Etleva Gjelaj

Department of Applied Mathematics, Faculty of Natural Sciences, University of Tirana.
Tirana, Albania
etleva.gjelaj@yahoo.com

ABSTRACT

This paper aims to solve a stochastic Real Business Cycles (RBC) model. Further, this work presents the dynamics of the business cycles, describing the cyclicality of capital, employment and consumption. The baseline method consists in computing the impulse-response functions based on simulated data and determine the effect of technological and government expenditure shocks on the main macroeconomic variables including consumption, output, employment and capital. The results show that capital, labor, consumption and output rise as a result of a technological shock. In contrast, capital and consumption fall in the case of government shock, while output and labor increase.

Keywords : business cycles; economic fluctuation; RBC; shocks; technology; government expenditure;

1 INTRODUCTION

THIS paper provides an empirical solution to the general form of the Real Business Cycle (RBC) model, i.e., without imposing strict assumptions on the mechanisms households, government and firms optimize their decisions. Under those assumptions (see Romer [1]; Gjelaj [2]), omitted in this model, one could solve the model analytically and determine the changes in the endogenous variables as a result of a shock in the exogenous variables. The main issue with the simplified or the special case of the RBC model refers to its non-realistic findings, resulting from the non-realistic features assumed to describe the economy in the model. According to Romer [1], most of the stochastic macroeconomic models, including the RBC models, do not have an analytical solution. This holds even after applying several linear transformations to the motion variables such as capital.

In essence, this paper shows the response of capital, technology, consumption, labor supply and output to 1 percent technological and government expenditure shocks. It is important to note that the data is simulated and the series are generated using the method of underdetermined coefficients.

Business cycles are periodic fluctuations of output and the other macroeconomic variables including consumption, capital stock, government expenditure, investment and labor supply observed in a certain economy [1], [3]. In general, the length of a business cycle lies within the interval of two quarters as the lower bound and 8 years as the upper bound Mankiw [4] argues that the only determinants of economic fluctuations, known to the RBC theory are those that distort the Walrasian equilibrium. Further, the author explains that these forces that cause the economic fluctuations are generally considered as macroeconomic disturbances, i.e., changes in government expenditure, investment tax-credit, and changes in the technological progress among others. Prescott [5] argues that technological shocks are the main drivers of business cycles. However, Rebelo [6] claims that the role of technological shocks in causing economic fluctuations is overestimated. Instead, oil price shocks as well as fiscal and monetary shocks could provide more intuition regarding the macroeconomic fluctuations. Rebelo [6] and Barro [7] explain that the nature of the shock should be exogenous. To this extent, the Total Factor Productivity (TFP) shocks, as used in Prescott [5] estimations, could not be fully exogenous. Nevertheless, this work considers both, fiscal and technological shocks.

Barro [7] explains that an increase in government expenditure leads to an upward shift in the aggregate demand for goods and services. The former does not necessarily tell that consumption will grow. In contrast, the increase in the demand for goods and services is associated with an increase in the real interest rate, which reduces both consumption and investment [6]. Hence, we expect consumption to move down. Regarding output and employment, Mankiw [4] and Romer [1] determine that the shock in government expenditure is expected to increase both output and employment.

With reference to technological shocks, the expected effects are intuitive. When the economy is in recession (economic boom), consumption falls (rises) and leisure rises (falls) [6]. Thus, consumption and leisure move in opposite directions. Mankiw [4] argues that both should fall/rise as both represent normal goods. Similarly, Romer [1] determines that wages will increase owing to the technological progress. If neither consumption, nor employment will change, households can improve their utility by working more hours. This would lead to an increase in the current level of consumption. Hence, leisure would fall, and consumption would rise. Unlike the opposite movements in the case of the fiscal shock, employment and consumption are expected to move to same direction in the case of a technological shock.

This work is organized as follows: Section (2) presents the

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theoretical solution of the General case of the RBC model; Section (3) presents the responses of the variables to a 1 percent technological shock; Section (4) present the responses to a 1 percent government expenditure shock. Section (5) presents the dynamics of the business cycles dynamics and Section (6) concludes.

2 The General Case of the RBC Model

In this section, we consider a stochastic RBC model with technological and government expenditure shocks. Further, let the stock of capital, inherited from the former period, and the actual government expenditure and technology be the state variables. The control or the endogenous variables, over which the representative agent optimizes are consumption and labor supply (employment). In general, the solution to a macro model consists in defining an expression for the choice variables as a function of the state variables [3]. This theoretical consideration applies also to the RBC model. Campbell [8] states that to solve the model, the most appropriate technique is to log-linearize the model around the balanced growth path (BGP) using Taylor's Approximation (usually at order one). In specific, Romer [1] defines consumption and labor supply as follows:

$$\tilde{C}_t \simeq \alpha_{CK}\tilde{K}_t + \alpha_{CA}\tilde{A}_t + \alpha_{CG}\tilde{G}_t \quad (1)$$

$$\tilde{L}_t \simeq \alpha_{LK}\tilde{K}_t + \alpha_{LA}\tilde{A}_t + \alpha_{LG}\tilde{G}_t \quad (2)$$

The α parameters are complicated functions of the parameters of the model. The tilde-defined variable represents the difference between the log of that variable with the log of its balanced growth value. Further consider the log-linearization of next-periods capital and output:

$$\tilde{K}_{t+1} \simeq b_{KK}\tilde{K}_t + b_{KA}\tilde{A}_t + b_{KG}\tilde{G}_t \quad (3)$$

$$\tilde{Y}_t = [\alpha + (1-\alpha)\alpha_{LK}]\tilde{K}_t + (1-\alpha)(1+\alpha_{LA})\tilde{A}_t + (1-\alpha)\alpha_{LG}\tilde{G}_t \quad (4)$$

The $\boldsymbol{b}$ parameters are complicated functions of the parameters of the model and the vector of α parameters. Numerically, it is not hard to compute the parameters (see [1] & [8]). In essence, (1)-(4) present the responses of left hand-side variables as a result of unexpected shocks in the right-hand side variables. In the following sections, we estimate [1]-[4]

3 The Response to 1 % Technological Shock

This section presents the impulse-response functions in the case of technological shocks. We omit the computation of the parameters (displayed in Table (1)).

Figure (1) presents the effect of a 1 percent technological shock on capital, employment and technology. After the shock occurs, capital remains constant, employment rises by 35 percent, and technology increases by 100 percent. Afterwards, the response of A or the Total Factor Productivity (TFP) to its own shock is a monotonically decreasing function. In contrast, the response of capital is concave. In specific, capital increases by 23 percent after 5 quarters, peaks at 60 percent after 19 quarters and declines gradually after 25 quarters, thus returning to its normal state. Regarding the employment, we observe that the effect of the shock becomes negative after 15 quarters, and gradually the shock looses persistence, i.e., employment returns to its normal state.

TABLE 1
Choice of Parameters

Parameters	Description	Calibration
$\boldsymbol{\alpha}$	share of capital	0,5
$\boldsymbol{\rho_A}$	persistency of A	0,95
$\boldsymbol{\rho_G}$	persistency of G	0,95
α_{ij}		
α_{CK}	response parameter of C to K	0,59
α_{CA}	response parameter of C to A	0,38
α_{CG}	response parameter of C to G	-0,13
α_{LK}	response parameter of L to K	-0,31
α_{LA}	response parameter of L to K	0,35
α_{LG}	response parameter of L to K	0,15
b_{ij}		
b_{KK}	response parameter of K to K	0,95
b_{KA}	response parameter of K to A	0,08
b_{KG}	response parameter of K to G	-0,004

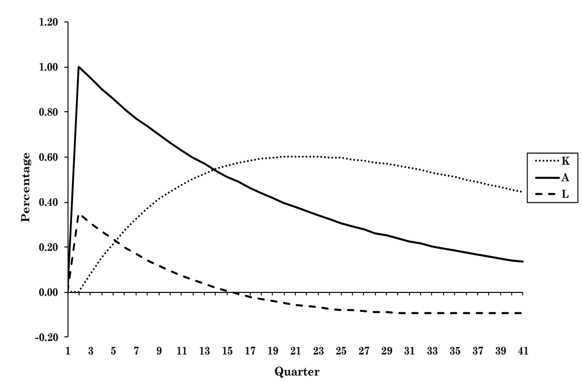

Fig. 1. Response of Capital (K), Employment (E) and Technology (A)

Figure (2) presents the effect of a 1 percent technological shock on output and consumption. During the timing of the shock, consumption increases by 38 percent and output remains constant. This is owing to the null effect on capital. One can alternatively derive the effect on output using the effects on the production factors and technology, under the Cobb-Douglas specification. In the second quarter, output rises by 68 percent. After the second quarter, the response curve of output is strictly decreasing. In contrast, the response function of consumption is concave, i.e., peaks at the roughly 50 percent after 15 quarters and declines afterwards, thus reaching its normal state. The results are highly consistent with the theoretical considerations, i.e., both employment and consumption are positively affected by an unexpected change in technology.

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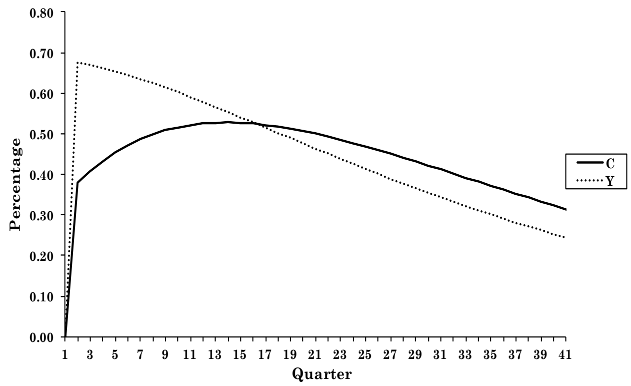

Fig. 2. Response of Consumption (C) and Output (Y)

4 The Response to 1 % Government Expenditure Shock

This section presents the effect of a 1 percent government expenditure shock on consumption, output, employment and capital. To isolate the effect of the technological progress, we set TFP at zero, and generate a sequence of government expenditures as follows: in period one, G equals 1, in period two G equals $\rho_G G_{t-1}$, and so on. Figure (3) presents the response functions of capital, employment and technology to a 1 percent shock in G. We observe that after the shock occurs, employment rises by 15 percent in the second quarter. Afterwards, the response function is monotonically decreasing and convex. Compared to the response to the technological shock, in this case, employment does not fall at any quarter below its normal state. Regarding the response of capital, the effect is low in magnitude and peaks at - 3 percent after 20 quarters.

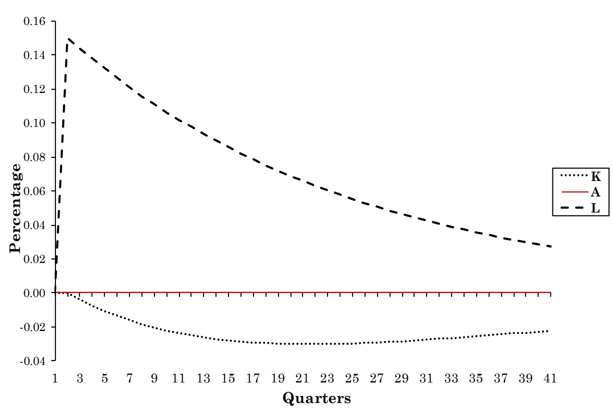

Fig. 3. Response of Capital (K) and Employment (L)

Given that TFP is zero and the low changes in capital, we expect that changes in output will mainly depend on the changes in employment.

Figure (4) presents the responses of output and consumption to 1 percent shock in government expenditure. In the second quarter, or after the shock occurrence period, as output declines by 0.13 percent as a result of the shock, consumption rises by 8 percent. After the second quarter, the response of output is convex and the response of consumption is concave. Further, both variables tend to return to their normal state after 40 quarters. Hence, the responses of both output and employment are similar in shape and magnitude.

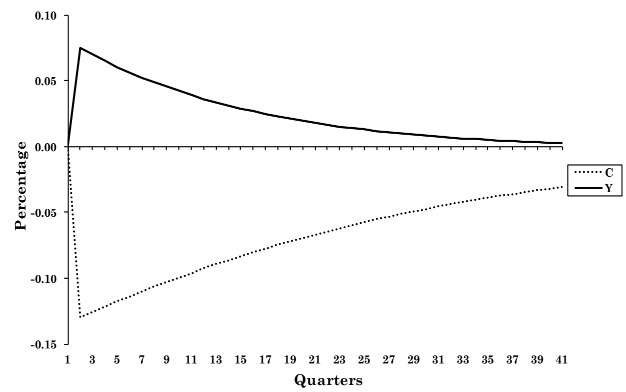

Fig. 4. Response of Capital (K) and Employment (L)

5 The Dynamics of Business Cycles

This section describes the Kaldor's facts in the context of the RBC model based on simulated data of quarterly. Figure (5) presents the simulated series of consumption and output (in logs). We observe that consumption is procyclical, i.e., moves along output or is positively correlated with output. This is consistent with Kaldor's facts (see [9]).

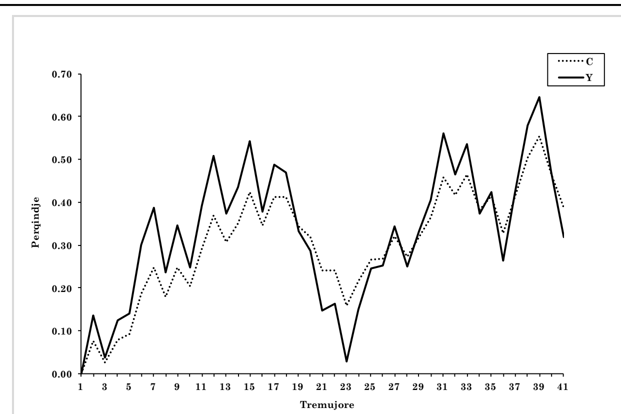

Fig. 5. Output (Y) & Consumption

Figure (6) presents the simulated series of capital, TFP, and employment. Clearly, capital is acyclical, i.e., is neither positively nor negatively correlated with output. The contrary is observed for employment.

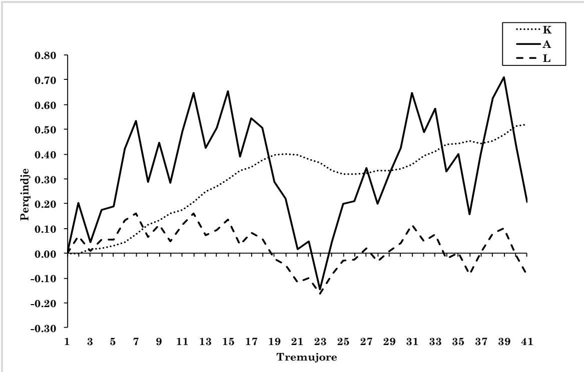

Fig. 6. Employment (E), Capital (K) and TFP (A)

However, the RBC model fails in predicting that employment or labor supply is constant. Stadler [10] provides a critical discussion related to the issues that the RBC models fail to represent the reality. First, the nature of the shocks is such that it affects all sectors and all production factors equally, regardless of their economic features. Second, the model is not subject to econometric tests, e.g., one cannot test the predictive power of the model. Third, the number of restrictions imposed is high. Fourth, the Walrasian features of the model are non realistic. Lastly, it is doubtable whether the RBC models can account for economic recessions.

5 Conclusion

This paper aims to solve empirically a simulated RBC model. In specific, this work highlights the effect of a one percent technological and government expenditure shock on consumption, output, employment and capital. The results show that in the case of the technological shock, consumption and output rise, while in the case of the fiscal shock, consumption falls and output rises. The movements in output are driven by the movements in the production factors. That is, both capital and labor rise after the technological shock occurs. In the case of a shock in government expenditure, capital falls and employment rises. However, the magnitude of the changes in the capital stock is low and thus the changes in output are driven by the changes in employment. The results are in line with the theoretical considerations of the RBC theory. Nevertheless, the theory is partly consistent with the Kaldor's or the stylized facts, i.e., the features of the business cycles do not fully match with the data.

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