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ON HSU- STRUCTURE MANIFOLD, NIJENHUIS TENSOR

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ABSTRACT

In the present paper, various forms of Nijenhuis tensor with respect to Hsu- structure has been defined and some properties of Nijenhuis tensor have also been discussed.

Keywords : Hsu–structure manifold; GF – structure ; Nijenhuis tensor.

1 INTRODUCTION

Consider a differentiable manifold M_n of differentiability class C^∞. Let there be in M_n a vector –valued linear function F of class C^∞,satisfying the algebraic equation

$$F^2=a^r I_n. \tag{1.1}$$

For arbitrary vector field X, equation (1.1) is expressed as

$$\bar{\bar{X}} = a^r X, \tag{1.2}$$

where $\bar{X} \overset{\text{def}}{=} FX$, 'r' is an integer and 'a' is a real or imaginary number. Then $\{F\}$ gives to M_n a Hsu– structure and the manifold M_n is called Hsu–structure manifold.The equation (1.1) gives different structures for different values of 'a' and 'r'.If r =0, it is an almost product structure; if a = 0, it is an almost tangent structure; if r = $\pm$1 and a = -1, it is an almost complex structure; if r = $\pm$1 and a = +1, it is an almost product structure; if r = 2 then it is a GF – structure which includes π - structure for a $\neq$ 0,an almost complex structure for a = $\pm$i, an almost product structure for a = $\pm$1, an almost tangent structure for a = 0.

Let the Hsu – structure be endowed with a metric tensor g, such that

$$g(\bar{X},\bar{Y}) + a^r g(X,Y) = 0.$$

Then (F,G) is said to give to M_n a metric Hsu – structure and M_n is called a metric Hsu – structure manifold.

2 NIJENHUIS TENSOR

A bilinear function B in Hsu- structure manifold is said to be pure in the two slots,if

$$B(\bar{X},\bar{Y}) - a^r B(X,Y) = 0. \tag{2.1}$$

It is said to be hybrid in the two slots,if

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$B(\bar{X},\bar{Y}) + a^r B(X,Y) = 0.$ (2.2)

The Nijenhuis tensor with respect to F is a vector valued bilinear function N, defined by

$N(X,Y) \overset{\text{def}}{=} [\bar{X},\bar{Y}] + a^r[X,Y] - \overline{[X,\bar{Y}]} - \overline{[\bar{X},Y]}.$ (2.3)

Where $[X,Y]= D_X Y - D_Y X$ and D is Riemannian connexion.

Theorem (2.1). In a Hsu-structure manifold, we have

$N(X,Y) = - N(Y,X) = [\bar{X},\bar{Y}] + a^r[X,Y] - \overline{[X,\bar{Y}]} - \overline{[\bar{X},Y]},$ (2.4)a

i.e. N is skew- symmetric in X and Y .

$\overline{N(X,Y)} = - \overline{N(Y,X)} = \overline{[\bar{X},\bar{Y}]} + a^r\overline{[X,Y]} - a^r[X,\bar{Y}] - a^r[\bar{X},Y],$ (2.4)b

$N(\bar{X},Y) = N(X,\bar{Y}) = a^r[X,\bar{Y}] + a^r[\bar{X},Y] - \overline{[\bar{X},\bar{Y}]} - a^r\overline{[X,Y]},$ (2.4)c

$\overline{N(\bar{X},Y)} = \overline{N(X,\bar{Y})} = a^r\overline{[X,\bar{Y}]} + a^r\overline{[\bar{X},Y]} - a^r[\bar{X},\bar{Y}] - a^{2r}[X,Y],$ (2.4)d

$N(\bar{X},\bar{Y}) = a^r N(X,Y) = a^{2r}[X,Y] + a^r[\bar{X},\bar{Y}] - a^r\overline{[X,\bar{Y}]} - a^r\overline{[\bar{X},Y]},$ (2.4)e

i.e. N is pure in X and Y.

$\overline{N(\bar{X},\bar{Y})} = a^r\overline{N(X,Y)} = a^{2r}\overline{[X,Y]} + a^r\overline{[\bar{X},\bar{Y}]} - a^{2r}[X,\bar{Y}] - a^{2r}[\bar{X},Y].$ (2.4)f

Consequently

$N(\bar{X},\bar{Y}) = a^r N(X,Y) = -\overline{N(\bar{X},Y)} = - \overline{N(X,\bar{Y})},$ (2.5)a

$N(\bar{X},Y) = N(X,\bar{Y}) = - \overline{N(X,Y)} = \overline{N(Y,X)}.$ (2.5)b

Proof. Interchanging X and Y in equation (2.3), we get the equation (2.4)a.Barring equation (2.4)a throughout and using the equation (1.1) , we get the equation (2.4)b. Similarly we can prove the equations (2.4)c, (2.4)d, (2.4)e and(2.4)f.

The equation (2.5)a is obtained from the equations (2.4)d and (2.4)e. The equations (2.4)b and (2.4)c yields the equation (2.5)b.

Theorem(2.2). Let us put

$P(X,Y) \overset{\text{def}}{=} [\bar{X},\bar{Y}] - \overline{[\bar{X},Y]}.$ (2.6)

Then

$\overline{P(X,\bar{Y})} = - a^r P(X,Y)$

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$$= a^r \left(\overline{\left[\overline{X}, Y\right]} - \left[\overline{X}, \overline{Y}\right]\right) \quad \text{(2.7)a}$$

$$\overline{P\left(\overline{X}, \overline{Y}\right)} = -\ a^r\ P\left(\overline{X}, Y\right)$$

$$= a^{2r}\ \left(\overline{\left[X, Y\right]} - \left[X, \overline{Y}\right]\right), \quad \text{(2.7)b}$$

$$\overline{P\left(X, Y\right)} = -P\left(X, \overline{Y}\right)$$

$$= \left(\overline{\left[\overline{X}, \overline{Y}\right]} - a^r \left[\overline{X}, Y\right]\right), \quad \text{(2.7)c}$$

$$P\left(\overline{X}, \overline{Y}\right) = -\ \overline{P\left(\overline{X}, Y\right)}$$

$$= a^r \left(a^r \left[X, Y\right] - \overline{\left[X, \overline{Y}\right]}\right). \quad \text{(2.7)d}$$

Consequently

$$P\left(\overline{X}, \overline{Y}\right) + a^r\ P(X,Y) = N(\bar{X}, \bar{Y}) = a^r N(X, Y), \quad \text{(2.8)a}$$

i.e. P is hybrid in X and Y.

$$\overline{P\left(\overline{X}, \overline{Y}\right)} + a^r\ \overline{P\left(X, Y\right)} = \overline{N\left(\overline{X}, \overline{Y}\right)}, \quad \text{(2.8)b}$$

$$P\left(\overline{X}, Y\right) + P\left(X, \overline{Y}\right) = N\left(X, \overline{Y}\right), \quad \text{(2.8)c}$$

$$\overline{P\left(\overline{X}, Y\right)} + \overline{P\left(X, \overline{Y}\right)} = \overline{N\left(\overline{X}, Y\right)}. \quad \text{(2.8)d}$$

Proof. Barring equation (2.6) throughout or different vectors in it and using equation (1.1), we get the equations (2.7)a,…. (2.7)d. Again subtracting equation (2.7)a from equation(2.7)d,we get the equation(2.8)a. Similarly, we can prove the equations (2.8)b,(2.8)c and (2.8)d.

Theorem(2.3). Let us put

$$Q(X,Y) \overset{\text{def}}{=} a^r \left[X, Y\right] - \overline{\left[X, \overline{Y}\right]}. \quad \text{(2.9)}$$

Then

$$Q\left(X, \overline{Y}\right) = -\overline{Q\left(X, Y\right)}$$

$$= a^r \left(\left[X, \overline{Y}\right] - \overline{\left[X, Y\right]}\right), \quad \text{(2.10)a}$$

$$\overline{Q\left(X, \overline{Y}\right)} = -\ a^r\ Q\left(X, Y\right)$$

$$= a^r \left(\overline{\left[X, \overline{Y}\right]} - a^r \left[X, Y\right]\right), \quad \text{(2.10)b}$$

$$\overline{Q\left(\overline{X}, Y\right)} = -\ Q\left(\overline{X}, \overline{Y}\right)$$

$$= a^r \left(\overline{\left[\overline{X}, Y\right]} - \left[\overline{X}, \overline{Y}\right]\right), \quad \text{(2.10)c}$$

$$\overline{Q\left(\overline{X}, \overline{Y}\right)} = -a^r Q\left(\overline{X}, Y\right)$$

$$= a^r \left(\overline{\left[\overline{X}, \overline{Y}\right]} - a^r \left[\overline{X}, Y\right]\right). \quad \text{(2.10)d}$$

Consequently

$$Q\left(\overline{X}, \overline{Y}\right) + a^r\ Q\left(X, Y\right) = N\ (\bar{X}, \bar{Y})$$

$$= a^r\ N\ (X, Y), \quad \text{(2.11)a}$$

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i.e. Q is hybrid in X and Y.

$$\overline{Q(\overline{X},\overline{Y})} + a^r\,\overline{Q(X,Y)} = \overline{N(\overline{X},\overline{Y})}, \quad (2.11)b$$

$$Q(\overline{X},Y) + Q(X,\overline{Y}) = N(X,\overline{Y}), \quad (2.11)c$$

$$\overline{Q(\overline{X},Y)} + \overline{Q(X,\overline{Y})} = \overline{N(\overline{X},Y)}. \quad (2.11)d$$

Proof. Barring equation (2.9) throughout or different vectors in it and using equation (1.1), we get the equations (2.10)a,- - - - - (2.10)d. Again adding equation (2.10)b and equation(2.10)c,we get the equation(2.11)a. Similarly, we can prove the equations (2.11)b,(2.11)c and (2.11)d.

Corollary (2.1). We have in a Hsu - structure manifold

$$P(X,\bar{Y}) = Q(\bar{X},Y), \quad (2.12)a$$

$$P(\bar{X},Y) = Q(X,\bar{Y}), \quad (2.12)b$$

$$P(\bar{X},\bar{Y}) = a^r Q(X,Y), \quad (2.12)c$$

$$a^r\overline{P(X,Y)} = \overline{Q(\bar{X},\bar{Y})}, \quad (2.12)d$$

Proof. The statement follows from equations (2.7) and (2.10) .

Corollary (2.2). We have in a Hsu - structure manifold

$$P(X,Y) + Q(X,Y) = N(X,Y), \quad (2.13)a$$

$$P(\overline{X},Y) + Q(\overline{X},Y) = N(X,\overline{Y}), \quad (2.13)b$$

$$\overline{P(\overline{X},Y)} + \overline{Q(\overline{X},Y)} = \overline{N(\overline{X},Y)}, \quad (2.13)c$$

$$P(\overline{X},\overline{Y}) - \overline{Q(\overline{X},Y)} = N(\overline{X},\overline{Y}). \quad (2.13)d$$

Proof. The statement follows from equations (2.7) , (2.10) and (2.5) .

Theorem(2.4). Let us put

$$V(X,Y) \stackrel{\text{def}}{=} [\overline{\bar{X},Y}] + [\overline{X,\bar{Y}}]. \quad (2.14)$$

Then

$$V(X,Y) = -V(Y,X)$$

$$= [\overline{\bar{X},Y}] + [\overline{X,\bar{Y}}]. \quad (2.15)a$$

i.e. V is skew-symmetric in X and Y.

$$V(\bar{X},Y) = V(X,\bar{Y})$$

$$= a^r[\overline{X,Y}] + [\overline{\bar{X},\bar{Y}}], \quad (2.15)b$$

$$\overline{V(\bar{X},Y)} = \overline{V(X,\bar{Y})},$$

$$= a^r(a^r[X,Y] + [\bar{X},\bar{Y}]) \quad (2.15)c$$

$$V(\bar{X},\bar{Y}) = a^r V(X,Y)$$

$$= a^r([\overline{X,\bar{Y}}] + [\overline{\bar{X},Y}]), \quad (2.15)d$$

i.e.V is pure in X and Y.

$$\overline{V(\bar{X},\bar{Y})} = a^r\overline{V(X,Y)}$$

$$= a^{2r}([X,\bar{Y}] + [\bar{X},Y]), \quad (2.15)e$$

Consequently

$$\overline{V(X,Y)} - V(\bar{X},Y) = N(X,\bar{Y}), \quad (2.16)a$$

$V(\bar{X},\bar{Y}) - \overline{V(X,\bar{Y})} = \overline{N(\bar{X},Y)},$ (2.16)b

$\overline{V(\bar{X},\bar{Y})} - a^r V(\bar{X},Y) = a^r N(\bar{X},Y),$ (2.16)c

$\overline{V(\bar{X},Y)} - a^r V(X,Y) = N(\bar{X},\bar{Y}).$ (2.16)d

Proof. Interchanging X and Y in equation (2.14), we get the equation (2.15)a. Barring equation (2.14) throughout or different vectors in it and using equation(1.1), we get the equations (2.15)b,….. (2.15)e.

Again, using equations (1.1) , (2.15) and (2.4) , we get the equations (2.16)a ,…., (2.16)d.

Theorem(2.5). Let us put

$R(X,Y) \stackrel{\text{def}}{=} [\bar{X},\bar{Y}] + a^r[X,Y],$ (2.17)

Then

$R(\bar{X},Y) = R(X,\bar{Y})$

$= a^r[\bar{X},Y] + a^r[X,\bar{Y}],$ (2.18)a

$\overline{R(\bar{X},Y)} = \overline{R(X,\bar{Y})}$

$= a^r\overline{[\bar{X},Y]} + a^r\overline{[X,\bar{Y}]},$ (2.18)b

$R(\bar{X},\bar{Y}) = a^r R[X,Y]$

$= a^{2r}[X,Y] + a^r[\bar{X},\bar{Y}].$ (2.18)c

i.e. R is pure in X and Y.

$\overline{R(\bar{X},\bar{Y})} = a^r\overline{R[X,Y]}$

$= a^{2r}\overline{[X,Y]} + a^r\overline{[\bar{X},\bar{Y}]},$ (2.18)d

$R(X,Y) = -R[Y,X]$

$= [\bar{X},\bar{Y}] + a^r[X,Y].$ (2.18)e

i.e. R is skew-symmetric in X and Y.

Consequently

$R(\bar{X},Y) - \overline{R(X,Y)} = N(\bar{X},Y),$ (2.19)a

$\overline{R(X,\bar{Y})} - R(\bar{X},\bar{Y}) = \overline{N(X,\bar{Y})},$ (2.19)b

$\overline{R(\bar{X},\bar{Y})} - a^r R(\bar{X},Y) = \overline{N(\bar{X},\bar{Y})},$ (2.19)c

$a^r R(X,Y) - \overline{R(\bar{X},Y)} = N(\bar{X},\bar{Y}).$ (2.19)d

Proof. The proof follows the pattern of the proof of the theorem(2.4).

Corollary(2.3). We have in a Hsu-structure manifold.

$\overline{V(\bar{X},Y)} = R(\bar{X},\bar{Y}),$ (2.20)a

$V(\bar{X},\bar{Y}) = \overline{R(X,\bar{Y})},$ (2.20)b

$\overline{V(X,Y)} = R(X,\bar{Y}),$ (2.20)c

$V(\bar{X},\bar{Y}) = \overline{R(X,Y)}.$ (2.20)d

Proof. The statement follows from equations (2.15),(2.18) and (1.1).

Corollary(2.4). We have in a Hsu-structure manifold.

$R(X,Y) - V(X,Y) = N(X,Y),$ (2.21)a

$R(\bar{X},Y) - V(X,\bar{Y}) = N(\bar{X},Y),$ (2.21)b

$\overline{R(\bar{X},\bar{Y})} - a^r\overline{V(X,Y)} = \overline{N(\bar{X},\bar{Y})},$ (2.21)c

$a^r R(X,Y) - V(\bar{X},\bar{Y}) = N(\bar{X},\bar{Y}).$ (2.21)d

Proof.From equation (2.3), we have

$$N(X,Y) = [\bar{X},\bar{Y}] + a^r[X,Y] - \overline{[X,\bar{Y}]} - \overline{[\bar{X},Y]}. \quad (2.22)$$

Using equations (2.14) and (2.17) in equation (2.22), we get the equation(2.21)a . Similarly, we can prove the equations (2.21)b,(2.21)c and (2.21)d.

Remark(2.1). From equations (2.13)a and (2.21)a, it is clear that the sum of two hybrid functions and the subtraction of two pure functions is a pure function.

Theorem(2.6). If we put

$$`N(X,Y,Z) = -a^r g(N(X,Y),Z) = -g(N(\bar{X},\bar{Y}),Z), \quad (2.23)$$

called associate Nijenhuis tensor of the type (0,3).

Then

$$`N(X,Y,Z) = -`N(Y,X,Z), \quad (2.24)a$$

i.e. ‘N is skew-symmetric in X and Y.

$$`N(\bar{X},Y,Z) = `N(X,\bar{Y},Z) = `N(X,Y,\bar{Z}) \quad (2.24)b$$

$$`N(\bar{X},\bar{Y},Z) = `N(\bar{X},Y,\bar{Z}) = `N(X,\bar{Y},\bar{Z}) = a^{r'}N(X,Y,Z). \quad (2.24)c$$

i.e. ‘N is pure in any two of the three slots.

Proof. From equations (2.4)a and (2.23), we get the equation (2.24)a, which shows that ‘N is skew-symmetric in X and Y. Using equation (2.23) in equation (2.5)b , we have the equation (2.24)b. From equations (2.5)a and (2.23), we get the equation (2.24)c. Which shows that ‘N is pure in any two of the three slots.

Corollary(2.5). Let us define.

$$`P(X,Y,Z) \stackrel{\text{def}}{=} g(P(X,Y),Z). \quad (2.25)$$

$$`P(X,\bar{Y},\bar{Z}) = a^{r\prime}P(X,Y,Z), \quad (2.26)a$$

i.e. ‘P is pure in Y and Z.

$$`P(\bar{X},\bar{Y},Z) = \quad 'P(\bar{X},Y,\bar{Z}), \quad (2.26)b$$

and

$$a^r`P(\bar{X},\bar{Y},Z) + a^{2r}`P(X,Y,Z) = -`N(\bar{X},\bar{Y},Z) = -a^r`N(X,Y,Z), \quad (2.27)a$$

$$a^r`P(\bar{X},\bar{Y},\bar{Z}) + a^{2r}`P(X,Y,\bar{Z}) = -`N(\bar{X},\bar{Y},\bar{Z}). \quad (2.27)b$$

Proof. Using equations (2.7)a, (2.7)d and (2.25) , we get the equations (2.26)a and

(2.26)b. From equation (2.26)a it is clear that ‘P is pure in Y and Z. Barring X and Y in equation (2.7)a and using equations (2.23) and (2.25), we get the equation (2.27)a. Barring Z in equation (2.27)a, we get the equation(2.27)b.

Remark(2.2). If a ≠0, using the fact that ‘N is pure in X and Y in equation (2.27)a, we get

$$‘P(\bar{X},\bar{Y},Z) = -a^{r}‘P(X,Y,Z). \qquad (2.28)$$

i.e.’P is hybrid in X and Y.

From equations (2.26)b and (2.28), we get

$$‘P(\bar{X},Y,\bar{Z}) = -a^{r}‘P(X,Y,Z).$$

i.e. ‘P is hybrid in X and Z.

Corollary (2 .6). Let us define

$$‘Q(X,Y,Z) \stackrel{\text{def}}{=} g(Q(X,Y),Z). \qquad (2.29)$$

Then

$$‘Q(X,\bar{Y},\bar{Z}) = a^{r}{}'Q(X,Y,Z). \qquad (2.30)\text{a}$$

i.e. ‘Q is pure in Y and Z.

$$‘Q(\bar{X},Y,\bar{Z}) = {}'Q(\bar{X},\bar{Y},Z). \qquad (2.30)\text{b}$$

and

$$a^{r}‘Q(\bar{X},\bar{Y},Z) + a^{2r}‘Q(X,Y,Z) =$$
$$-‘N(\bar{X},\bar{Y},Z) = -a^{r}‘N(X,Y,Z), \qquad (2.31)\text{a}$$
$$a^{r}‘Q(\bar{X},\bar{Y},\bar{Z}) + a^{2r}‘Q(X,Y,\bar{Z}) =$$
$$-‘N(\bar{X},\bar{Y},\bar{Z}), \qquad (2.31)\text{b}$$

Proof. Using equations (2.10)b, (2.10)c and (2.29), we get the equations (2.30). The equation (2.30)a shows that ‘Q is pure in Y and Z. Barring X and Y in equation (2.11)a and using equations (2.23) and (2.29), we get equation (2.31)a. Barring Z in equation (2.31)a, we get the equation (2.31)b.

Remark (2.3). If $a \neq 0$, using the fact that ‘N is pure in X and Y in equation (2.31)a then from the resulting equation and equation (2.30)b, we get

$$‘Q(\bar{X},\bar{Y},Z) = {}'Q(\bar{X},Y,\bar{Z})$$
$$= -a^{r}‘Q(X,Y,Z) \qquad (2.32)$$

i.e.’Q is hybrid in X,Y and X,Z.

Corollary (2 .7). Let us define

$$‘V(X,Y,Z) \stackrel{\text{def}}{=} g(V(X,Y),Z). \qquad (2.33)$$

Then

$$‘V(\bar{X},Y,\bar{Z}) = {}'V(X,\bar{Y},\bar{Z}), \qquad (2.34)\text{a}$$

$'V(\bar{X},\bar{Y},Z) = a^r{}'V(X,Y,Z).$ (2.34)b

i.e. 'V is pure in X and Y.

and

$'V(\bar{X},\bar{Y},\bar{Z}) + a^r{}'V(\bar{X},Y,Z)$

$= {}'N(\bar{X},Y,Z),$ (2.35)a

$a^r{}'V(\bar{X},Y,\bar{Z}) + a^{2r}{}'V(X,Y,Z) =$

$a^r{}'N(X,Y,Z) ={}' N(\bar{X},\bar{Y},Z).$ (2.35)b

Proof. Using equations (2.15)c, (2.15)d and (2.33), we get the equation (2.34)a and (2.34)b. The equation (2.34)b shows that 'V is pure in X and Y. Using equations (2.16)c, (2.16)d, (2.23) and (2.33) ,we get the equations (2.35)a and (2.35)b.

Remark(2.4). If $a \neq 0$, then using the fact that 'N is pure in any two of the three slots in equation (2.35)b, we get

$'V(\bar{X},Y,\bar{Z}) = -a^r{}'V\,(X,Y,Z),$ (2.36)

i.e. 'V is hybrid in X and Z.

From equations (2.34)a and (2.36), we get

$'V(X,\bar{Y},\bar{Z}) = -a^r{}'V\,(X,Y,Z),$ (2.37)

i.e . 'V is hybrid in Y and Z.

Corollary (2 .8) . Let us define

$'R(X,Y,Z) \overset{\text{def}}{=} g(R(X,Y),Z).$ (2.38)

Then

$'R(\bar{X},Y,\bar{Z}) = {}'R(X,\bar{Y},\bar{Z}),$ (2.39)a

$'R(\bar{X},\bar{Y},Z) = a^r{}'R(X,Y,Z).$ (2.39)b

i.e. 'R is pure in X and Y.

and

$'R(\bar{X},\bar{Y},\bar{Z}) + a^r{}'R(\bar{X},Y,Z) =$

$-'N\,(X,Y,\bar{Z}),$ (2.40)a

$a^{2r}{}'R(X,Y,Z) + a^r{}'R(\bar{X},Y,\bar{Z}) =$

$-'N\,(\bar{X},\bar{Y},Z) = -a^{r}{}'N(X,Y,Z).$ (2.40)b

Proof. Using equations (2.18)b, (2.18)c and (2.38) , we get the equations (2.39). Equation (2.39)b shows that 'R is pure in X and Y. Using equations (2.19)c, (2.19)d , (2.23) and (2.38) ,we get the equations(2.40).

Remark(2.5). If $a \neq 0$ then using the fact that 'N is pure in X and Y in equation (2.40)b, we get

$'R(\bar{X},Y,\bar{Z}) = -a^r{}'R\,(X,Y,Z).$ (2.41)

i.e. 'R is hybrid in X and Z. From equation (2.39)a and (2.41), we get

$'R(X,\bar{Y},\bar{Z}) = -a^r{}'R\,(X,Y,Z).$

i.e. ‘R is hybrid in Y and Z.

Corollary(2.9). In the Hsu-structure manifold the associate Nijenhuis tensor ‘N(X,Y,Z) can be put in the form

$$'P(\bar{X},\bar{Y},Z) + a^r{}'P(X,Y,Z) = -'N(X,Y,Z), \quad (2.42)a$$

$$'Q(\bar{X},\bar{Y},Z) + a^r{}'Q(X,Y,Z) = -'N(X,Y,Z), \quad (2.42)b$$

$$-'P(\bar{X},\bar{Y},Z) - 'Q(\bar{X},Y,\bar{Z}) = 'N(X,Y,Z), \quad (2.42)c$$

$$'V(\bar{X},Y,\bar{Z}) + a^r{}'V(X,Y,Z) = 'N(X,Y,Z), \quad (2.42)d$$

$$-a^r{}'R(X,Y,Z) - 'R(\bar{X},Y,\bar{Z}) = 'N(X,Y,Z), \quad (2.42)e$$

$$'V(\bar{X},\bar{Y},Z) - a^r{}'R(X,Y,Z) = 'N(X,Y,Z). \quad (2.42)f$$

Proof. Using equations (2.25),(2.23) in equation (2.8)a, we get the equation (2.42)a. Using equations (2.29),(2.23) in equation (2.11)a, we get equation (2.42)b. Using equations (2.25), (2.29) and (2.23) in equation (2.13)d, we get equation(2.42)c.From equations (2.16)d, (2.33) and (2.23), we get the equation (2.42)d. Using equation(2.38) and (2.23) in equation(2.19)a, we get the equation (2.42)e. From equations (2.21)d, (2.33), (2.38) and (2.23), we get the equation (2.42)f.

Corollary(2.10). In the Hsu-structure manifold, we have

$$'P(\bar{X},\bar{Y},\bar{Z}) + a^r{}'P(X,Y,\bar{Z}) = a^r{}'R(X,Y,\bar{Z}) + a^r{}'R(\bar{X},Y,Z), \quad (2.43)a$$

$$a^r{}'Q(X,\bar{Y},Z) + a^r{}'Q(\bar{X},Y,Z) + a^r{}'V(X,Y,\bar{Z}) + a^r{}'V(\bar{X},Y,Z) = 0, \quad (2.43)b$$

$$'P(\bar{X},\bar{Y},\bar{Z}) + a^r{}'P(X,Y,\bar{Z}) = a^r{}'Q(X,\bar{Y},Z) + a^r{}'Q(\bar{X},Y,Z), \quad (2.43)c$$

$$a^r{}'V(X,Y,\bar{Z}) + a^r{}'V(\bar{X},Y,Z) + a^r{}'R(X,Y,\bar{Z}) + a^r{}'R(\bar{X},Y,Z) = 0. \quad (2.43)d$$

Proof. Using equations (2.25) and (2.23) in equation (2.8)b, we get

$$'P(\bar{X},\bar{Y},\bar{Z}) + a^r{}'P(X,Y,\bar{Z}) = -'N(X,Y,\bar{Z}). \quad (2.44)$$

Using equations (2.38) and (2.23) in equation (2.19)b, we get

$$a^{r\prime}R(X,Y,\bar{Z}) + a^{r\prime}R(\bar{X},Y,Z)$$

$= -'N(X, Y, \bar{Z})$. (2.45)

From equations (2.44) and (2.45), we get the equation (2.43)a. Similarly, we can prove the remaining equations.

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