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Non-Split Restrained Dominating Set of an Interval Graph Using an Algorithm

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ABSTRACT

Among the various applications of the theory of Restrained domination, the most often discussed is communication network. There has been persistent in the Algorithmic aspects of interval graphs in past decades spurred much by their numerous application of an interval graphs corresponding to an interval family I. A set $D \subseteq V(G)$ is a Restrained dominating set of a graph G, if every vertex not in D is adjacent to a vertex in D and to a vertex in $V-D$. In graph theory, a connected component of an undirected graph is a subgraph in which any two vertices are connected to each other by paths. For a graph G, if the induced subgraph of G itself is a connected component then the graph G is called connected. A Restrained dominating set RDS of a graph $G(V,E)$ is a Non-split restrained dominating set, if the induced subgraph $<V-RDS>$ is connected. In this paper we introduce an Algorithm to find a Non-split Restrained dominating set of an interval graph .

Key words:

Interval family, interval graph, connected graph, restrained dominating set, Non-split restrained dominating set.

1. INTRODUCTION

The research of the domination in graphs has been an evergreen of the graph theory. Its basic concept is the dominating set and the domination number. The theory of domination in graphs was introduced by Ore [1] and Berge [2]. A survey on results and applications of dominating sets was presented by E.J.Cockayane and S.T.Hedetniemi [3]. In 1997 Kulli et.al introduced the concept of Non-split domination [4] and studied these parameters for various standard graphs and obtained the bounds for these parameters.

In general an undirected graph $G=(V,E)$ is an interval graph(IG), if the vertex set V can be put into one-to-one correspondence with a set of intervals I on the real line R, such that two vertices are adjacent in G, if and only if their corresponding intervals have non-empty intersection. The set I is called an interval representation of G and G is referred to as the intersection graph I. Let $I = I_1, I_2, I_3, I_4, ...I_n$ be any interval family where, each I_i is an interval on the real line and $I_i = a_i, b_i$ for $i=1,2,3,4,..........n$. Here a_i is called the left end point labeling and b_i is the right end point labeling of I_i .Without loss of generality we assume that all end points of the intervals in I are distinct numbers between 1 and $2n$. Two intervals i and j are said to be intersect each other if they have non empty intersection. Also we say that the intervals contains both its end points and that no two intervals share a common end point. The intervals and vertices of an interval graph are one and the same thing. The graph G is connected, and the list of sorted end point is given and the intervals in I are indexed by increasing right end points, that is $b_1 < b_2 < b_3 < .... < b_n$.

Let $G=(V,E)$ be a graph. A set $D \subseteq V(G)$ is a dominating set of G if every vertex in V/D is adjacent to some vertex in D. A set $S \subseteq V$ is a restrained dominating set (RDS) if every vertex not in S is adjacent to a vertex in S and to a vertex in $V-S$. Every graph has a RDS, since $S=V$ is such a set. The Restrained domination number of G, denoted by $\gamma_r(G)$, is the minimum cardinality of a RDS of G. A RDS S is called a $\gamma_r(G)$-set of G if $|S| = \gamma_r(G)$.

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The concept of Restrained domination was introduced by Telle and Proskurowski [5], albeit indirectly, as a vertex partitioning problem. One application of domination is that of prisoners and guards. For security, each prisoner must be seen by some guard; the concept is that of domination. However, in order to protect the rights of prisoners, we may also require that each prisoner is seen by another prisoner; the concept is that of restrained domination.

A Restrained dominating set RDS of G is connected Restrained dominating set, if the induced subgraph $<V-RDS>$ is connected. i.e., A Restrained dominating set RDS of a graph $G(V,E)$ is a Non-split Restrained dominating set, if the induced subgraph $<V-RDS>$ is connected.

In this connection introduce the Restrained dominating set using an Algorithm [6,7,8,9]. For finding the Restrained domination [10], through an algorithm, we consider a connected Interval graph. In this Connected Interval graph the vertices are ordered by IG ordering. First of all we treat none of a vertex of $V(G)$ is a member of Restrained dominating set RDS. Then insert vertices one by one by tasting their consistency. If a vertex v is dominated by at least two vertices then leave it, otherwise take the highest numbered adjacent vertex from $N[v]$ as a member of RDS if it is not adjacent to the next member of $N[v]$ or v is not the last vertex.

Let us associate a new term $M_i(v)$ for a vertex $v \in V$, for all $i = 0,1,2,\ldots\ldots,k(k = |N(v)|)$ to each adjacent vertices of v in order to IG ordering of intervals in the following way: $M_i(v) = \max \; N[v] - \bigcup_{j=0}^{i-1} M_i(v)$

With $M_0(v) = \max \; N(v)$

In connection with the highest adjacent vertex of v, we call this $M_i(v)$ as the p - th numbered adjacent vertex of v. Let $u, v \in V$. If for some $i \; (i = 0,1,2,\ldots\ldots,|N(v)|)$, $|N(v)| - i = p$ such that $u = M_i(v)$, then u is called the p - th numbered adjacent vertex of v.

The purpose of this paper is to find the Non-split Restrained dominating set of an Interval graph.

2 MAIN THEOREMS

2.1 THEOREM : Let $I = \{i_1, i_2, \ldots\ldots, i_n\}$ be an n Interval family and G is an Interval graph corresponding to I. If i and j are any two intervals in I such that $i \in RDS$,where RDS is a Restrained Dominating Set, $j \neq 1$ and j is contained in i, if there is at least one interval to the left of j that intersect j and there is at least one interval $k \neq i$ to the right of j that intersect j. Then the Restrained domination occurs in G and the non-split restrained dominating set $<V-RDS>$ is connected as $|RDS| = 3$.

Proof : Let $I = \{i_1, i_2, \ldots\ldots, i_n\}$ be the given n Interval family and G is an interval graph corresponding to I . First we will find the Restrained dominating set corresponding to G. Suppose there is at least one interval $k \neq i$ to the right of j that intersect j. Then it is obvious that j is adjacent to k in $<V-RDS>$, so that there will not be any disconnection in $<V-RDS>$. Since, there is at least one interval to the left of j that intersect j, there will not be any disconnection in $<V-RDS>$, to its left. Thus we get Non-split Restrained domination in G. In this procedure we also find Restrained dominating set of an interval graph towards an algorithm with an illustration as follows,

2.2 AN ALGORITHM FOR RESTAINED DOMINATING SET OF AN INTERVAL GRAPH

Input: An Interval graph $G = (V, E)$ with IG ordering vertex set $V = \{1,2,\ldots\ldots,n\}$.

Output: Restrained Dominating Set RDS

Step 1: Set $f(j) = 0, \forall j = 1,2,\ldots\ldots,n;$

Step 2: Set $i = 1, D = \phi;$

Step 2.1: Compute $W_i(f) = \sum_{v \in N[i]} f(v)$

Step 2.2: If $W_i(f) = 0$ then

Set $f(M_0(i)) = 1,\; f(M_1(i)) = 1;$

take RDS = $\{M_0(i)\}$.

Step 2.3: else if $W_i(f) = 1$, i is not the last vertex, then

Step 2.3.1: if $f(M_0(i)) = 0$, $M_0(i)$ is adjacent to $M_1(i)$

RDS remains unchanged.

end if;

Step 2.3.2: otherwise if $f(M_0(i)) = 0$, $M_0(i)$ is not adjacent to $M_1(i)$

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Set $f(M_0(i)) = 1$
take RDS = RDS $\cup \{M_0(i)\}$
end if;
else if $W_i(f) = 1$, i is the last vertex, then
RDS remains unchanged.
end if;
Step 2.4: else if $W_i(f) \geq 2$, then
RDS remains unchanged.
end if;
Step 2.5: Calculate $i = i + 1$ and go to Step 2.1 and continue until the last vertex. end RDS.

Now we will find the Restrained dominating set of an interval graph with an illustration using the above algorithm as follows,

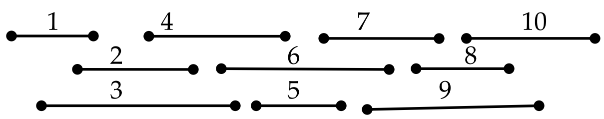

Fig.1: Interval family I

nbd [1] = {1,2,3}, nbd [2] = {1,2,3,4},
nbd [3] = {1,2,3,4,6}, nbd [4] = {2,3,4,5,6},
nbd [5] = {4,5,6,7}, nbd [6] = {3,4,5,6,7,9},
nbd [7] = {5,6,7,8,9}, nbd [8] = {7,8,9,10},
nbd [9] = {6,7,8,9,10}, nbd [10] = {8,9,10}

To find the Restrained Dominating Set, we have to compute all p - th numbered adjacent vertices.

TABLE 1

$M_i\ v \setminus v$	1	2	3	4	5	6	7	8	9	10
$M_0\ v$	3	4	6	6	7	9	9	10	10	10
$M_1\ v$	2	3	4	5	6	7	8	9	9	9
$M_2\ v$	1	2	3	4	5	6	7	8	8	8
$M_3\ v$	-	1	2	3	4	5	6	7	7	-
$M_4\ v$	-	-	1	2	-	4	5	-	6	-
$M_5\ v$	-	-	-	-	-	3	-	-	-	-

First set $f(j) = 0, \forall j \in V$. In Step 2, set $i = 1$, RDS = ϕ, that is initially RDS is empty. Step 2 repeats for n times. Here $n = 10$, the number of vertices in the interval graph G.

We follow the iterations of an illustration through the table.

Iteration (1):

For the first iteration $i = 1$

$$N\ 1 = 1,2,3$$

$$w_1\ f = f(N[1])$$

$$w_1\ f = f\ 1 + f\ 2 + f(3) = 0$$

The first condition of if-end if is satisfied.Since $w_1\ f = 0$, we find $M_0\ 1 = 3, M_1\ 1 = 2$

Then set $f\ 3 = 1, f\ 2 = 1$

Also set RDS = $\phi \cup \{3\}$

$\Rightarrow$RDS = $\{3\}$

Iteration (2):

For the second iteration $i = 2$

$$N\ 2 = 1,2,3,4,$$

$$w_2\ f = f(N[2])$$

$$w_2\ f = f\ 1 + f\ 2 + f\ 3 + f\ 4 = 0 + 1 + 1 + 0 = 2$$

So, in this iteration RDS could not be calculated. Hence RDS remains same and i is being increased to 3.

Iteration (3):

For the third iteration $i = 3$

$$N\ 3 = 1,2,3,4,6$$

$$w_3\ f = f(N[3])$$

$$w_3\ f = f\ 1 + f\ 2 + f\ 3 + f(4) + f(6)$$

$$= 0 + 1 + 1 + 0 + 0 = 2$$

In this iteration RDS remains unchanged. The iteration number i is being increased to 4.

Iteration (4):

For the fourth iteration $i = 4$

$$N\ 4 = 2,3,4,5,6$$

$$w_4\ f = f(N[4])$$

$$w_4\ f = f\ 2 + f\ 3 + f\ 4 + f\ 5 + f(6)$$

$$= 1 + 1 + 0 + 0 + 0 = 2$$

In this iteration RDS remains unchanged. The iteration number i is being increased to 5.

Iteration (5):

For the fifth iteration $i = 5$

$$N\ 5 = 4,5,6,7$$

$$W_5 \;\; f \;\; = f(N[5])$$

$$W_5 \;\; f \;\; = f \;\; 4 \; + f \;\; 5 \; + f \;\; 6 \; + f \;\; 7$$

$$= 0 + 0 + 0 + 0 = 0$$

The first condition of if-end if is satisfied. Since $w_5 \;\; f \;\; = 0,$ we find $M_0 \;\; 5 \;\; = 7, M_1 \;\; 5 \;\; = 6$

We find $M_0(5) = 7, M_1(5) = 6$

Then set $f \;\; 7 \;\; = 1, f \;\; 6 \;\; = 1$

Also set RDS = RDS $\cup \{7\}$

$\Rightarrow$RDS = $\{3,7\}$

The iteration number i is being increased to 6.

Iteration (6):

For the sixth iteration $i = 6$

$$N \;\; 6 \;\; = \;\; 3,4,5,6,7,9$$

$$W_6(f) = f(N[6])$$

$$W_6(f) = f \;\; 3 \; + f \;\; 4 \; + f \;\; 5 \; + f \;\; 6 \; + f \;\; 7 \; + f \;\; 9$$

$$= 1 + 0 + 0 + 1 + 1 + 0 = 3$$

In this iteration RDS remains unchanged. The iteration number i is being increased to 7.

Iteration (7):

For the seventh iteration $i = 7$

$$N \;\; 7 \;\; = \;\; 5,6,7,8,9$$

$$W_7 \;\; f \;\; = f(N[7])$$

$$W_7 \;\; f \;\; = f \;\; 5 \; + f \;\; 6 \; + f \;\; 7 \; + f \;\; 8 \; + f \;\; 9$$

$$= 0 + 1 + 1 + 0 + 0 = 2$$

In this iteration RDS remains unchanged. The iteration number i is being increased to 8.

Iteration (8):

For the eighth iteration $i = 8$

$$N \;\; 8 \;\; = \;\; 7,8,9,10$$

$$w_8 \;\; f \;\; = f(N[8])$$

$$w_8 \;\; f \;\; = f \;\; 7 \; + f \;\; 8 \; + f(9) + f(10)$$

$$= 1 + 0 + 0 + 0 = 1$$

Here the Restrained domination criteria is not satisfied. The else-if condition of if-end if is satisfied. Now $f(M_0(8)) = f \;\; 10 \;\; = 0$ and $M_0(8)$ is adjacent to $M_1(8).$ So RDS remains unchanged. The iteration number i is being increased to 9.

Iteration (9):

For the ninth iteration $i = 9$

$$N \;\; 9 \;\; = \;\; 6,7,8,9,10$$

$$w_9 \;\; f \;\; = f(N[9])$$

$$w_9 \;\; f \;\; = f \;\; 6 \; + f \;\; 7 \; + f \;\; 8 \; + f \;\; 9 \; + f(10)$$

$$= 1 + 1 + 0 + 0 + 0 = 2$$

In this iteration RDS could not be calculated. Hence RDS remains unchanged and i is being increased to 10.

Iteration (10):

For the tenth iteration $i = 10$

$$N \;\; 10 \;\; = \;\; 8,9,10$$

$$W_{10} \;\; f \;\; = f(N[10])$$

$$W_{10} \;\; f \;\; = f \;\; 8 \; + f \;\; 9 \; + f \;\; 10$$

$$= 0 + 0 + 0 = 0$$

The first condition of if-end if is satisfied. Since $w_{10} \;\; f \;\; = 0,$ we find $M_0 \;\; 10 \;\; = 10, M_1 \;\; 10 \;\; = 9$

Then set $f \;\; 10 \;\; = 1, f \;\; 9 \;\; = 1$

Also set RDS = RDS $\cup \{10\}$

$\Rightarrow$RDS = $\{3,7\} \cup \{10\} = \{3,7,10\}$

$\therefore$ RDS = $\{3,7,10\}$

$|RDS|$ = The cardinality of RDS = 3.

Thus we get the Non-spilt Restrained dominating set $< V - RDS >$ as follows,

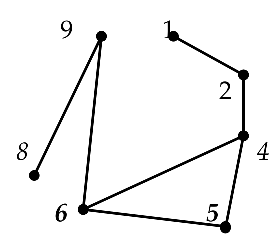

Fig.2: Vertex induced subgraph

$< V - RDS >$ **- Connected graph from G**

2.3 THEOREM : Let G be an Interval graph corresponding to an n Interval family $I = \{i_1, i_2, ..., i_n\}.$ If i and j are any two intervals in I such that $i \in RDS, j = 1, j$ intersects i and if there is one more interval that intersects j or contains j. Then Restrained domination occurs in G and the non-split restrained dominating set $< V - RDS >$ is connected as $|RDS| = 2.$

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Proof : Let $I=\{i_1,i_2,.....,i_n\}$ be the given n Interval family and G is an interval graph corresponding to I. First we will find the Restrained dominating set corresponding to G. Now let $j=1$ be an interval contained in an interval $k\neq i$ or intersects k which is not in the Restrained dominating set. Suppose j intersects i, since $i\in RDS, <V-RDS>$ does not contain i. Further in $<V-RDS>$, the vertex j is adjacent to the vertex k, since j is contained in k or j intersects k and hence there will not be any disconnection in $<V-RDS>$. Therefore we get Non-split dominating in G.

Next we will find the Restrained dominating set as follows from an interval family using Algorithm as explained in section 2.2.

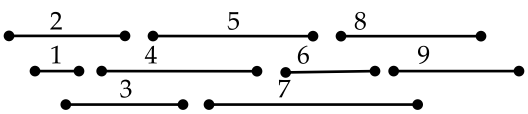

Fig.3: Interval Family I

nbd [1] = {1,2,3}, nbd [2] = {1,2,3,4},
nbd [3] = {1,2,3,4,5}, nbd [4] = {2,3,4,5,7},
nbd [5] = {3,4,5,6,7}, nbd [6] = {5,6,7,8},
nbd [7] = {4,5,6,7,8,9}, nbd [8] = {6,7,8,9},
nbd [9] = {7,8,9}

To find the Restrained Dominating Set, we have to compute all p - th numbered adjacent vertices.

TABLE 2

$M_i\ v\ \backslash v$	1	2	3	4	5	6	7	8	9
$M_0\ v$	3	4	5	7	7	8	9	9	9
$M_1\ v$	2	3	4	5	6	7	8	8	8
$M_2\ v$	1	2	3	4	5	6	7	7	7
$M_3\ v$	-	1	2	3	4	5	6	6	-
$M_4\ v$	-	-	1	2	3	-	5	-	-
$M_5\ v$	-	-	-	-	-	-	4	-	-

First set $f(j)=0, \forall j\in V$. In Step 2, set $i=1$, RDS = ϕ, that is initially RDS is empty. Step 2 repeats for n times. Here $n=9$, the number of vertices in the interval graph G.

As follows the iterations through the table,

Iteration (1) :

For the first iteration $i=1$

$$N\ 1 = 1,2,3$$

$$w_1\ f\ = f(N[1])$$

$$w_1\ f\ = f\ 1\ + f\ 2\ + f\ 3\ = 0+0+0=0$$

The first condition of if-end if is satisfied. Since $w_1\ f\ =0$, we find $M_0\ 1\ =3, M_1\ 1\ =2$

Then set $f\ 3\ =1, f\ 2\ =1$

Also set RDS = $\phi\cup\{3\}$

$\Rightarrow$RDS = $\{3\}$

Iteration (2):

For the second iteration $i=2$

$$N\ 2 = 1,2,3,4,$$

$$w_2\ f\ = f(N[2])$$

$$w_2\ f\ = f\ 1\ + f\ 2\ + f\ 3\ + f\ 4$$

$$=0+1+1+0=2$$

So, in this iteration RDS could not be calculated. Hence RDS remains same and i is being increased to 3.

Iteration (3)

For the third iteration $i=3$

$$N\ 3 = 1,2,3,4,5$$

$$w_3\ f\ = f(N[3])$$

$$w_3\ f\ = f\ 1\ + f\ 2\ + f\ 3\ + f(4)+f(5)$$

$$=0+1+1+0+0=2$$

In this iteration RDS remains unchanged.

The iteration number i is being increased to 4.

Iteration (4):

For the fourth iteration $i=4$

$$N\ 4 = 2,3,4,5,7$$

$$w_4\ f\ = f(N[4])$$

$$w_4\ f\ = f\ 2\ + f\ 3\ + f\ 4\ + f\ 5\ + f(7)$$

$$=1+1+0+0+0=2$$

In this iteration RDS remains unchanged. The iteration number i is being increased to 5.

Iteration (5):

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For the fifth iteration $i = 5$

$$N\ 5\ =\ 3,4,5,6,7$$

$$W_5\ f\ = f(N[5])$$

$$W_5\ f\ = f\ 3\ + f\ 4\ + f\ 5\ + f\ 6\ + f\ 7$$

$$= 1+0+0+0+0 = 1$$

Here the Restrained domination criteria is not satisfied. The else-if condition of if-end if is satisfied. Now $f(M_0(5)) = f\ 7\ = 0$ and $M_0(5)$ is adjacent to $M_1(5)$. So RDS remains unchanged. The iteration number i is being increased to 6.

Iteration (6):

For the sixth iteration $i = 6$

$$N\ 6\ =\ 5,6,7,8$$

$$W_6(f) = f(N[6])$$

$$W_6(f) = f\ 5\ + f\ 6\ + f\ 7\ + f\ 8$$

$$= 0+0+0+0 = 0$$

The first condition of if-end if is satisfied. Since $w_6\ f\ = 0,$ we find $M_0\ 6\ = 8, M_1\ 6\ = 7$

Then set $f\ 8\ = 1, f\ 7\ = 1$

Also set

$$\text{RDS} = \text{RDS} \cup \{8\}$$

$$\Rightarrow \text{RDS} = \{3\} \cup \{8\} = \{3,8\}$$

The iteration number i is being increased to 7.

Iteration (7):

For the seventh iteration $i = 7$

$$N\ 7\ =\ 4,5,6,7,8,9$$

$$W_7\ f\ = f(N[7])$$

$$W_7\ f\ = f\ 4\ + f\ 5\ + f\ 6\ + f\ 7\ + f\ 8\ + f\ 9$$

$$= 0+0+0+1+1+0 = 2$$

In this iteration RDS remains unchanged. The iteration number i is being increased to 8.

Iteration (8):

For the eighth iteration $i = 8$

$$N\ 8\ =\ 6,7,8,9$$

$$w_8\ f\ = f(N[8])$$

$$w_8\ f\ = f\ 6\ + f\ 7\ + f\ 8\ + f(9) = 0+1+1+0 = 2$$

In this iteration RDS could not be calculated. The iteration number i is being increased to 9.

Iteration (9):

For the ninth iteration $i = 9$

$$N\ 9\ =\ 7,8,9$$

$$w_9\ f\ = f(N[9])$$

$$w_9\ f\ = f\ 7\ + f\ 8\ + f\ 9\ = 1+1+0 = 2$$

In this iteration RDS could not be calculated. Hence RDS remains unchanged.

$$\therefore \text{RDS} = \{3,8\}$$

$$|RDS| = \text{The cardinality of RDS} = 2.$$

Thus we get the Non-spilt restrained dominating set $< V - RDS >$ as follows,

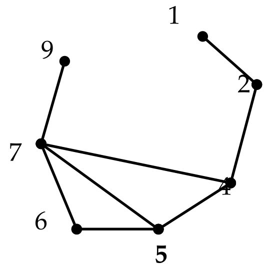

Fig.4: Vertex induced subgraph $< V - RDS >$ - Connected graph from G

2.4 THEOREM : Let us consider an n interval family $I = \{i_1, i_2, \ldots.., i_n\}$ and G be an interval graph of I. If i, j, k are three consecutive intervals such that $i < j < k$ and $j \in RDS,$, i intersects j, j intersect k and i intersect k. Then Restrained domination occurs in G and the non-split restrained dominating set $< V - RDS >$ is connected as $|RDS| = 2.$

Proof : Let $I = \{i_1, i_2, \ldots.., i_n\}$ be an n interval family and G be an interval graph of I. Let i, j, k be three consecutive intervals satisfy the hypothesis. Now i and k intersect implies that i and k are adjacent in $< V - RDS >$. So that there will not be any disconnection in $< V - RDS >$. Now we will find Restrained dominating set using Algorithm as given in section 2.2 as follows. For this consider the following interval family,

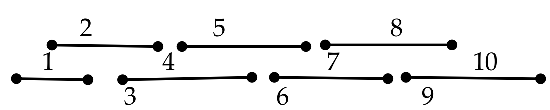

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Fig.5: Interval Family I

nbd [1] = {1,2,3}, nbd [2] = {1,2,3,4},
nbd [3] = {1,2,3,4,5}, nbd [4] = {2,3,4,5,6},
nbd [5] = {3,4,5,6,7}, nbd [6] = {4,5,6,7,8},
nbd [7] = {5,6,7,8,9}, nbd [8] = {6,7,8,9,10},
nbd [9] = {7,8,9,10}, nbd [10] = {8,9,10}.
To find the Restrained Dominating Set,
we have to compute all p - th numbered adjacent vertices.

TABLE 3

$M_i\ v\ \backslash v$	1	2	3	4	5	6	7	8	9	10
$M_0\ v$	3	4	5	6	7	8	9	10	10	10
$M_1\ v$	2	3	4	5	6	7	8	9	9	9
$M_2\ v$	1	2	3	4	5	6	7	8	8	8
$M_3\ v$	-	1	2	3	4	5	6	7	7	-
$M_4\ v$	-	-	1	2	3	4	5	6	-	-

First set $f(j)=0, \forall j \in V.$ In Step 2, set $i=1,$ RDS = $\phi,$ that is initially RDS is empty. Step 2 repeats for n times. Here $n=10,$ the number of vertices in the interval graph $G.$

As follows iterations,

Iteration (1) :

For the first iteration $i=1$

$$N\ 1 = 1,2,3$$
$$w_1\ f = f(N[1])$$
$$w_1\ f = f\ 1 + f\ 2 + f(3)$$
$$= 0+0+0=0$$

The first condition of if-end if is satisfied. Since $w_1\ f = 0,$ we find $M_0\ 1 = 3, M_1\ 1 = 2$

Then set $f\ 3 = 1, f\ 2 = 1$

Also set RDS = $\phi \cup \{3\}$

$\Rightarrow$RDS = $\{3\}$

Iteration (2):

For the second iteration $i=2$

$$N\ 2 = 1,2,3,4,$$
$$w_2\ f = f(N[2])$$
$$w_2\ f = f\ 1 + f\ 2 + f\ 3 + f\ 4$$
$$= 0+1+1+0=2$$

So, in this iteration RDS could not be calculated. Hence RDS remains same and i is being increased to 3.

Iteration (3)

For the third iteration $i=3$

$$N\ 3 = 1,2,3,4,5$$
$$w_3\ f = f(N[3])$$
$$w_3\ f = f\ 1 + f\ 2 + f\ 3 + f(4) + f(5)$$
$$= 0+1+1+0+0=2$$

In this iteration RDS remains unchanged.
The iteration number i is being increased to 4.

Iteration (4):

For the fourth iteration $i=4$

$$N\ 4 = 2,3,4,5,6$$
$$w_4\ f = f(N[4])$$
$$w_4\ f = f\ 2 + f\ 3 + f\ 4 + f\ 5 + f(6)$$
$$= 1+1+0+0+0=2$$

In this iteration RDS remains unchanged.
The iteration number i is being increased to 5.

Iteration (5):

For the fifth iteration $i=5$

$$N\ 5 = 3,4,5,6,7$$
$$W_5\ f = f(N[5])$$
$$W_5\ f = f\ 3 + f\ 4 + f\ 5 + f\ 6 + f\ 7$$
$$= 1+0+0+0+0=1$$

Here the Restrained domination criteria is not satisfied. The else-if condition of if-end if is satisfied. Now $f(M_0(5)) = f\ 7 = 0$ and $M_0(5)$ is adjacent to $M_1(5).$ So RDS remains unchanged. The iteration number i is being increased to 6 .

Iteration (6):

For the sixth iteration $i=6$

$$N\ 6 = 4,5,6,7,8$$
$$W_6(f) = f(N[6])$$
$$W_6(f) = f\ 4 + f\ 5 + f\ 6 + f\ 7 + f\ 8$$
$$= 0+0+0+0+0=0$$

The first condition of if-end if is satisfied. Since $w_6\ f = 0,$ we find $M_0\ 6 = 8, M_1\ 6 = 7$

Then set $f\ 8 = 1, f\ 7 = 1$

Also set RDS = RDS$\cup\{8\}$

$\Rightarrow$RDS = $\{3\} \cup \{8\} = \{3,8\}$

The iteration number i is being increased to 7.

Iteration (7):

For the seventh iteration $i = 7$

$$N[7] = \{5,6,7,8,9\}$$

$$W_7[f] = f(N[7])$$

$$W_7[f] = f[5] + f[6] + f[7] + f[8] + f[9]$$

$$= 0+0+1+1+0 = 2$$

In this iteration RDS remains unchanged. The iteration number i is being increased to 8.

Iteration (8):

For the eighth iteration $i = 8$

$$N[8] = \{6,7,8,9,10\}$$

$$w_8[f] = f(N[8])$$

$$w_8[f] = f(6) + f[7] + f[8] + f(9)$$

$$= 0+1+1+0 = 2$$

In this iteration RDS remains unchanged. The iteration number i is being increased to 9.

Iteration (9):

For the ninth iteration $i = 9$

$$N[9] = \{7,8,9,10\}$$

$$w_9[f] = f(N[9])$$

$$w_9[f] = f[7] + f[8] + f[9] + f(10)$$

$$= 1+1+0+0 = 2$$

In this iteration RDS could not be calculated. Hence RDS remains unchanged and i is being increased to 10.

Iteration (10):

For the tenth iteration $i = 10$

$$N[10] = \{8,9,10\}$$

$$W_{10}[f] = f(N[10])$$

$$W_{10}[f] = f[8] + f[9] + f[10] = 1+0+0 = 1$$

$$\Rightarrow W_{10}[f] = 1$$

10 is the last vertex, then RDS remains unchanged.

$\therefore$ RDS = $\{3,8\}$, $|RDS|$ = The cardinality of RDS = 2.

Thus we get the Non-spilt Restrained dominating set $<V - RDS>$ as follows,

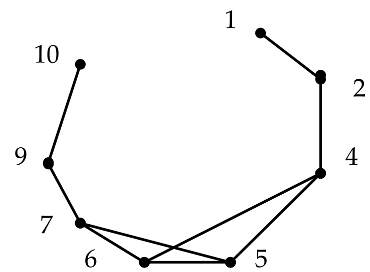

Fig.6: Vertex induced subgraph
$<V - RDS>$- Connected graph from G

3 CONCLUSION

We study the Non-split restrained dominating set problem on an interval graph corresponding to an interval family I. Given an interval model with end points sorted. We presented an algorithm to solve the Restrained domination problem on interval graphs. We extended the results to solve the Non- split restrained domination problem on interval graphs using an algorithm.

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