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Non-Homogeneous Unsteady-State Problem of an inverse of thin annular disc with internal heat source

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ABSTRACT

The present paper deal with the determination of an inverse unsteady state thermoelastic problem of thin annular disc with internal heat source applied for upper plane surface By applying Marchi-zgrablich and Laplace integral transform technique .To study inverse response of finite length thin annular disc with internal heat sources with third kind boundary conditions to determined linear temperature, displacement and stress function. The results are obtained in terms of infinite series and the numerical calculations are carried out by using MATHCAD -7 software and shown graphically.

Keywords: Inverse thermoelastic problem, finite length thin annular disc, Marchi-zgrablich and Laplace integral transform

1. INTRODUTION

In [2] Grysa K and Cialkowski,M. J. has determine one dimensional transient thermoelastic problem heating temperature and heat flux on surface of an isotropic infinite slab. [5] Durge, M.H and Khobragade, N.W has determined an inverse steady state thermoelastic problem of thin annular disc in Marchi-Zgrablich Transform Domain. [7] N.W.Khobragade and R.T.Walde has determined direct problem with constant temp. of thermal deflection of a clamped annular disc due to heat generation [8] V.Verghese and L. Khalsa has discuss transient thermoelastic problem for thick annular disc with radiation type boundary conditions .

In the present paper, an attempt has been made completely the inverse unsteady state thermoelastic problem of thin annular disc with internal heat source applied for upper plane surface with third kind boundary conditions. To determine the temperature, displacement and thermal stresses on upper plane surface of finite length thin annular disc with internal heat sources.

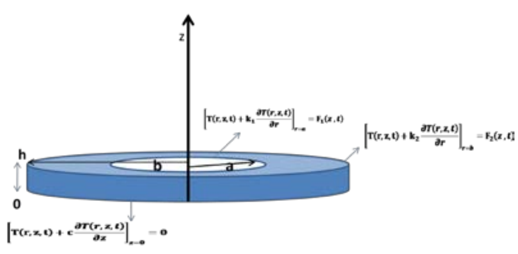

Fig.1 Thin annular disc with third kind boundary conditions

2. STATEMENT OF THE PROBLEM

Consider a thin annular isotropic disc from fig.1 of thickness h occupying the space $D: a \le r \le b, 0 \le z \le h$ the differential equation governing the displacement U(r, z, t) as in [1] is

$$\frac{\partial^2 U}{\partial r^2} + \frac{1}{r}\frac{\partial U}{\partial r} = (1+v)a_t T \qquad (2.1)$$

With U=0 at r=a and r=b (2.2)

V and a_t are the poisson's ratio and the linear coefficient of thermal expansion of the material of the disc respectively and T(r, z, t) is the temperature of the satisfying the differential equation

$$\frac{\partial^2 T}{\partial r^2} + \frac{1}{r}\frac{\partial T}{\partial r} + \frac{\partial^2 T}{\partial z^2} + \frac{\theta(r,z,t)}{k} = \frac{1}{\propto}\frac{\partial T}{\partial t} \qquad (2.3)$$

where $\propto$ the thermal diffusivity of the material of the cylinder, k is thermal conductivity of material and $\theta(r,z,t)$ is internal heat source

Subject to the initial condition,

T(r, z, 0) =0 (2.4)

The boundary conditions

$$\left[T(r,z,t) + k_1\frac{\partial T(r,z,t)}{\partial r}\right]_{r=a} = F_1(z,t) \qquad (2.5)$$

$$\left[T(r,z,t) + k_2\frac{\partial T(r,z,t)}{\partial r}\right]_{r=a} = F_2(z,t) \qquad (2.6)$$

$$\left[T(r,z,t) + c\frac{\partial T(r,z,t)}{\partial z}\right]_{z=0} = 0 \qquad (2.7)$$

$$\left[T(r,z,t) + c\frac{\partial T(r,z,t)}{\partial z}\right]_{z=\xi} = f(r,t)(\text{Known}) \qquad (2.8)$$

$$[T(r,z,t)]_{z=h} = g(r,t) \quad (\text{unknown}) \qquad (2.9)$$

The stress function σ_{rr} and $\sigma_{\theta\theta}$ are given by

$$\sigma_{rr} = -2\mu\frac{1}{r}\frac{\partial U}{\partial r} \qquad (2.10)$$

$$\sigma_{\theta\theta} = -2\mu\frac{\partial^2 U}{\partial r^2} \qquad (2.11)$$

Where μ is the Lame' constant, while each of the stress function are zero within the disc in the plane state of stress

The equations 2.1 to 2.11 constitute the mathematical formation of the problem under consideration.

3. SOLUTION OF THE PROBLEM

Applying the finite Marchi-Zgrablich integral transform to (2.3), (2.4), (2.7), (2.8), (2.9) and using (2.5), (2.6) one obtains

$$\frac{d^2\bar{T}}{dz^2} - \mu_n^2\,\bar{T} + \frac{\bar{\theta}}{k} = \frac{1}{\propto}\frac{dT}{dt} + Q \qquad (3.1)$$

Where $Q = \frac{a}{k_1}S_0(k_1,k_2,\mu_n a)F_1(z,t) - \frac{b}{k_2}S_0(k_1,k_2,\mu_n b)F_2(z,t)$

and p=0

$$\bar{T}(\mu_n, z, 0) = 0 \qquad (3.2)$$

$$\left[\bar{T}(\mu_n, z, t) + c\frac{d\bar{T}(\mu_n, z, t)}{dz}\right]_{z=0} = 0 \quad (3.3)$$

$$[\bar{T}(\mu_n, z, t)]_{z=h} = \bar{g}(\mu_n, t) \quad (3.4)$$

$$\left[\bar{T}(\mu_n, z, t) + c\frac{d\bar{T}(\mu_n, z, t)}{dz}\right]_{z=\xi} = \bar{f}(\mu_n, t) \quad (3.5)$$

where $\bar{T}$ denotes the Marchi-Zgrablich integral transform of T and μ_n is parameter.

Applying Laplace transform to (3.1), (3.3), (3.4), (3.5) and using (3.2) one obtains

$$\frac{d^2\bar{T}^*}{dz^2} - q^2\bar{T}^* = \left(Q^* - \frac{\bar{\theta}^*}{k}\right) \quad (3.6)$$

Where $q^2 = {\mu_n}^2 + \frac{s}{\propto}$

$$Q^* = \frac{a}{k_1}S_0(k_1, k_2, \mu_n a)F1^*(z,t) - \frac{b}{k_2}S_0(k_1, k_2, \mu_n b){F_2}^*(z,t)$$

$$\left[\bar{T}^*(\mu_n, z, t) + c\frac{d\bar{T}^*(\mu_n, z, t)}{dz}\right]_{z=0} = 0 \quad (3.7)$$

$$[\bar{T}^*(\mu_n, z, t)]_{z=h} = \bar{g}^*(\mu_n, t) \quad (3.8)$$

$$\left[\bar{T}^*(\mu_n, z, t) + c\frac{d\bar{T}^*(\mu_n, z, t)}{dz}\right]_{z=\xi} = \bar{f}^*(\mu_n, t) \quad (3.9)$$

The equation (3.6) is a second order differential equation whose solution is in form

$$\bar{T}^* = Ae^{qz} + Be^{-qz} + PI \quad (3.10)$$

Where $PI = \frac{\propto(Q^* - \frac{\bar{\theta}^*}{k})}{D^2 - q^2}$, $D \equiv \frac{d}{dz}$ and A, B are constant. Using equation (3.7), (3.9) in (3.10) we obtain the values of A and B substituting these values (3.10) and then apply inverse of Laplace transform and Marchi-Zgrablich integral transform. We obtain

$$T(r,z,t) = \frac{2k\pi}{\xi^2}\sum_{n=1}^{\infty}\frac{S_0(k_1,k_2,\mu_n r)}{C_n(1-c^2{\lambda_n}^2)} \times \left\{\sum_{m=1}^{\infty}(-1)^{m+1}m[\sin(\lambda_m z) - c\lambda_m\cos(\lambda_m z)]\right\} \times \int_0^t\left[\bar{f}^*(\mu_n, t) - [PI]_{z=\xi} - c\left[\frac{dPI}{dz}\right]_{z=\xi}\right]e^{-k({\mu_n}^2+{\lambda_m}^2)(t-t')}dt' - \frac{2k\pi}{\xi^2}\sum_{n=1}^{\infty}\frac{S_0(k_1,k_2,\mu_n r)}{C_n(1-c^2{\lambda_n}^2)} \times \left\{\sum_{m=1}^{\infty}(-1)^{m+1}m[\sin\lambda_m(z-\xi) - c\lambda_m\cos\lambda_m(z-\xi)]\right\} \times \int_0^t\left[-[PI]_{z=0}c\left[\frac{dPI}{dz}\right]_{z=0}\right]e^{-k({\mu_n}^2+{\lambda_m}^2)(t-t')}dt' + \sum_{n=1}^{\infty}\frac{S_0(k_1,k_2,\mu_n r)}{C_n}\int_0^t[L^{-1}\{PI\}]e^{-k({\mu_n}^2+{\lambda_m}^2)(t-t')}dt' \quad (3.11)$$

Where $\lambda_m = \frac{\pi m}{\xi}$

$$g(r,t) = \frac{2k\pi}{\xi^2}\sum_{n=1}^{\infty}\frac{S_0(k_1,k_2,\mu_n r)}{C_n(1-c^2{\lambda_n}^2)} \times \left\{\sum_{m=1}^{\infty}(-1)^{m+1}m[\sin(\lambda_m h) - c\lambda_m\cos(\lambda_m h)]\right\} \times \int_0^t\left[\bar{f}^*(\mu_n, t) - [PI]_{z=\xi} - c\left[\frac{dPI}{dz}\right]_{z=\xi}\right]e^{-k({\mu_n}^2+{\lambda_m}^2)(t-t')}dt' - \frac{2k\pi}{\xi^2}\sum_{n=1}^{\infty}\frac{S_0(k_1,k_2,\mu_n r)}{C_n(1-c^2{\lambda_n}^2)} \times \left\{\sum_{m=1}^{\infty}(-1)^{m+1}m[\sin\lambda_m(h-\xi) - c\lambda_m\cos\lambda_m(h-\xi)]\right\} \times \int_0^t\left[-[PI]_{z=0} - c\left[\frac{dPI}{dz}\right]_{z=0}\right]e^{-k({\mu_n}^2+{\lambda_m}^2)(t-t')}dt' + \sum_{n=1}^{\infty}\frac{S_0(k_1,k_2,\mu_n r)}{C_n}\int_0^t[L^{-1}\{PI\}]e^{-k({\mu_n}^2+{\lambda_m}^2)(t-t')}dt' \quad (3.12)$$

4. DETERMINATION OF THERMO ELASTIC DISPLACEMENT

Substituting the value of T(r, z t) from (3.11) in (2.1), one obtain the thermo elastic displacement function $U(r,z,t)$ as

$$U(r,z,t) = -(1+\nu)a_t\frac{k\pi}{2\xi^2}\sum_{n=1}^{\infty}\frac{r^2S_0(k_1,k_2,\mu_n r)}{C_n(1-c^2{\lambda_n}^2)} \times \left\{\sum_{m=1}^{\infty}(-1)^{m+1}m[\sin(\lambda_m z) - c\lambda_m\cos(\lambda_m z)]\right\} \times \int_0^t\left[\bar{f}^*(\mu_n, t) - [PI]_{z=\xi} - c\left[\frac{dPI}{dz}\right]_{z=\xi}\right]e^{-k({\mu_n}^2+{\lambda_m}^2)(t-t')}dt' + (1+\nu)a_t\frac{k\pi}{2\xi^2}\sum_{n=1}^{\infty}\frac{r^2S_0(k_1,k_2,\mu_n r)}{C_n(1-c^2{\lambda_n}^2)} \left\{\sum_{m=1}^{\infty}(-1)^{m+1}m[\sin\lambda_m(z-\xi) - c\lambda_m\cos\lambda_m(z-\xi)]\right\} \times \int_0^t\left[-[PI]_{z=0} - c\left[\frac{dPI}{dz}\right]_{z=0}\right]e^{-k({\mu_n}^2+{\lambda_m}^2)(t-t')}dt' - (1+\nu)a_t\sum_{n=1}^{\infty}\frac{r^2S_0(k_1,k_2,\mu_n r)}{C_n}\int_0^t[L^{-1}\{PI\}]e^{-k({\mu_n}^2+{\lambda_m}^2)(t-t')}dt' \quad (4.1)$$

5. DETERMINATION OF STRESS FUNCTIONS

Using (4.1) in (2.10),(2.11) the stress functions are obtained as

$$\sigma_{rr} = 2\mu\frac{1}{r}(1+\nu)a_t\frac{k\pi}{2\xi^2}\sum_{n=1}^{\infty}\frac{[2rS_0 + r^2\mu_n S'_0]}{C_n(1-c^2{\lambda_n}^2)} \times \left\{\sum_{m=1}^{\infty}(-1)^{m+1}m[\sin(\lambda_m z) - c\lambda_m\cos(\lambda_m z)]\right\} \times \int_0^t\left[\bar{f}^*(\mu_n, t) - [PI]_{z=\xi} - c\left[\frac{dPI}{dz}\right]_{z=\xi}\right]e^{-k({\mu_n}^2+{\lambda_m}^2)(t-t')}dt' - 2\mu\frac{1}{r}(1+\nu)a_t\frac{k\pi}{2\xi^2}\sum_{n=1}^{\infty}\frac{[2rS_0 + r^2\mu_n S'_0]}{C_n(1-c^2{\lambda_n}^2)} \left\{\sum_{m=1}^{\infty}(-1)^{m+1}m[\sin\lambda_m(z-\xi) - c\lambda_m\cos\lambda_m(z-\xi)]\right\} \times \int_0^t\left[-[PI]_{z=0} - c\left[\frac{dPI}{dz}\right]_{z=0}\right]e^{-k({\mu_n}^2+{\lambda_m}^2)(t-t')}dt' + 2\mu\frac{1}{4r}(1+\nu)a_t\sum_{n=1}^{\infty}\frac{[2rS_0 + r^2\mu_n S'_0]}{C_n} \times \int_0^t[L^{-1}\{PI\}]e^{-k({\mu_n}^2+{\lambda_m}^2)(t-t')}dt' \quad (5.1)$$

$$\sigma_{\theta\theta} = 2\mu(1+\nu)a_t\frac{k\pi}{2\xi^2}\sum_{n=1}^{\infty}\frac{[2\mu_n S'_0 + S'_0 + r\mu_n S''_0]}{C_n(1-c^2{\lambda_n}^2)} \left\{\sum_{m=1}^{\infty}(-1)^{m+1}m[\sin(\lambda_m z) - c\lambda_m\cos(\lambda_m z)]\right\} \times \int_0^t\left[\bar{f}^*(\mu_n, t) - [PI]_{z=\xi} - c\left[\frac{dPI}{dz}\right]_{z=\xi}\right]e^{-k({\mu_n}^2+{\lambda_m}^2)(t-t')}dt' - 2\mu(1+\nu)a_t\frac{k\pi}{2\xi^2}\sum_{n=1}^{\infty}\frac{[2\mu_n S'_0 + S'_0 + r\mu_n S''_0]}{C_n(1-c^2{\lambda_n}^2)} \times \left\{\sum_{m=1}^{\infty}(-1)^{m+1}m[\sin\lambda_m(z-\xi) - c\lambda_m\cos\lambda_m(z-\xi)]\right\} \times \int_0^t\left[-[PI]_{z=0} - c\left[\frac{dPI}{dz}\right]_{z=0}\right]e^{-k({\mu_n}^2+{\lambda_m}^2)(t-t')}dt' + \frac{\mu}{2}(1+\nu)a_t \sum_{n=1}^{\infty}\frac{[2\mu_n S'_0 + S'_0 + r\mu_n S''_0]}{C_n}\int_0^t[L^{-1}\{PI\}]e^{-k({\mu_n}^2+{\lambda_m}^2)(t-t')}dt' \quad (5.2)$$

6. SPECIAL CASE AND NUMERICAL RESULTS

Setting

$$f(r,t) = (1-e^{-t})\frac{\delta(r-r_0)}{2\pi r}\xi \quad (6.1)$$

Where δ Dirac-delta function

Applying the finite Marchi-Zgrablich integral transform to (6.1) one obtains

$$\bar{f}(\mu_n, t) = \int_a^b r(1-e^{-t})\frac{\delta(r-r_0)}{r}\xi S_0(k_1, k_2, \mu_n r)dr$$

$\theta(r,z,t) = \frac{\delta(r)}{r} g(t)\delta(z-\mathrm{h})$,Φ=0 ,

where $g(t) = constant$ (6.2)

,a=1cm ,b=2cm h=3cm, $\xi = 1.5cm$, $\mathrm{k}_1 = \mathrm{k}_2 = 1$, k=0.86 (for copper metal), t=1 sec.μ_n is the root of transcendental equation.

$[\mathrm{PI}]_{z=\xi} + \mathrm{c}\left[\frac{\mathrm{dPI}}{\mathrm{dz}}\right]_{z=\xi} = \mathrm{A, cons.}$

$[\mathrm{PI}]_{z=0} = 0 \ , \left[\frac{\mathrm{dPI}}{\mathrm{dz}}\right]_{z=0} = 0$

$L^{-1}[PI] = fun\ of\ z = z$

$$\mathrm{T(r,z,t)} = \frac{2k\pi}{\xi^2}\sum_{n=1}^{\infty}\frac{S_0(\mathrm{k_1,k_2,\mu_n} r)}{C_n(1-c^2{\lambda_n}^2)} \times \left\{\sum_{m=1}^{\infty}(-1)^{m+1}m[\sin(\lambda_m z) - c\lambda_m\cos(\lambda_m z)]\right\} \times \int_0^t\left[(1-e^{-t}) - [\mathrm{PI}]_{z=\xi} - \mathrm{c}\left[\frac{\mathrm{dPI}}{\mathrm{dz}}\right]_{z=\xi}\right]e^{-k({\mu_n}^2+{\lambda_m}^2)(t-t')}dt' \times \sum_{n=1}^{\infty}\frac{S_0(\mathrm{k_1,k_2,\mu_n} r)}{C_n}\int_0^t[\ \mathrm{L^{-1}\{PI\}}]e^{-k({\mu_n}^2+{\lambda_m}^2)(t-t')}dt' \quad (6.3)$$

Where $\lambda_m = \frac{\pi m}{\xi}$

$$\mathrm{g(r,t)} = \frac{2k\pi}{\xi^2}\sum_{n=1}^{\infty}\frac{S_0(\mathrm{k_1,k_2,\mu_n} r)}{C_n(1-c^2{\lambda_n}^2)} \times \left\{\sum_{m=1}^{\infty}(-1)^{m+1}m[\sin(\lambda_m h) - c\lambda_m\cos(\lambda_m h)]\right\} \times \int_0^t\left[(1-e^{-t}) - [\mathrm{PI}]_{z=\xi} - \mathrm{c}\left[\frac{\mathrm{dPI}}{\mathrm{dz}}\right]_{z=\xi}\right]e^{-k({\mu_n}^2+{\lambda_m}^2)(t-t')}dt' + \sum_{n=1}^{\infty}\frac{S_0(\mathrm{k_1,k_2,\mu_n} r)}{C_n}\int_0^t[\ \mathrm{L^{-1}\{PI\}}]e^{-k({\mu_n}^2+{\lambda_m}^2)(t-t')}dt' \quad (6.4)$$

DETERMINATION OF THERMO ELASTIC DISPLACEMENT

$$U(r,z,t) = -(1+\nu)a_t\frac{k\pi}{2\xi^2}\sum_{n=1}^{\infty}\frac{r^2 S_0(\mathrm{k_1,k_2,\mu_n} r)}{C_n(1-c^2{\lambda_n}^2)} \times \left\{\sum_{m=1}^{\infty}(-1)^{m+1}m[\sin(\lambda_m z) - c\lambda_m\cos(\lambda_m z)]\right\} \times \int_0^t\left[(1-e^{-t}) - [\mathrm{PI}]_{z=\xi} - \mathrm{c}\left[\frac{\mathrm{dPI}}{\mathrm{dz}}\right]_{z=\xi}\right]e^{-k({\mu_n}^2+{\lambda_m}^2)(t-t')}dt' \times -(1+\nu)a_t\sum_{n=1}^{\infty}\frac{r^2 S_0(\mathrm{k_1,k_2,\mu_n} r)}{C_n} \times \int_0^t[\ \mathrm{L^{-1}\{PI\}}]e^{-k({\mu_n}^2+{\lambda_m}^2)(t-t')}dt' \quad (6.5)$$

DETERMINATION OF STRESS FUNCTIONS

$$\sigma_{\mathrm{rr}} = 2\mu\frac{1}{r}(1+\nu)a_t\frac{k\pi}{2\xi^2}\sum_{n=1}^{\infty}\frac{[2rS_0 + r^2\mu_n {S'}_0]}{C_n(1-c^2{\lambda_n}^2)} \times \left\{\sum_{m=1}^{\infty}(-1)^{m+1}m[\sin(\lambda_m z) - c\lambda_m\cos(\lambda_m z)]\right\} \times \int_0^t\left[(1-e^{-t}) - [\mathrm{PI}]_{z=\xi} - \mathrm{c}\left[\frac{\mathrm{dPI}}{\mathrm{dz}}\right]_{z=\xi}\right]e^{-k({\mu_n}^2+{\lambda_m}^2)(t-t')}dt' + 2\mu\frac{1}{4r}(1+\nu)a_t\sum_{n=1}^{\infty}\frac{[2rS_0 + r^2\mu_n {S'}_0]}{C_n} \times \int_0^t[\ \mathrm{L^{-1}\{PI\}}]e^{-k({\mu_n}^2+{\lambda_m}^2)(t-t')}dt' \quad (6.6)$$

$$\sigma_{\theta\theta} = 2\mu(1+\nu)a_t\frac{k\pi}{2\xi^2}\sum_{n=1}^{\infty}\frac{[2\mu_n {S'}_0 + {S'}_0 + r\mu_n {S''}_0]}{C_n(1-c^2{\lambda_n}^2)} \times \left\{\sum_{m=1}^{\infty}(-1)^{m+1}m[\sin(\lambda_m z) - c\lambda_m\cos(\lambda_m z)]\right\} \times \int_0^t\left[(1-e^{-t}) - [\mathrm{PI}]_{z=\xi} - \mathrm{c}\left[\frac{\mathrm{dPI}}{\mathrm{dz}}\right]_{z=\xi}\right]e^{-k({\mu_n}^2+{\lambda_m}^2)(t-t')}dt' + \frac{\mu}{2}(1+\nu)a_t\sum_{n=1}^{\infty}\frac{[2\mu_n {S'}_0 + {S'}_0 + r\mu_n {S''}_0]}{C_n}$$

$$\int_0^t[\ \mathrm{L^{-1}\{PI\}}]e^{-k({\mu_n}^2+{\lambda_m}^2)(t-t')}dt' \quad (6.7)$$

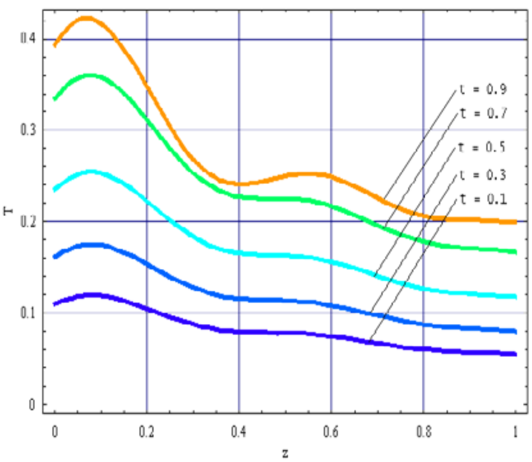

Fig.2 Temperature verses z with different time t

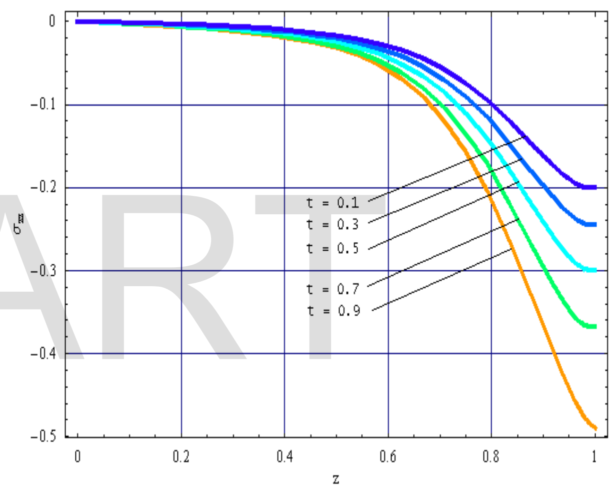

Fig.3 Radial stress verses z with different time t

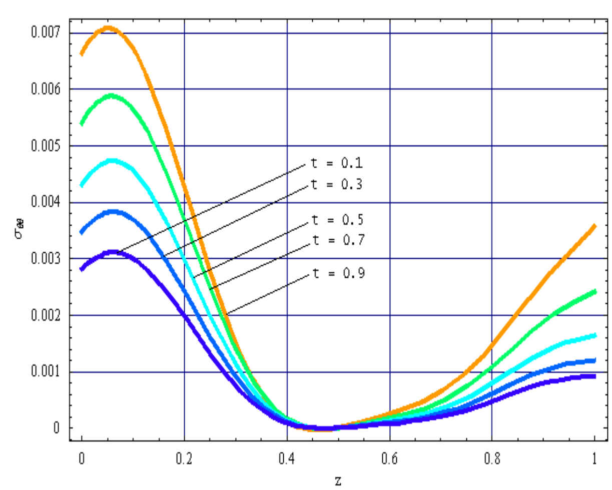

Fig.4 Tangential stress verses z with different time t

7. CONCLUSION

In this paper, we discussed completely the inverse unsteady state thermoelastic problem of thin annular disc with internal heat source applied for upper plane surface where the heat is dissipated by convention from the boundary surfaces at r=a and r=b in to surrounding varies position and time on curved surfaces and at lower plane surface heat is dissipated to surrounding at zero temperature. The finite Marchi-Zgrablich and Laplace transforms are used to obtain the numerical results. The series solution convergent since the length of annular disc is very small the temperature, Displacement and thermal stresses that are obtained can be applied to the design of useful structure or machines in engineering application. Any particular case of special interest can be derived by assigning suitable value of the parameters and function in the series expression.

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