Modified Super Convergent Line Series Method for Selection of Optimal Crop Combination for Intercropping

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ABSTRACT

Farmers are generally confronted with problems of determining optimal crop combinations when interactive effects are present between crops grown in intercropping scheme. This, therefore informs the varying margin of profitability in farming exercise. In order to aid farmers obtain optimum yield, this paper proposes a completely new method to optimally select crop combinations for their intercropping scheme. Numerical illustration given shows that the method is adequate for this purpose.

Keywords: Optimal designs, modified super convergent line series, quadratic programming, crop combinations, farmland.

1 Introduction

A rising from the high population growth rate and the need for increased food production, both small and large scale farmers are faced with the problem of selection of optimal crop combinations for their intercropping scheme that will yield maximum profit. Onwueme and Sinha [1] and Igbozurike [2] defined intercropping scheme as a deliberate practice of cultivating two or more crops simultaneously on the same parcel of land.

The practice of intercropping is more popular in the economically under developed nations and occupies about ninety percent of cropped area in most countries, particularly in the Tropical Rain Forest and Semi Arid Tropics, [3] and [4]. According to [1], there is a yield advantage in growing crops together rather than growing each one separately because of the fact that crops complement one another in their use of field time. Again, the spread of disease and pests is considerably less rapid in intercropping than in sole cropping.

Etukudo and Umoren in [5] have already developed a quadratic programming model to solve this problem and the solution technique adopted was the modified simplex method. However, a new algorithm known as modified super convergent line series (MSCLS_Q) has been developed for solving quadratic programming problems, [6]. Meanwhile, Etukudo and Umoren in [7] compared the two methods of quadratic programming problems namely, modified simplex method (MSM) and modified super convergent line series method (MSCLS_Q) and concluded that MSCLS_Q method is more efficient than MSM in handling quadratic programming problems based on well known measures of efficiency of an algorithm.

This paper, therefore focuses on the MSCLSQ approach in determining the optimal selection of crop combinations in intercropping scheme. The super convergent line series algorithm is a line search algorithm which makes use of the principles of optimal designs of experiment to get to the optimizer.

2 A QUADRATIC PROGRAMMING MODEL FOR SELECTING OPTIMAL CROP COMBINATIONS IN INTERCROPPING

The quadratic programming model for crop combinations in intercropping scheme is given as follows

Maximize
$$f(x) = \sum_{j=1}^{n} c_j x_j + \frac{1}{2} \sum_{j=1}^{n} \sum_{k=1}^{n} q_{jk} x_j x_k$$
 (1)

subject to

$$\begin{split} \sum_{j=1}^{n} l_{j} x_{j} & \leq & L \\ \sum_{j=1}^{n} s_{j} x_{j} & \leq & S \\ \sum_{j=1}^{n} f_{j} x_{j} & \leq & F \\ \sum_{j=1}^{n} p_{j} x_{j} & \leq & P \\ \sum_{j=1}^{n} m_{j} x_{j} & \leq & M \\ \sum_{j=1}^{n} v_{j} x_{j} & \leq & V \\ \sum_{j=1}^{n} h_{j} x_{j} & \leq & K \\ \sum_{j=1}^{n} p_{j} x_{j} & \leq & K \\ \sum_$$

where

 l_j = cost of preparation of land per hectare with respect to crop j, j = 1, 2, ..., n

 s_j = cost of seeds/seedlings per hectare of crop j, j = 1, 2, ..., n

 f_j = cost of fertilizer needed per hectare of crop j, j = 1, 2, ..., n

 p_j = cost of planting per hectare with respect to crop j, j = 1, 2, ..., n

 m_j = cost of farm management from planting to harvesting with respect to crop j, j = 1, 2, ..., n

 v_j = cost of harvesting per hectare with respect to crop j, j = 1, 2, ..., n

 g_j = cost of insurance cover per hectare of crop j, j = 1, 2, ..., n

 h_j = cost of post harvest handling per hectare of crop j, j = 1, 2, ..., n

L = total funds available for preparing farmland for intercropping scheme

S = total funds available for purchase of seedlings with respect to all crops in the intercropping scheme

F = total funds available for procurement of fertilizer for all the crops in the intercropping scheme

P = total funds available for planting all the crops in the intercropping scheme

M = total funds available for management of the intercropping scheme from planting to harvest time

V = total funds available for harvesting of all the crops in the intercropping scheme

G = total funds available for obtaining insurance cover for all the crops in the intercropping scheme

H = total funds available for post harvest handling of the intercropping scheme

3 METHODOLOGY

The sequential procedure for the MSCLSQ given below requires that the optimal support points that form the initial design matrix obtained from the entire experimental region be partitioned into k^* groups, $k^* = 2$, 3, \cdots so that optimal starting points are obtained for each group. However, [8] showed that with $k^* = 2$ for quadratic programming problems, optimal solutions are obtained. The sequential steps involved in MSCLSQ are given as follows:

Step 1: Given the response surface
$$f(x) = \mathbf{C} x + \frac{1}{2} \mathbf{x}^{\mathrm{T}} \mathbf{Q} \mathbf{x}$$

Select N support points such that

$$k^*(n+1) \le N \le \frac{1}{2} k^*n(n+1) + k^*$$

where k* is the number of partitioned groups desired and n is the number of variables. Hence, by arbitrarily choosing the support points as long as they do not violate any of the constraints, make up an initial design matrix

$$X = \begin{bmatrix} 1 & x_{11} & x_{12} & \dots & x_{1n} \\ 1 & x_{21} & x_{22} & \dots & x_{2n} \\ 1 & x_{31} & x_{32} & \dots & x_{3n} \\ \vdots & & & \vdots \\ 1 & x_{N1} & x_{N2} & \dots & x_{Nn} \end{bmatrix}$$
(3)

Step 2:

Step 3:

Step 4:

(2)

Partition X into k^* groups with equal number of support points and obtain the design matrix, X_i , $i = 1, 2, \dots, k^*$ for each group.

Obtain the information matrices $M_i = X_i^T X_i$, $i = 1, 2, \dots, k^*$ and their inverses M_i^{-1} , $i = 1, 2, \dots, k^*$

, i = 1, 2, \cdots , k^* and their inverses $\,M_i^{-1}$, i = 1, 2, $\,\cdots$, k^*

Compute the matrices of the interaction effect of the variables for the groups, X_{ii} where $i = 1, 2, ..., k^*$ and the vector of the interaction parameters obtained from f(x) is given by

$$\mathbf{g} = \begin{bmatrix} \mathbf{q}_1 \\ \mathbf{q}_2 \\ \mathbf{q}_3 \end{bmatrix}.$$

The interaction vectors for the groups are given by $\mathbf{I_i} = \mathbf{M_i^{-1}} \mathbf{X_i^T} \mathbf{X_{iI}} \mathbf{g}$ and the matrices of the mean square error of the groups are $\overline{\mathbf{M}_i} = \mathbf{M_i^{-1}} + \mathbf{I_i} \mathbf{I_i^T}$.

The matrices of coefficient of convex combinations of the matrices of mean square

$$H_{i} = \operatorname{diag} \left\{ \frac{\overline{v}_{i11}}{\sum \overline{v}_{i11}}, \frac{\overline{v}_{i22}}{\sum \overline{v}_{i22}}, \frac{\overline{v}_{i33}}{\sum \overline{v}_{i33}} \right\}$$

$$= \operatorname{diag}\{h_{i1}, h_{i2}, h_{i3}\}$$

$$i = 1, 2, \dots, k^{*}$$
(4)

By normalizing H_i such that $\sum H_i^* H_i^{*T} = I$, we have

$$H_{i}^{*} = diag \left\{ \frac{h_{i1}}{\sqrt{\sum h_{i1}^{2}}}, \frac{h_{i2}}{\sqrt{\sum h_{i2}^{2}}}, \frac{h_{i3}}{\sqrt{\sum h_{i3}^{2}}} \right\}$$
 (5)

The average information matrix is given by $M(\xi_{\rm N})$

$$\sum_{i} H_i^* X_i^T X_i H_i^{*T}$$
 (6)

Step 5: From f(x), obtain the response vector

$$\mathbf{z} = \begin{bmatrix} \mathbf{Z}_0 \\ \mathbf{Z}_1 \\ \mathbf{Z}_2 \\ \vdots \\ \mathbf{Z}_n \end{bmatrix},$$

where

$$z_{0} = f(\overline{m}_{21}, \overline{m}_{31}, ..., \overline{m}_{[n+1]1})$$

$$z_{1} = f(\overline{m}_{22}, \overline{m}_{32}, ..., \overline{m}_{[n+1]2})$$
.....
$$z_{n} = f(\overline{m}_{2[n+1]}, \overline{m}_{3[n+1]}, ..., \overline{m}_{[n+1][n+1]})$$

Hence, we define the direction vector

$$\mathbf{d} = \begin{bmatrix} \frac{\mathbf{d}}{0} \\ \mathbf{d}_{1} \\ \mathbf{d}_{2} \\ \vdots \\ \mathbf{d}_{n} \end{bmatrix} = \mathbf{M}^{-1}(\xi_{N})\mathbf{z}$$
 (7)

and by normalizing **d** such that $\mathbf{d}^{*T}\mathbf{d}^* = 1$, we have

$$\mathbf{d}^{*} = \begin{bmatrix} \mathbf{d}_{1}^{*} \\ \mathbf{d}_{1}^{*} \\ \mathbf{d}_{2}^{*} \\ \vdots \\ \mathbf{d}_{n}^{*} \end{bmatrix} = \begin{bmatrix} \frac{\mathbf{d}_{1}}{\sqrt{\mathbf{d}_{1}^{2} + \mathbf{d}_{2}^{2} + \dots + \mathbf{d}_{n}^{2}}} \\ \frac{\mathbf{d}_{2}}{\sqrt{\mathbf{d}_{1}^{2} + \mathbf{d}_{2}^{2} + \dots + \mathbf{d}_{n}^{2}}} \\ \vdots \\ \mathbf{d}_{n} \\ \hline \sqrt{\mathbf{d}_{1}^{2} + \mathbf{d}_{2}^{2} + \dots + \mathbf{d}_{n}^{2}} \\ \end{bmatrix}$$
(8)

Step 6: Compute the optimal starting point, \mathbf{x}_{1}^{*} from

$$\mathbf{x}_{1}^{*} = \sum_{m=1}^{N} \mathbf{u}_{m}^{*} \mathbf{x}_{m}^{T} ; \mathbf{u}_{m}^{*} > 0; \sum_{m=1}^{N} \mathbf{u}_{m}^{*} = \mathbf{1}$$

$$\mathbf{u}_{m}^{*} = \frac{\mathbf{a}_{m}^{-1}}{\sum_{m=1}^{N} \mathbf{a}_{m}^{-1}}, m = 1, 2, \dots, N$$

$$a_{m} = \mathbf{x}_{m} \mathbf{x}_{m}^{T}$$
, m = 1, 2, ..., N.

Step 7: Obtain the step length, ρ_1^* from

$$\rho_1^* = \max_{i} \left\{ \frac{\mathbf{A_i} \mathbf{x_1^*} - \mathbf{b_i}}{\mathbf{A_i} \mathbf{d^*}} \right\}$$
(9)

for a maximization problem or from

$$\rho_1^* = \min_{i} \left\{ \frac{\mathbf{A}_i \mathbf{x}_1^* - \mathbf{b}_i}{\mathbf{A}_i \mathbf{d}^*} \right\}$$
 (10)

for a minimization problem,

where $\mathbf{A_i} \mathbf{X} = \mathbf{b_i}$, $i = 1, 2, \dots$, m is the ith

constraint of the quadratic programming problem.

Step 8: Make a move to the point

$$\mathbf{x}_{2}^{*} = \mathbf{x}_{1}^{*} - \rho_{1}^{*} \mathbf{d}^{*} \tag{11}$$

Step 9: Compute $f(\mathbf{x}_{2}^{*})$ and $f(\mathbf{x}_{1}^{*})$. Is $\left| f(\mathbf{x}_{2}^{*}) - f(\mathbf{x}_{1}^{*}) \right|$ < ϵ where ϵ = 0.0001, then stop for the current

solution is optimal, otherwise, replace $old x_1^*$ by

 \mathbf{X}_{2}^{*} and return to step 7. If the new step length, ρ_{2}^{*} is negligibly small, then the optimizer had been located at the first move.

4 PROCEDURE FOR OBTAINING AN OPTIMIZER USINGTHE METHOD

The assumption here is that the soil analysis of the farmland to be used for the intercropping scheme had been carried out with respect to all the crops to be included in the scheme and that the crops can thrive very well based on the analysis.

If there are t crops to be cultivated on the farmland, there

are
$$\sum_{n=2}^{t} {}^{t}C_{n}$$
 possible groups of crops from which the optimal

combination can be selected where n is the number of crops to be taken from t for combination.

As an illustration, let us assume that we have four crops, namely maize, yam, pepper and okro denoted respectively as crops 1, 2, 3 and 4 for this exercise. We assume further that soil analysis favours the four crops on parcel of land acquired for farming. The data for this illustration obtained from [5] are given in Tables 1-5.

Table 1: Cost of each farming operation per crop per hectare

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Cost (N'000)	Crops (j)			
	1	2	3	4
l _i	8	9	7	6
si	4	3	6	5
f_j	4	2	3	5
p _i	1	4	4	3
m _j	3	2	4	1
v _i	1	4	3	2
h _i	5	3	1	2
$g_{\rm j}$	3	4	1	2

Table 2: The per hectare effects of one crop and the other when planted together

F				
Crop j	1	2	3	4
1	0	-1	-4	-3
2	1	0	-2	-1
3	-3	2	0	-4
4	3	2	-5	0

Table 3: Optimal coefficient symmetric matrix of per hectare effects of one crop and the other

Crop j	1	2	3	4	
1	0	1	-4	-3	
2	1	0	-2	-1	
3	-3	2	0	-4	
4	3	2	-5	0	7

Table 4: Value of resource (monetary) constraints

Resource	Value (N'000)
L	20
S	5
F	10
P	6
M	18
V	30
Н	15
G	22

Table 5: Expected profit per crop per hectare

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Crop j	1	2	3	4	_
Profit (N'00)	60	70	60	40	

A = 600 hectares

The decision variables are

 x_i = hectares of land allocated to crop j, j = 1, 2, ..., n

 x_k = hectares of land allocated to crop k, k = 1, 2, ..., n

From the four available crops, there are eleven different combinations such as (1, 2), (1, 3), (1, 4), (2, 3), (2, 4), (3, 4), (1, 2, 3), (1, 2, 4), (1, 3, 4), (2, 3, 4), (1, 2, 3, 4). Substituting the data for each of the combinations, quadratic programming model 1 (QP1) for combination (1, 2) is given by

$$\operatorname{Max} f(x) = 60x_1 + 70x_2 + \frac{1}{2} \begin{bmatrix} x_1 & x_2 \end{bmatrix} \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} \qquad \operatorname{M}_{1}^{-1} = \left(X_{1}^{T} X_{1} \right)^{-1}$$

$$+ 60x_1 + 70x_2$$

(12)Subject to

$$\begin{array}{l} 8x_1 + 9x_2 \leq 20 \\ 4x_1 + 3x_2 \leq 5 \\ 4x_1 + 2x_2 \leq 10 \\ x_1 + 4x_2 \leq 6 \\ 3x_1 + 2x_2 \leq 18 \\ x_1 + 4x_2 \leq 30 \\ 5x_1 + 3x_2 \leq 15 \\ 3x_1 + 4x_2 \leq 22 \\ x_1 \leq 0.6 \\ x_2 \leq 0.6 \end{array}$$

We now obtain the solution of QP1 by super convergent line series method as follows:

Step 1

Let X be the area defined by the constraints. Hence,

$$X = \{x_1, x_2; C\}$$

Select N support points such that

$$k*(n+1) \le N \le \frac{1}{2} k*n(n+1) + k*,$$

where k* is the number of partitioned groups desired and n is the number of variables. By choosing $k^* = 2$ and n = 2, we have $6 \le N \le 8$. Hence, by arbitrarily choosing 6 support points as long as they do not violate the constraints, the initial design matrix is

$$X = \begin{bmatrix} 1 & 0.6 & 0.1 \\ 1 & 0.5 & 0.2 \\ 1 & 0.4 & 0.3 \\ 1 & 0.3 & 0.4 \\ 1 & 0.2 & 0.5 \\ 1 & 0.1 & 0.6 \end{bmatrix}$$
 (13)

Step 2:

Partition X into two groups such that

$$\mathbf{X}_1 = \begin{bmatrix} 1 & 0.6 & 0.1 \\ 1 & 0.5 & 0.2 \\ 1 & 0.4 & 0.3 \end{bmatrix}$$

$$\mathbf{X}_2 = \begin{bmatrix} 1 & 0.3 & 0.4 \\ 1 & 0.2 & 0.5 \\ 1 & 0.1 & 0.6 \end{bmatrix}.$$

The information matrices are $M_1 = X_1^T X_1$ and $M_2 = X_2^T X_2$ and their inverses are respectively

$$\mathbf{M}_{1}^{-1} = \left(\mathbf{X}_{1}^{\mathrm{T}}\mathbf{X}_{1}\right)^{-1}$$

$$= \begin{bmatrix} -1.7654 & 2.5220 & 2.5220 \\ 2.5220 & -3.6029 & -3.6029 \\ 2.5220 & -3.6029 & -3.6029 \end{bmatrix} \times 10^{16}$$

and

$$\mathbf{M}_{2}^{-1} = \left(\mathbf{X}_{2}^{T} \mathbf{X}_{2}\right)^{-1}$$

$$= \begin{bmatrix} 0.39231 & -0.56045 & -0.56045 \\ -0.56045 & 0.80064 & 0.80064 \\ -0.56045 & 0.80064 & 0.80064 \end{bmatrix} \times 10^{16}$$

Step 3: Obtain the matrices of coefficients of convex combinations from M_1^{-1} and M_2^{-1} . These are

$$H_1 = diag\{-0.2857, -0.2857,$$

$$H_2 = I - H_1 = diag\{1.2857, 1.2857, 1.2857\}$$
 (15)

and by normalizing H_i such that $\sum H_i^* H_i^{*T} = I$, i = 1, 2, we have

$$H_{1}^{*} = diag\{-0.2169, -0.2169, -0.2169\}$$
 (16)

$$H_{2}^{*} = \text{diag}\{0.9762, 0.9762, 0.9762\}$$
 (17)

The average information matrix is given by

$$\begin{split} M(\xi_N) &= \sum_{i=1}^2 \boldsymbol{H}_i^* \boldsymbol{X}_i^T \boldsymbol{X}_i \boldsymbol{H}_i^{*T} \\ &= \begin{bmatrix} 3.0000 & 0.6423 & 1.4012 \\ 0.6423 & 0.1696 & 0.2537 \\ 1.4012 & 0.2537 & 0.7404 \end{bmatrix} \end{split}$$

Step 4: From $f(\mathbf{x})$, obtain the response vector

$$\mathbf{z} = \begin{bmatrix} z_0 \\ z_1 \\ z_2 \end{bmatrix} = \begin{bmatrix} 136.622 \\ 27.935 \\ 67.05 \end{bmatrix}$$

Hence, we define the direction vector

$$\mathbf{d} = \begin{bmatrix} \frac{d_0}{d_1} \\ d_2 \end{bmatrix} = \mathbf{M}^{\text{-1}}(\xi_N)\mathbf{z} = \begin{bmatrix} \frac{-0.3723}{60.7176} \\ 70.4650 \end{bmatrix}$$

and by normalizing \mathbf{d} such that $\mathbf{d}^{*T}\mathbf{d}^{*} = 1$, we have

$$\mathbf{d}^* = \begin{bmatrix} \mathbf{d}_1^* \\ \mathbf{d}_2^* \end{bmatrix} = \begin{bmatrix} \frac{60.7176}{\sqrt{60.7176^2 + 70.4650^2}} \\ \frac{70.4650}{\sqrt{60.7176^2 + 70.4650^2}} \end{bmatrix} = \begin{bmatrix} 0.6528 \\ 0.7576 \end{bmatrix}.$$

Step 3: Obtain the optimal starting point, \mathbf{x}_{1}^{*} as follows:

$$a_{1} = x_{1} x_{1}^{T} = \begin{bmatrix} 1 & 0.6 & 0.1 \end{bmatrix} \begin{bmatrix} 1 \\ 0.6 \\ 0.1 \end{bmatrix} = 1.37,$$

$$a_1^{-1} = 0.7299$$

Similarly,

$$a_2^{-1} = 0.7752, \ a_3^{-1} = 0.8000, \ a_4^{-1} = 0.8000,$$

$$a_5^{-1} = 0.7752, \ a_6^{-1} = 0.7299$$

$$\sum_{m=1}^{N} a_{m}^{-1} = 4.6102$$

Since

$$\mathbf{u}_{m}^{*} = \frac{\mathbf{a}_{m}^{-1}}{\sum_{m=1}^{N} \mathbf{a}_{m}^{-1}}, m = 1, 2, \dots, N, \text{ then}$$

$$\sum_{m=1}^{N} \mathbf{a}_{m}^{-1}$$

$$\mathbf{u}_{1}^{*} = 0.1583, \mathbf{u}_{2}^{*} = 0.1681, \mathbf{u}_{3}^{*} = 0.1735, \mathbf{u}_{4}^{*} = 0.1735,$$

$$\mathbf{u}_{5}^{*} = 0.1681, \mathbf{u}_{6}^{*} = 0.1583$$

Hence, the optimal starting point is

$$\mathbf{x_1^*} = \sum_{m=1}^{N} \mathbf{u_m^* x_m} = \begin{bmatrix} \underline{0.9998} \\ 0.3499 \\ 0.3499 \end{bmatrix}$$

Step 6: Obtain the step length, ρ_1^* from

$$\rho_1^* = \max_{i} \left\{ \frac{\mathbf{A}_i \mathbf{x}_1^* - \mathbf{b}_i}{\mathbf{A}_i \mathbf{d}^*} \right\}$$
(18)

where $\mathbf{A_i} \mathbf{x} = \mathbf{b_i}$, $i = 1, 2, \cdots$, m is the ith constraint of the linear programming problem.

For
$$\mathbf{A}_1 = \begin{bmatrix} 8 & 9 \end{bmatrix}$$
 and $b = 20$, we have

$$\rho_{1}^{*} = \begin{cases} \boxed{8 & 9} \boxed{0.3499 \\ 0.3499} - 20 \\ \hline \boxed{8 & 9} \boxed{0.6528 \\ 0.7576 \end{bmatrix}}$$

$$= -1.1670$$

Similarly, the step lengths for the remaining constraints are respectively -0.5223, -1.9146, -1.1540, -4.6783, -7.6701, -2.2036, -3.9189, -0.3831, -0.3301.

We choose the maximum step length, $\rho_1^* = -0.3301$.

Step 7: Make a move to the point

$$\mathbf{x}_{2}^{*} = \mathbf{x}_{1}^{*} - \rho_{1}^{*}\mathbf{d}^{*}$$

$$= \begin{bmatrix} 0.3499 \\ 0.3499 \end{bmatrix} - (-0.3301) \begin{bmatrix} 0.6528 \\ 0.7576 \end{bmatrix} = \begin{bmatrix} 0.5654 \\ 0.6000 \end{bmatrix}$$

Step 8: Now, f(\mathbf{x}_{2}^{*}) = 60(0.5654) + 70(0.6000) = 75.9240

$$f(\mathbf{X_1^*}) = 60(0.3499) + 70(0.3499) = 45.4870.$$

Since
$$|f(\mathbf{x}_{2}^{*}) - f(\mathbf{x}_{1}^{*})| = |75.9240 - 45.4870| = 30.4370,$$

make a second move by replacing

$$\mathbf{x}_{1}^{*} = \begin{bmatrix} 0.3499 \\ 0.3499 \end{bmatrix} \text{ by } \mathbf{x}_{2}^{*} = \begin{bmatrix} 0.5654 \\ 0.6000 \end{bmatrix}.$$

By using the constraint matrix that gave the maximum $\,\rho_1^*\,$

we obtain ρ_2^* as follows:

$$\rho_{2}^{*} = \begin{cases} \boxed{0 & 1 \begin{bmatrix} 0.5654 \\ 0.6000 \end{bmatrix} - 0.6} \\ \boxed{0 & 1 \begin{bmatrix} 0.6528 \\ 0.7576 \end{bmatrix}} \end{cases} = 0.$$

Since $\rho_2^* = 0$, then the optimizer was located at the first move,

$$\mathbf{x}_{2}^{*} = \begin{bmatrix} 0.5654 \\ 0.6000 \end{bmatrix}$$

and

$$f(\mathbf{x_2^*}) = 75.9240.$$

By making similar computations, we have the results as displayed on Table 6 below for all the crop combinations.

Table 6: Summary of optimal solution obtained by $MSCLS_Q$

	metnod						
	Model	Crop	Value of decision	Objective function			
		Combination	variables	value			
	QP1	1, 2	$x_1 = 565.40,$	N75,924.00			
			$x_2 = 600$				
	QP 2	1, 3	$x_1 = 529.16,$	N59,692.00			
4			$x_3 = 480.54$				
1	QP 3	1, 4	$x_1 = 600,$	N56,665.60			
			$x_4 = 516.64$				
	QP 4	2, 3	$x_2 = 600,$	N75,856.20			
			$x_3 = 564.27$				
	QP 5	2, 4	$x_2 = 599.98,$	N61,714.97			
			$x_4 = 489.24$				
	QP 6	3, 4	$x_3 = 600,$	N54,871.56			
			$x_3 = 505.94$				
	QP 7	1, 2, 3	$x_1 = 600,$	N99,589.80			
			$x_2 = 592.30, x_3 =$				
			382.19				
	QP 8	1, 2, 4	$x_1 = 433.64, x_2 =$	N74,963.30			
			$600, x_4 = 172.33$				
	QP9	1, 3, 4	$x_1 = 556.02, x_2 =$	N72,977.80			
			$599.97, x_3 = 128.31$				
	QP 10	2, 3, 4	$x_2 = -2156.11, x_3 =$				
			$-1228.53, x_4 =$				
			599.99				
	QP 11	1, 2, 3, 4	$x_1 = 3799.70, x_2 =$				
			2792.70,				
			$x_3 = -149.02, x_4 =$				
			600				

Models QP 10 and QP 11 are not permissible since the variables must take only positive values not greater than 600. Hence, the optimal solution is obtained from model QP7 with the objective function value of N99,589.80.

5 CONCLUSION

The primary objective of this study, namely, optimal selection of

crop combinations in intercropping scheme by modified super convergent line series method has been successfully carried out. As could be seen in Table 6, an intercropping scheme consisting of crops 1, 2 and 3 yields the highest profit of N99,589.80 followed by the scheme consisting of crops 1 and 2 with a profit of N75,924.00. Therefore, in order to have a maximum profit for his farming business, the farmer should adopt intercropping scheme consisting of crops 1, 2 and 3 and cultivate 600 hectares of maize, 592.30 hectares of yam and 382.19 hectares of pepper on the same farmland.

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