Assuming the supporting medium with intensity q; deflection y; at a point can be written as:

$$q = \beta y \quad (9)$$

Considering load (P) and time (t) as factors in potholes deflection (y), we consider the Euler-Lagrange equation for dynamic beam

$$EI\frac{\partial^4 y}{\partial x^4} + \frac{\partial^2 y}{\partial t^2} = P \quad (10)$$

$$EI\frac{\partial^4 y}{\partial x^4} = -\beta y \quad (11)$$

$$\frac{\partial^2 y}{\partial t^2} = P \quad (12)$$

$$y = y_{(x)}.y_{(t)} \quad (13)$$

First considering Eq. 11

$$EI\frac{d^4 y}{dx^4} = -\beta y$$

$$y_{(x)} = \frac{Pe^{-\lambda x}Cos\lambda x}{2EI\lambda^3} \quad (14)$$

Now considering Eq. 12

$$\frac{d^2 y}{dt^2} = P$$

Integrating equation (12), and further integrating the result gives Eq. 15 and 16

$$\int \frac{d^2 y}{dt^2} = \int P\,\partial t$$

$$\frac{dy}{dt} = Pt + F \quad (15)$$

$$y_{(t)} = \frac{Pt^2}{2} + Ft + G \quad (16)$$

Initial conditions include $y(x,0) = \frac{dy}{dt}(x,1) = 0$

Considering y = 0, when t = 0; substituting in Eq. 16

G = 0

Considering $\frac{dy}{dt} = 0, when\ t = 1$; substituting in Eq. 15

0 = P + F

F = -P

Substitute for F and G in Eq. 16

$$y_{(t)} = \frac{Pt^2}{2} - Pt \quad (17)$$

Recalling Eq. (13), and substituting Eqs (14) and (17)

$y = y_{(x)}.y_{(t)}$

The final critical deflection is given by;

$$y_{(c)} = \frac{Pe^{-\lambda x}Cos\lambda x}{2EI\lambda^3} * (\frac{Pt^2}{2} - Pt) \quad (18)$$

Since $E = E_s + E_p$ (eqn.6)

Eqn. (18) becomes $y_{(c)} = \frac{Pe^{-\lambda x}Cos\lambda x}{2(E_s+E_p)I\lambda^3} * (\frac{Pt^2}{2} - Pt)$ (19)

$$\boldsymbol{y_{(c)} = \frac{Pe^{-\lambda x}Cos\lambda x}{2(2\rho V_s^2(1+\mu)+E_p)I\lambda^3} * (\frac{Pt^2}{2} - Pt)} \quad (20)$$

4.0 Verification of Model

Wheel loading is acting on the edge of the crack. The following data is considered:

Thickness of the pavement layer, h = 0.10 m

Wheel load, P= 44.8 KN

Tire pressure, p = 480KN/m^2

Wheel load acts on the pavement surface, it has a contact area depending upon the tire pressure. The contact area is calculated as follows:

Contact area is load divided by the tire pressure. The contact area is assumed to be rectangular.

Contact area = 0.093 m^2

Width of tire = 0.34 m

Length of contact area = 0.27 m

Consider a pavement strip of 0.025m and width 0.27mlength,

Load on the strip = 3.33KN

Breadth of beam, b= Width of tire = 0.34 m

Elastic Modulus of pavement, E_p = 600.5 x 10^3KN/m^2

Elastic Modulus of pavement, E_s = 89 x 10^3KN/m^2

$E = E_s + E_p$ =689.5 x 10^3KN/m^2

Modulus of beam, k = 68346.6 N/m^3

Reaction of foundation, β = 23.27 x 10^3KN/m^2

Cross - section of pavement layer

w = 0.025m and h = 0.10m

Moment of inertia, I= $\frac{wh^3}{12} = 0.0000022m^4$

$\lambda = \sqrt[4]{\frac{\beta}{4EI}}$ = 7.87m^{-1}

Deflection, $y_{(c)} = \frac{Pe^{-\lambda x}Cos\lambda x}{2(2\rho V_s^2(1+\mu)+E_p)I\lambda^3} * (\frac{Pt^2}{2} - Pt) = \frac{Pe^{-\lambda x}Cos\lambda x}{2(E_s+E_p)I\lambda^3} * (\frac{Pt^2}{2} - Pt)$

Verifying for the deflection, taking t = 2 - 21 days after crack for a 20 sample, assuming that the period of formation of cracks is considered as 1 and that there's a constant application of axle loads on the crack. This verification was carried out using SciLab software, and the results are presented below.

Table 1: Verification of the model

S/N	Time (days)	Deflection (mm)
1	2	0
2	3	0.162864387
3	4	0.434305033
4	5	0.814321937
5	6	1.302915099
6	7	1.900084519
7	8	2.605830198
8	9	3.420152135
9	10	4.34305033
10	11	5.374524783
11	12	6.514575495
12	13	7.763202464
13	14	9.120405692
14	15	10.58618518
15	16	12.16054092
16	17	13.84347293
17	18	15.63498119
18	19	17.53506571

19	20	19.54372648
20	21	21.66096352

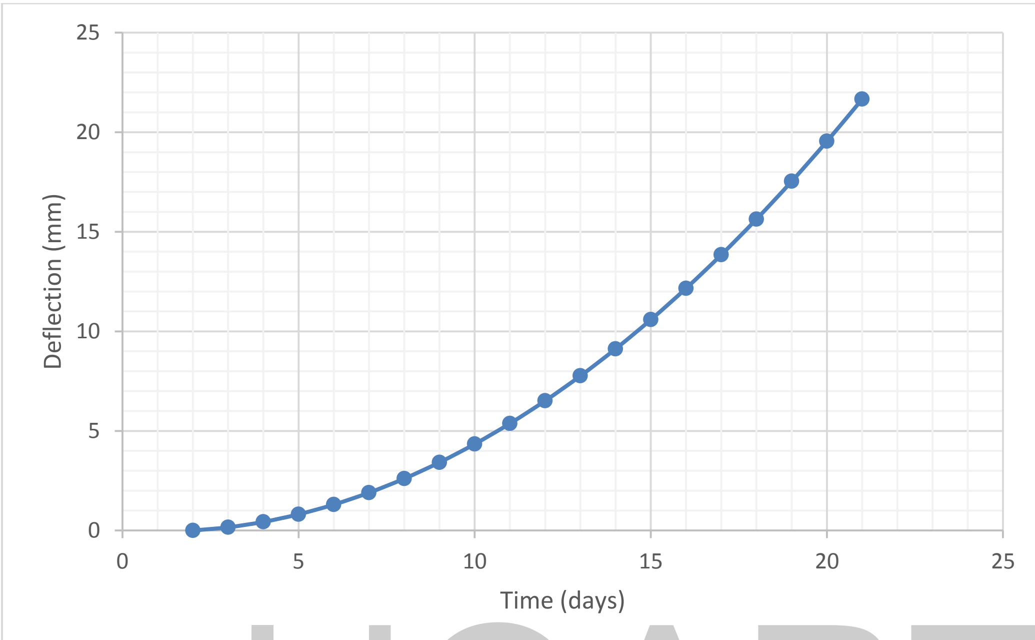

Fig. 1: Graph of Deflection (mm) versus time (days) for constant load, varying time

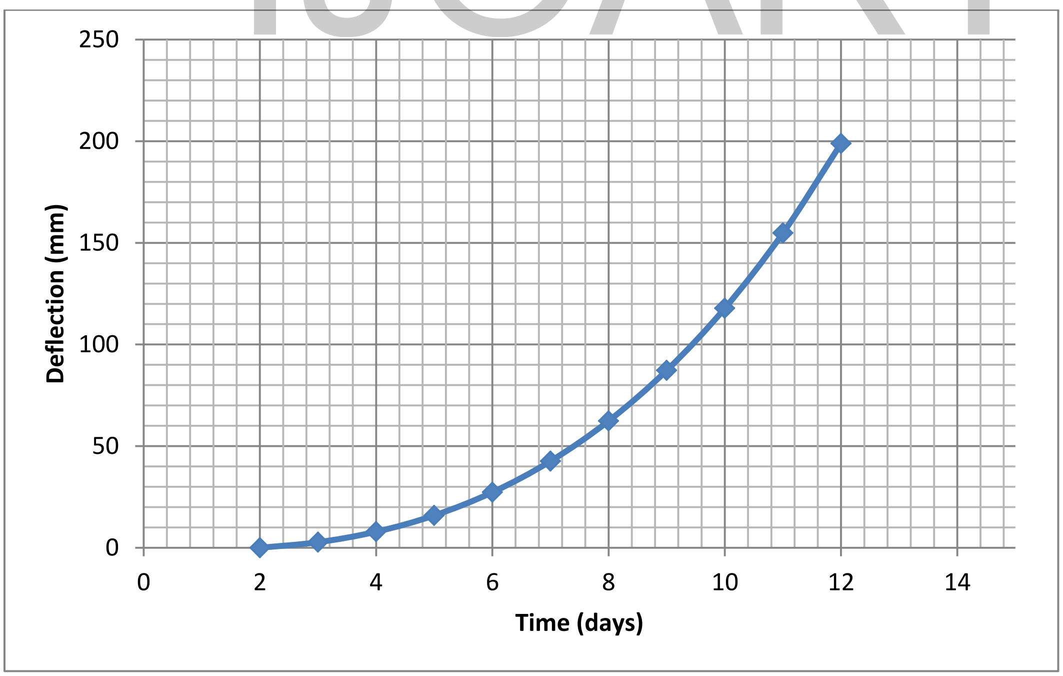

Fig. 2: Graph of Deflection (mm) versus time (days) for varying load, varying time

This model in comparison to the pothole deflection model without time (t) as a variable gave a deflection of 0.002mm with the same parameters as above, while we can see a progression in the values of the deflection as time (t) increases.

6.0 CONCLUSION

A framework for the prediction of the performance of potholes developed from transverse crack is formulated incorporating time as a factor for the development and the further deepening of the pothole, while also taking into account the elastic modulus of the supporting subgrade. It is important to have such a model to understand the rate of development of potholes as wheel load applies on the cracked pavement and the progression of the deflection as time passes by. This model helps to predict what happens to a road as it ages under a regular loading and prepares the road managers for adequate maintenance before roads become death trap.

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