

Inverse Thermoelastic Problem of An Elliptical Crown of Thin Plate

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ABSTRACT

This paper is concerned with inverse thermoelastic problem of an elliptical plate to determine the temperature distribution and unknown temperature gradient at point $\xi = b$ for all time ($t > 0$) with the help of Mathieu transform and integral transform techniques.

Keywords: Inverse thermoelastic problem, Mathieu transform, Marchi-Fasulo transform.

AMS SUBJECT CLASSIFICATION NO. 35-XX, 44-XX, 80-XX

1 INTRODUCTION

The small transverse vibrations produced in a circular crown of thin plate with certain boundary condition is solved by **Marchi** [1] with the help of Hankel and Laplace transforms. This problem is solved by Mathieu transform and Marchi-Fasulo transform techniques.

2 STATEMENT OF THE PROBLEM

The differential equation which governs the phenomenon in elliptical co-ordinates is given by

$$b^2 \left[\left(\frac{\partial^2 \omega(\xi, \eta, z, t)}{\partial \xi^2} + \frac{\partial^2 \omega(\xi, \eta, z, t)}{\partial \eta^2} \right) \frac{2d^{-2}}{(\cosh 2\xi - \cos 2\eta)} \right] + \frac{\partial^2 \omega(\xi, \eta, z, t)}{\partial z^2} + \frac{\partial^2 \omega(\xi, \eta, z, t)}{\partial t^2} = \frac{P(\xi, \eta, z, t)}{2\rho d}$$

$$a \leq \xi \leq b, \quad -h \leq z \leq h, \quad 0 \leq \eta \leq 2\pi, \quad (1)$$

Where

ω = Displacement, ξ, η are elliptical co-ordinates

$b^2 = \frac{D}{2\rho n}$ = flexural rigidity

ρ = Density of the material

$D = \frac{2Ed^3}{3(1-\sigma^2)}$, E = Young's Modulus

h = thickness of the plate

p = forcing function

σ = coefficient of Poisson

$2d$ = Interfocal length of elliptic crown

The solution of equation (1) is given by

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$$\omega(a, \eta, z, t) = \left[\left(\frac{\partial^2}{\partial \xi^2} + \frac{\partial^2}{\partial \eta^2} \right) \frac{2d^{-2}}{(\cosh 2\xi - \cos 2\eta)} \right]_{\xi=a} = 0$$

$$0 \leq \eta \leq 2\pi \text{ for all } t. \quad (2)$$

$$\omega(b, \eta, z, t) = \left[\left(\frac{\partial^2}{\partial \xi^2} + \frac{\partial^2}{\partial \eta^2} \right) \frac{2d^{-2}}{(\cosh 2\xi - \cos 2\eta)} \right]_{\xi=b} = 0 \quad (3)$$

$$\omega(a, \eta, z, 0) = f(\xi, \eta, z), \quad \frac{\partial \omega}{\partial t} = g(\xi, \eta, z),$$

$$0 \leq \eta \leq 2\pi \text{ for all } t \quad (4)$$

$$\left[\omega + k_1 \frac{\partial \omega}{\partial z} \right]_{z=h} = 0, \quad \left[\omega + k_2 \frac{\partial \omega}{\partial z} \right]_{z=-h} = 0 \quad (5)$$

The equations (1) – (5) constitute the mathematical formulation of the problem under consideration.

3 REQUIRED RESULT

FINITE MATHIEU TRANSFORM

The Mathieu transform of $[f(\xi, \eta)]$ is defined as

$$M[f(\xi, \eta)] = \bar{f}(q_{2n, m}) = \int_0^{2\pi} \int_a^b (\cosh 2\xi - \cos 2\eta) B_{2n}(\xi, q_{2n, m}) \times c_{e_{2n}}(\eta, q_{2n, m}) d\xi d\eta \quad (6)$$

$$B_{2n}(\xi, q_{2n, m}) = [\{ FeY_{2n}(a, q_{2n, m}) - FeY_{2n}(b, q_{2n, m}) \} \times$$

$$Ce_{2n}(\xi, q_{2n, m}) - \{ Ce_{2n}(a, q_{2n, m}) - Ce_{2n}(b, q_{2n, m}) \} \times FeY_{2n}(\xi, q_{2n, m})] \quad (7)$$

$q_{2n,m}$ are the roots of the equation

$$Ce_{2n}(b, q)FeY_{2n}(a, q) - FeY_{2n}(b, q)Ce_{2n}(a, q) = 0$$

And

$$FeY_{2n}(\xi, q) = \sum_{r=0}^{\infty} A_{2r}^{(2n)} \frac{Ce_{2n}(\xi, q)}{A_0^{2n}} Y_{2r}(2k' \sinh \xi), |\sinh \xi| > 0, R(\xi) > 0$$

Where $k = q$, Y is Bessel function.

Property of the transform:

$$M \left[\left(\frac{\partial^2}{\partial \xi^2} + \frac{\partial^2}{\partial \eta^2} \right) \frac{2d^{-2}}{(\cosh 2\xi - \cos 2\eta)} \right]^2 \omega(\xi, \eta) = \frac{-4q_{2n,m}}{d^2} \bar{\omega} \tag{8}$$

Inversion of Mathieu transform is

$$\omega(\xi, \eta) = \frac{1}{\pi} \sum_{n=0}^{\infty} \sum_{m=1}^{\infty} \frac{\bar{\omega} B_{2n}(q_{2n,m})}{\int_a^b Ce_{2n}^2(\xi, q_{2n,m}) [\cosh 2\xi - \theta_{2n,m}] d\xi} \tag{9}$$

Where

$$\theta_{2n,m} = \frac{1}{\pi} \int_0^{2\pi} ce_{2n}^2(\eta, q_{2n,m}) \cos 2\eta d\eta \tag{10}$$

4 SOLUTION

By applying finite Mathieu transform defined in (5) repeatedly and finite Marchi-Fasulo transform and Laplace transform to the equation (2), and then inversion of transforms, we get

$$\omega(\xi, \eta, z, t) = \sum_{l=0}^{\infty} \frac{P_l(z)}{\mu_l} \sum_{n=0}^{\infty} \sum_{m=1}^{\infty} C_{2n,m} B_{2n}(\xi, z, q_{2n,m}) ce_{2n}(\eta, z, q_{2n,m}, t) \tag{11}$$

and

$$C_{2n,m} = \frac{1}{\pi} \frac{\bar{\omega}(q_{2n,m}, z, t)}{b \int_a^b B_{2n}^2(\xi, z, q_{2n,m}) [\cosh 2\xi - \theta_{2n,m}] d\xi} \tag{12}$$

Hence the solution is given by

$$\omega(\xi, \eta, z, t) = \frac{1}{\pi} \sum_{l=0}^{\infty} \frac{P_l(z)}{\mu_l} \sum_{n=0}^{\infty} \sum_{m=1}^{\infty} \frac{\bar{\omega}(q_{2n,m}, z, t) B_{2n}(\xi, z, q_{2n,m}) ce_{2n}(\eta, z, q_{2n,m}, t)}{b \int_a^b B_{2n}^2(\xi, z, q_{2n,m}) [\cosh 2\xi - \theta_{2n,m}] d\xi} \tag{13}$$

Where,

$$\bar{\omega} = \bar{f}(\lambda_{2n,m}) \cos b \lambda_{2n,m}^2 t + \frac{\bar{g}(\lambda_{2n,m}^2)}{b \lambda_{2n,m}^2} \sin b \lambda_{2n,m}^2 t$$

$$+ \frac{1}{2 \rho d b \lambda_{2n,m}^2} \int_0^1 \sin b \lambda_{2n,m}^2 (t - \tau) \cdot \bar{P}(\lambda_{2n,m}^2, \tau) d\tau \tag{14}$$

5 CONCLUSION

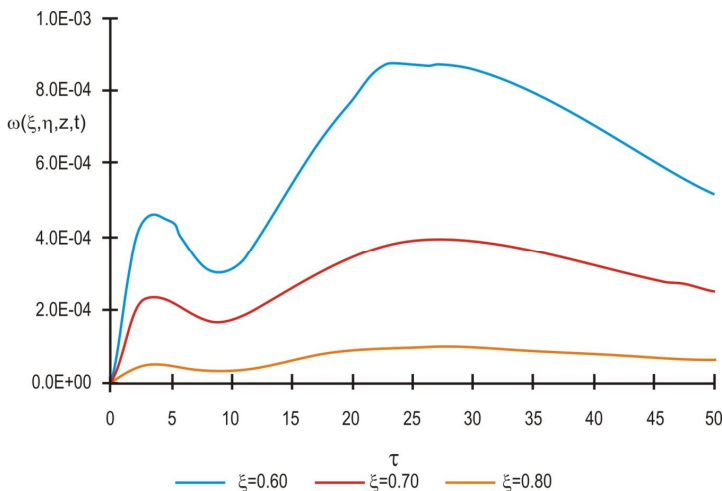
In this paper, an elastic vibration of elliptic plate have been determined with the help of finite Mathieu transform and finite Marchi-Fasulo transform and Laplace transform techniques. The expression is represented graphically. The results that are obtained can be useful to the design of structures or machines in engineering applications.

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Graph $\omega(\xi, \eta, z, t)$ Versus t for different values of ξ