Inverse Thermoelastic Problem of An Elliptical Crown of Thin Plate

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ABSTRACT

This paper is concerned with inverse thermoelastic problem of an elliptical plate to determine the temperature distribution and unknown temperature gradient at point \( \xi = b \) for all time \((t > 0)\) with the help of Mathiue transform and integral transform techniques.

Keywords: Inverse thermoelastic problem, Mathiue transform, Marchi-Fasulo transform.

AMS SUBJECT CLASSIFICATION NO. 35-XX, 44-XX, 80-XX

1 INTRODUCTION

The small transverse vibrations produced in a circular crown of thin plate with certain boundary condition is solved by Marchi [1] with the help of Hankel and Laplace transforms. This problem is solved by Mathieu transform and Marchi-Fasulo transform techniques.

2 STATEMENT OF THE PROBLEM

The differential equation which governs the phenomenon in elliptical co-ordinates is given by

\[
b^2 \left( \frac{\partial^2 \omega(\xi, \eta, z,t)}{\partial \xi^2} + \frac{\partial^2 \omega(\xi, \eta, z,t)}{\partial \eta^2} \right) \frac{2d^2}{(\cosh 2\xi - \cos 2\eta)} - \frac{\partial^2 \omega(\xi, \eta, z,t)}{\partial z^2} \frac{2d}{\rho d} = P(\xi, \eta, z,t) \quad a \leq \xi \leq b, \quad -h \leq z \leq h, \quad 0 \leq \eta \leq 2\pi.
\]

Where

\( \omega \) =Displacement, \( \xi, \eta \) are elliptical co-ordinates

\( b^2 = \frac{D}{2 \rho n} \) = flexural rigidity

\( \rho \) = Density of the material

\( D = \frac{2Ed^3}{3(1-\sigma^2)} \), \( E \) = Young’s Modulus

\( h \) = thickness of the plate

\( p \) = forcing function

\( \sigma \) = coefficient of Poisson

\( 2d \) = Interfocal length of elliptic crown

The solution of equation (1) is given by

\[
\omega(a, \eta, z,t) = \left[ \frac{\partial^2}{\partial \xi^2} + \frac{\partial^2}{\partial \eta^2} \right] \frac{2d^2}{(\cosh 2\xi - \cos 2\eta)} \bigg|_{\xi=a} = 0
\]

\( 0 \leq \eta \leq 2\pi \) for all \( t \).

\[
\omega(b, \eta, z,t) = \left[ \frac{\partial^2}{\partial \xi^2} + \frac{\partial^2}{\partial \eta^2} \right] \frac{2d^2}{(\cosh 2\xi - \cos 2\eta)} \bigg|_{\xi=b} = 0
\]

\( 0 \leq \eta \leq 2\pi \) for all \( t \)

\[
\omega(a, \eta, z,0) = f(\xi, \eta, z), \quad \frac{\partial \omega}{\partial t} = g(\xi, \eta, z),
\]

\( 0 \leq \eta \leq 2\pi \) for all \( t \)

\[
\left[ \omega + k_1 \frac{\partial \omega}{\partial z} \right]_{z=h} = 0, \quad \left[ \omega + k_2 \frac{\partial \omega}{\partial z} \right]_{z=-h} = 0 \quad (5)
\]

The equations (1) – (5) constitute the mathematical formulation of the problem under consideration.

3 REQUIRED RESULT

FINITE MATHIEU TRANSFORM

The Mathieu transform of \( [f(\xi, \eta)] \) is defined as

\[
M[f(\xi, \eta)] = \int_{a}^{b} \int_{-h}^{h} f(\xi, \eta, z) B_{2n}(\xi, \eta, z) \cos 2\eta d\xi d\eta
\]

\[
B_{2n}(\xi, \eta, z) = \left[ \{ FeY_{2n}(a, \eta, z) - FeY_{2n}(b, \eta, z) \} \times CeY_{2n}(\xi, \eta, z) \right] F_{2n}(\xi, \eta, z)
\]

\[
Ce_{2n}(\xi, \eta, z) = \left[ \{ CeY_{2n}(a, \eta, z) - CeY_{2n}(b, \eta, z) \} \times FeY_{2n}(\xi, \eta, z) \right] F_{2n}(\xi, \eta, z)
\]
$q_{2n,m}$ are the roots of the equation

$$Ce_{2n}(b,q)Fe_{2n}(a,q) - Fe_{2n}(b,q)Ce_{2n}(a,q) = 0$$

And

$$Fe_{2n}(\xi,q) = \sum_{r=0}^{\infty} A_{2r} \frac{Ce_{2n}(\xi,q)}{A_{2n}^{2}} Y_{2r}(2k'sinh\xi), |sinh\xi| > 0, R(\xi) > 0$$

Where $k = q$, $Y$ is Bessel function.

**Property of the transform:**

$$M \left[ \frac{\varrho^2}{\varrho^2 + \varrho^2} - \frac{2d^2}{(cosh2\varrho - cos2\eta)} \right] = \frac{-4q_{2n,m}}{d^2}$$

(8)

**Inversion of Mathieu transform is**

$$\omega(\xi, \eta) = \frac{1}{\pi} \sum_{n=0}^{\infty} \sum_{m=1}^{\infty} b B_{2n}(q_{2n,m})$$

(9)

Where

$$\theta_{2n,m} = \frac{1}{\pi} \int_{0}^{2\pi} ce_{2n}(\eta, q_{2n,m}) cos2\eta d\eta$$

(10)

**4 SOLUTION**

By applying finite Mathieu transform defined in (5) repeatedly and finite Marchi-Fasulo transform and Laplace transform to the equation (2), and then inversion of transforms, we get

$$\omega(\xi, \eta, t) = \frac{1}{\pi} \sum_{l=0}^{\infty} \frac{P_{l}(z)}{\mu_{l}} \sum_{n=0}^{\infty} \frac{C_{2n,m}(\xi, \eta, q_{2n,m})}{\varrho^{2}(q_{2n,m}, \xi, \eta, q_{2n,m}, t)}$$

(11)

and

$$C_{2n,m} = \frac{1}{\pi} b \int_{a}^{b} B_{2n}(\xi, \eta, q_{2n,m})|cosh2\xi - \theta_{2n,m}| d\xi$$

(12)

Hence the solution is given by

$$\omega(\xi, \eta, t) = \frac{1}{\pi} \sum_{l=0}^{\infty} \frac{P_{l}(z)}{\mu_{l}} \sum_{n=0}^{\infty} \frac{\varrho^{2}(q_{2n,m}, \xi, \eta, q_{2n,m})}{\varrho^{2}(q_{2n,m}, \xi, \eta, q_{2n,m}, t)} B_{2n}(\xi, \eta, q_{2n,m})|cosh2\xi - \theta_{2n,m}| d\xi$$

(13)

Where,

$$\varrho^{2} = \int \lambda_{2n,m} b \lambda_{2n,m}^{2} \sin b \lambda_{2n,m}^{2} \lambda_{2n,m}^{2}$$

$$+ \frac{1}{2} \int_{0}^{\infty} 1 \int b \lambda_{2n,m}^{2} \lambda_{2n,m}^{2} \sin b \lambda_{2n,m}^{2} \lambda_{2n,m}^{2}$$

(14)

**5 CONCLUSION**

In this paper, an elastic vibration of elliptic plate have been determined with the help of finite Mathieu transform and finite Marchi-Fasulo transform and Laplace transform techniques. The expression is represented graphically. The results that are obtained can be useful to the design of structures or machines in engineering applications.

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**REFERENCES**