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Inverse Thermoelastic Problem of An Elliptical Crown of Thin Plate

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ABSTRACT

This paper is concerned with inverse thermoelastic problem of an elliptical plate to determine the temperature distribution and unknown temperature gradient at point $\xi = b$ for all time (t > 0) with the help of Mathiue transform and integral transform techniques.

Keywords : Inverse thermoelastic problem, Mathiue transform, Marchi-Fasulo transform.

AMS SUBJECT CLASSIFICATION NO. 35-XX, 44-XX, 80-XX

1 INTRODUCTION

The small transverse vibrations produced in a circular crown of thin plate with certain boundary condition is solved by **Marchi** [1] with the help of Hankel and Laplace transforms. This problem is solved by Mathieu transform and Marchi-Fasulo transform techniques.

2 STATEMENT OF THE PROBLEM

The differential equation which governs the phenomenon in elliptical co-ordinates is given by

$$b^2\left[\left(\frac{\partial^2\omega(\xi,\eta,z,t)}{\partial\xi^2}+\frac{\partial^2\omega(\xi,\eta,z,t)}{\partial\eta^2}\right)\frac{2d^{-2}}{(cosh2\xi - cos2\eta}\right]^2 + \frac{\partial^2\omega(\xi,\eta,z,t)}{\partial z^2}+\frac{\partial^2\omega(\xi,\eta,z,t)}{\partial t^2}=\frac{P(\xi,\eta,z,t)}{2\rho d}$$

$$a \le \xi \le b, \quad -h \le z \le h, \quad 0 \le \eta \le 2\pi, \tag{1}$$

Where

ω =Displacement, ξ,η are elliptical co-ordinates

$b^2 = \dfrac{D}{2\rho n}$ = flexural rigidity

ρ = Density of the material

$D = \dfrac{2Ed^3}{3(1-\sigma^2)}$, E = Young's Modulus

h = thickness of the plate

p= forcing function

σ = coefficient of Poisson

2d= Interfocal length of elliptic crown

The solution of equation (1) is given by

$$\omega(a,\eta,z,t)=\left[\left(\frac{\partial^2}{\partial\xi^2}+\frac{\partial^2}{\partial\eta^2}\right)\frac{2d^{-2}}{(cosh2\xi - cos2\eta}\right]_{\xi=a}=0$$

$0 \le \eta \le 2\pi$ for all t. (2)

$$\omega(b,\eta,z,t)=\left[\left(\frac{\partial^2}{\partial\xi^2}+\frac{\partial^2}{\partial\eta^2}\right)\frac{2d^{-2}}{(cosh2\xi - cos2\eta}\right]_{\xi=b}=0 \tag{3}$$

$$\omega(a,\eta,z,0)=f(\xi,\eta,z), \quad \frac{\partial\omega}{\partial t}=g(\xi,\eta,z),$$

$0 \le \eta \le 2\pi$ for all t (4)

$$\left[\omega + k_1\frac{\partial\omega}{\partial z}\right]_{z=h}=0, \left[\omega + k_2\frac{\partial\omega}{\partial z}\right]_{z=-h}=0 \tag{5}$$

The equations (1) – (5) constitute the mathematical formulation of the problem under consideration.

3 REQUIRED RESULT

FINITE MATHIEU TRANSFORM

The Mathieu transform of $[f(\xi,\eta)]$ is defined as

$$M[f(\xi,\eta)]=\bar{f}(q_{2n,m})=\int_0^{2\pi}\int_a^b(\cosh 2\xi-\cos 2\eta)\, B_{2n}(\xi,q_{2n,m})\times ce_{2n}(\eta,q_{2n,m})d\xi\, d\eta \tag{6}$$

$$B_{2n}(\xi,q_{2n,m})=[\{FeY_{2n}(a,q_{2n,m})-FeY_{2n}(b,q_{2n,m})\}\times Ce_{2n}(\xi,q_{2n,m})-\{Ce_{2n}(a,q_{2n,m})-Ce_{2n}(b,q_{2n,m})\}\ x\ FeY_{2n}(\xi,q_{2n,m})] \tag{7}$$

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$q_{2n,m}$ are the roots of the equation

$$Ce_{2n}(b,q)FeY_{2n}(a,q) - FeY_{2n}(b,q)Ce_{2n}(a,q) = 0$$

And

$$FeY_{2n}(\xi,q) = \sum_{r=0}^{\infty} A_{2r}^{(2n)} \frac{Ce_{2n}(\xi,q)}{A_0^{2n}} Y_{2r}(2k'sinh\xi), \left|sinh\xi\right| > 0, R(\xi) > 0$$

Where k = q, Y is Bessel function.

Property of the transform:

$$M\left[\left(\frac{\partial^2}{\partial\xi^2}+\frac{\partial^2}{\partial\eta^2}\right)\frac{2d^{-2}}{(cosh2\xi - cos2\eta)}\right]^2 \omega(\xi,\eta) = \frac{-4q_{2n,m}}{d^2}\overline{\omega} \tag{8}$$

Inversion of Mathieu transform is

$$\omega(\xi,\eta) = \frac{1}{\pi}\sum_{n=0}^{\infty}\sum_{m=1}^{\infty}\frac{\overline{\omega}B_{2n}(q_{2n,m})}{\int_a^b Ce_{2n}^2(\xi,q_{2n,m})[\cosh 2\xi - \theta_{2n,m}]d\xi} \tag{9}$$

Where

$$\theta_{2n,m} = \frac{1}{\pi}\int_0^{2\pi} ce_{2n}^2(\eta, q_{2n,m})\cos 2\eta \, d\eta \tag{10}$$

4 SOLUTION

By applying finite Mathieu transform defined in (5) repeatedly and finite Marchi-Fasulo transform and Laplace transform to the equation (2), and then inversion of transforms, we get

$$\omega(\xi,\eta,z,t) = \sum_{l=0}^{\infty}\frac{P_l(z)}{\mu_l}\sum_{n=0}^{\infty}\sum_{m=1}^{\infty} C_{2n,m}B_{2n}(\xi,z,q_{2n,m})ce_{2n}(\eta,z,q_{2n,m},t) \tag{11}$$

and

$$C_{2n,m} = \frac{1}{\pi}\frac{\overline{\omega}(q_{2n,m},z,t)}{\int_a^b B_{2n}^2(\xi,z,q_{2n,m})[\cosh 2\xi - \theta_{2n,m}]d\xi} \tag{12}$$

Hence the solution is given by

$$\omega(\xi,\eta,z,t) = \frac{1}{\pi}\sum_{l=0}^{\infty}\frac{P_l(z)}{\mu_l}\sum_{n=0}^{\infty}\sum_{m=1}^{\infty}\frac{\overline{\omega}(q_{2n,m},z,t)B_{2n}(\xi,z,q_{2n,m})ce_{2n}(\eta,z,q_{2n,m},t)}{\int_a^b B_{2n}^2(\xi,z,q_{2n,m})[\cosh 2\xi - \theta_{2n,m}]d\xi} \tag{13}$$

Where,

$$\overline{\omega} = \bar{f}(\lambda_{2n,m})\cos b\lambda_{2n,m}^2 \, t + \frac{\bar{g}(\lambda^2{}_{2n,m})}{b\lambda^2{}_{2n,m}}\sin b\lambda^2{}_{2n,m}\,.\,t$$

$$+\frac{1}{2\rho db\lambda_{2n,m}^2}\int_0^1 \sin b\lambda^2{}_{2n,m}\,.\,(t-\tau).\,\overline{P}\,(\lambda^2{}_{2n,m},\tau)d\tau \tag{14}$$

5 CONCLUSION

In this paper, an elastic vibration of elliptic plate have been determined with the help of finite Mathiue transform and finite Marchi-Fasulo transform and Laplace transform techniques. The expression is represented graphically. The results that are obtained can be useful to the design of structures or machines in engineering applications.

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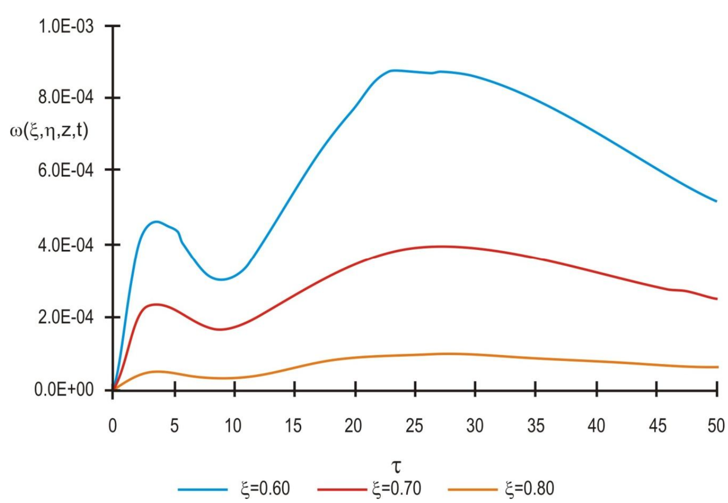

Graph $\omega(\xi,\eta,z,t)$ Versus **t** for different values of ξ