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Improved iterative method for solving parametric linear systems.

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ABSTRACT

Consider linear systems whose matrix and right-hand side vector depend affine-linearly on parameters varying within prescribed intervals. In this paper a new C-XSC(C for eXtended Scientific Computing) software for the symmetric single step method with generalized interval arithmetic for computing an enclosure for the solution set is proposed. Numerical examples illustrating the applicability of the proposed methods are solved..

Keywords : Parametric linear systems; C-XSC; generalized interval; validated interval software; symmetric single step method.

1 INTRODUCTION

SOLVING parametric linear systems involving uncertainties in the parameters is an important part of the solution to many scientific and engineering problems. Usually, in most engineering design problems, models in operational research, linear prediction problems, etc. [17] there are complicated dependencies between coefficients. The main reason for this dependency is that the errors in several different coefficients may be caused by the same factor [3, 8, 15]. More precisely, consider a parametric system

$$A(p) \cdot x = b(p) \qquad (1)$$

where the matrix $A(p) \in \mathbb{R}^{n\times n}$ and the vector $b(p) \in \mathbb{R}^n$ depend affine linearly on a parameter vector $p \in \mathbb{R}^k$.

Since, each individual component of $A(p)$, $b(p)$ is an affine-linear combination of the k parameters [16]

$$a_{ij}(p) := a_{ij}^{(0)} + \sum_{\nu=1}^{k} p_\nu a_{ij}^{(\nu)}, \quad b_i(p) := b_i^{(0)} + \sum_{\nu=1}^{k} p_\nu b_i^{(\nu)}$$

Denote the $k+1$ numerical matrices $A^{(0)} := (a_{ij}^{(0)}), A^{(1)} := (a_{ij}^{(1)}), \cdots, A^{(k)} := (a_{ij}^{(k)}) \in \mathbb{R}^{n\times n}$and the corresponding numerical vectors $b^{(0)} := (b_i^{(0)}), b^{(1)} := (b_i^{(1)}), \cdots, b^{(k)} := (b_i^{(k)}) \in \mathbb{R}^n$. Hence, the parametric matrix and the right-hand side vector can be represented by

$$A(p) := A^{(0)} + \sum_{\nu=1}^{k} p_\nu A^{(\nu)}, \quad b(p) := b^{(0)} + \sum_{\nu=1}^{k} p_\nu b^{(\nu)}$$

and the parametric system (1) can be rewritten in the following form

$$(A^{(0)} + \sum_{\nu=1}^{k} p_\nu A^{(\nu)})x = b^{(0)} + \sum_{\nu=1}^{k} p_\nu b^{(\nu)} \qquad (2)$$

where the parametric vector p varies within the range $[p] \in \mathbb{IR}^k$.

The solution set of (2), called parametric solution set, and is defined as

$\sum^p := \sum(A(p), b(p), [p]) := \{x \in \mathbb{R}^n | A(p) \cdot x = bp$ for some $p \in [p]\}$

Since the solution set has a complicated structure (it does not even need to be convex), which is difficult to find, one looks for the interval hull $\boxdot(\sum)$ here $\sum$ is a nonempty bounded subset of $\mathbb{R}^n$. For $\sum \subseteq \mathbb{R}^n$, define $\mathbb{PR}^n \longrightarrow \mathbb{IR}^n$ by1

$$\boxdot(\sum) := [\inf \sum, \sup \sum] = \bigcap\{[x] \in \mathbb{IR}^n | \sum \subseteq [x]\}$$

The calculation of $\boxdot(\sum)$ is also quite expensive.

The non-parametric interval matrix and vector, are obtained from the parametric matrix and vector, by

$$A([p]) := \boxdot(A(p) \in \mathbb{R}^{n\times n} | \ p \in [p]),$$
$$b([p]) := \boxdot(b(p) \in \mathbb{R}^n | \ p \in [p])$$

respectively.

The system of linear interval equations,

$$A([p]) \cdot x = b([p])$$

Corresponds to the parametric one (the elements of $A([p]), b([p])$ are assumed to be independent), and

$\sum^g := \sum(A([p]), b([p])) := \{x \in \mathbb{R}^n | A \cdot x = b$ for some $A \in A([p]), b \in b([p])\}$ is the interval solution set corresponding to the parametric one. The parametric solution set is a subset of the corresponding interval solution set and has often a much smaller volume than the latter [18]:-

$$\sum(A(p), b(p), [p]) \subseteq \sum(A([p]), b([p])).$$

Since it is quite expensive to obtain $\sum^p$ or $\boxdot(\sum^p)$, it would be a more realistic task to find an interval vector $[y] \in \mathbb{IR}^n$ which tightly encloses $\sum^p$.

Probably the first general purpose method for computing outer (and inner) bounds for the interval hull of ($\sum^p$) is based on the fixed-point interval iteration theory developed by S. Rump. In [21] he applies the general verification theory for system of nonlinear equations to the solution of parametric linear systems involving affine-linear dependencies. This method was generalized in [17] by proving that a sharp enclosure

[1] PR^n is the power set over R^n . Given a set S, the power set of S is the setoff all subset of S

of the iteration matrix expands the scope of application of the method over problems involving the so-called column-dependent matrices. Meanwhile, there were many attempts to construct suitable methods for solving parameter dependent interval linear systems [3, 8, 9, 10, 13, 15, 16, 17]. We do not intend to give here a complete overview of methods used for solving linear systems with dependent data.

In practice it is usually required that the matrix $A(p)$ is an H-matrix.

In this paper, we propose a new C-XSC software (C- for Extended Scientific Computing)[5] of the symmetric single step method for the solution of the systems of parametric linear equations with the use of „generalized interval". We will compare our method to other methods. The rest of the paper is set as follows. In Section 2, some Basic notation is introduced. In section 3, we recall the fundamentals of „generalized interval" and present its interpretation as the modified of interval arithmetic. The main results of this paper is presented in Section 4. Another modification for the symmetric single step method is introduced in Section 5. Numerical and practical examples illustrating the features of the proposed method are provided in Section 6. Last section concludes with some remarks.

2 Basic notations

We use the following notations $\mathbb{R}, \mathbb{R}^n, \mathbb{R}^{n\times n}, \mathbb{IR}, \mathbb{IR}^n, \mathbb{IR}^{n\times n}$, to denote the set of real numbers, the set of real vectors with n components, the set of real n×n matrices, the set of intervals, the set of interval vectors with n components and the set of n×n interval matrices, respectively. By interval we mean a real compact interval[1, 11]

$$[x] := [a, b] := \{x \in \mathbb{R} \mid \ a \le x \le b\}$$

For $[x], [y] := [c, d]$ we define

- The mid-point $\operatorname{mid}([x]) := (a+b)/2$,
- the Radius $\operatorname{rad}([x]) := (b-a)/2$,
- the absolute value $|[x]| := \max\{\ |a|, |b|\ \}$,
- the distance $q([x],[y]) := \max\{|a-c|, |b-d|\}$,
- minimal absolute value (mignitude)

$$\prec [x] \succ := \min\{|x| : x \in [x]\} = \begin{cases} \min\{|a|, |b|\} & \text{if} \quad 0 \notin [x] \\ 0 & \text{else} \end{cases} \tag{3}$$

For interval vectors and interval matrices, these quantities are defined componentwise. If for two interval vectors $[u], [v] \in \mathbb{IR}^n$ we have $[u_i] \cap [v_i] \neq \emptyset$, $i = 1,2,\cdots,n$, then $[u] \cap [v] := ([u_i] \cap [v_i])$ otherwise $[u] \cap [v] = \emptyset$. In addition, for $[u], [v] \in \mathbb{IR}^n$we define $[u] \subseteq [v$ iff $[u_i] \subseteq [v_i]$, $i = 1,2\cdots,n$. Furthermore, we repeat some relations concerning the distance:

$$q([u],[v]) \le q([u],[w]) + q([w],[v])$$
$$q([u]+[w],[v]+[w]) = q([u],[v])$$
$$q([u]+[v],[w]+[z]) \le q([u],[w]) + q([v],[z])$$

if $[u], [u], [w], [z] \in \mathbb{IR}^n$.

For square interval matrices we define the comparison matrix (Ostrowsky matrix) $\langle [A] \rangle := (C_{ij}) \in \mathbb{R}^{n\times n}$ using (3) by setting

$$c_{ij} := \begin{cases} -|a_{ij}| & \text{if} \quad i \neq j \\ \langle [a_{ij}] \rangle & \text{if} \quad i = j \end{cases}$$

A square matrix $[A] \in \mathbb{IR}^{n\times n}$ is called regular if all $A \in [A]$ are nonsingular.

If $\operatorname{mid}([A]) \cdot [A]$ is regular then $[A]$ is strongly regular. An interval matrix $[A]$ is an H-matrix iff there exist a vector $v > 0$ such that $\prec [A] \succ v > 0$.

Definition 1 [23] *Let $A, B, C \in \mathbb{R}^{n\times n}$. Then $A = B - C$ is a regular splitting of A if $C \ge 0$ and B is nonsingular with $B^{-1} \ge 0$.*

Theorem 1 [23] *Assume that is $A \in \mathbb{R}^{n\times n}$ nonsingular, that $A^{-1} \ge 0$ and that $A = B - C$ is a regular splitting of A. Then $\rho(B^{-1}C) < 1$, where $\rho(\ )$ denotes the spectral radius of a matrix.*

Regular splitting was introduced in [23], where one can also find the proof of Theorem 1.

3 Generalized intervals

Generalized intervals are intervals whose bounds are not constrained to be ordered, for example [-2,2] and [2,-2] are generalized intervals. They have been introduced in[6, 14, 22] so as to improve the algebraic structure of intervals, while maintaining the inclusion monotonicity. The set of generalized intervals is denoted by $\mathbb{KR}$ and is divided into three subset:

- The set of proper intervals with bounds ordered increasingly. These proper intervals are identified with classical intervals. The set of proper intervals is denoted $\mathbb{IR} := \{[a, b] | a \le b\}$. Strictly proper intervals satisfy $a < b$.
- The set of improper intervals with bounds ordered decreasingly. It is denoted by $\overline{\mathbb{IR}} := \{[a, b] | a \ge b\}$. Strictly improper intervals satisfy $a > b$.
- The set of degenerated intervals $\{[a, b] | a = b\} = \mathbb{IR} \cap \overline{\mathbb{IR}}$. Degenerated intervals are identified to reals.

Therefore, form a set of reals $\{x \in \mathbb{R}^n | a \le x \le b\}$, one can build the two generalized intervals [a,b] and [b,a]. It will be convenient to switch from one to the other keeping the underlying set of reals unchanged. To this purpose, the following three operations are introduced:

- The dual operation is defined by dual([a, b]) = [b, a].
- The proper projection is defined by
  pro([a, b]) = [min{a, b},max{a, b}].
- The improper projection is defined by
  imp([a, b]) = [max{a, b},min{a, b}].

The generalized intervals are partially ordered by an inclusion which extends the inclusion of classical intervals. Given two generalized intervals $[x] = [\underline{x}, \overline{x}]$ and $[y] = [\underline{y}, \overline{y}]$, the inclusion is defined by $[x] \subseteq [y] \Leftrightarrow \underline{y} \le \underline{x} \wedge \overline{x} \le \overline{y}$. For example, $[-1,1] \subseteq [-1.1,1.1]$ (this matches the set inclusion), $[-1.1,1.1] \subseteq [-1,1]$ (the inclusion between the underlying set of real is reversed

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for improper intervals) and $[2,0.9] \subseteq [-1,1]$. As degenerated intervals are identified to reals, if $[x]$ proper then $x \in [x] \Leftrightarrow x \subseteq [x]$. On the other hand, if $[x]$ is strictly improper then for all $x \in \mathbb{R}$ the inclusion $x \subseteq [x]$ is false.

The generalized interval arithmetic (Kaucher arithmetic) extends the classical interval arithmetic. Its definition can be found in[7, 14]. When only proper intervals are involved, this arithmetic coincides with the interval arithmetic: $[x] \circ [y] = \{x \circ y \in \mathbb{R} | x \in [x], y \in [y]\}$. When proper and improper intervals are involved, some new expressions are used . For example, $[a,b] + [c,d] = [a+c, b+d]$ and if $a,b,c,d \geq 0$ then $[a,b] \cdot [c,d] = [a \cdot c, b \cdot d]$. The following useful property provides some bounds on the proper projection of the results of the generalized interval arithmetic. Let us consider $[x], [y] \in \mathbb{KR}$ and $\circ \in \{+,-,\cdot,/\}$. If pro$[x] \circ$ pro$[y]$ is defined then $[x] \circ [y]$ is defined and it satisfies

$$\text{pro}([x] \circ [y]) \subseteq (\text{pro}[x]) \circ (\text{pro}[y])$$

Generalized interval arithmetic has better algebraic properties than the classical interval arithmetic. The addition in $\mathbb{KR}$ is a group. The opposite of an interval $[x]$ is – dual$[x]$, i.e. $[x] + (-\text{dual}[x]) = [x] - \text{dual}[x] = [0,0]$.

The multiplication in $\mathbb{KR}$ restricted to generalized intervals whose proper projection does not contain 0 is also a group. The inverse of such a generalized interval $[x]$ is $1/\text{dual}[x]$, i.e., $[x] \cdot (1/\text{dual}[x]) = [x]/(\text{dual}[x]) = [1,1]$.

Although addition and multiplication in $\mathbb{KR}$ are associative, they are not distributive. The addition and multiplication in $\mathbb{KR}$ are linked by the following distributivity laws[23,26]. Whatever are $[x], [y], [z] \in \mathbb{KR}$

- Conditional distributivity:

$$[x] \cdot [y] + (\text{imp}[x]) \cdot [z] \subseteq [x] \cdot ([y] + [z]) \subseteq [x] \cdot [y] + (\text{pro}[x]) \cdot [z].$$

The three following particular cases will be of practical interest in this paper.

- Subdistributivity: if $[x] \in \mathbb{KR}$ then
  $[x] \cdot ([y] + [z]) \subseteq [x] \cdot [y] + [x] \cdot [z]$;
- Superdistributivity: if $[x] \in \overline{\mathbb{KR}}$ then
  $[x] \cdot ([y] + [z]) \supseteq [x] \cdot [y] + [x] \cdot [z]$;
- Distributivity: if $x \in \mathbb{R}$ then
  $x \cdot ([y] + [z]) = x \cdot [y] + x \cdot [z]$.

Another useful property of the Kaucher arithmetic is its monotonicity with respect to the inclusion. Whatever are $\circ \in \{+,-,\cdot,/\}$ and $[x], [y], [xx], [yy] \in \mathbb{KR}$,

$[x] \subseteq [xx] \cap [y] \subseteq [yy] \Longrightarrow ([x] \circ [y]) \subseteq ([xx] \circ [yy])$.

The next example illustrates the way these properties will be used in the sequel.

Example 1: *Consider the expression* $[x] + [u][v] \subseteq [y]$. *Subtracting* $\text{dual}([u][v]) = (\text{dual}[u])(\text{dual}[v])$ *to each side preserves the inclusion:* $[x] + [u][v] - \text{dual}([u][v]) \subseteq [y] - (\text{dual}[u])(\text{dual}[v])$. *As* $-\text{dual}([u][v])$ *is the opposite of* $[u][v]$ *, the following inclusion is eventually proved to hold:* $[x] \subseteq [y] - (\text{dual}[u])(\text{dual}[v])$.

Finally, generalized interval vectors $[x] \in \mathbb{KR}^n$ and generalized interval matrices $[A] \in \mathbb{KR}^{n \times n}$ together with their additions and multiplications are defined similarly to their real and classical interval counterparts.

4 Generalized symmetric single step method

In this section we assume that the reader is familiar with the concept P contractions for proving the convergence of a fixed point iteration to a unique fixed point for an arbitrary starting vector. For the details please see [1,2].

We assume throughout that the matrix $A(p)$ is nonsingular, and moreover that its diagonal entries $a_{ii}(p)$ are all nonzero. We can express the matrix $A(p)$ as the matrix sum [1,20]

$$A(p) = D(p) + L(p) + U(p),$$

Where $D(p)$ is a diagonal matrix, and $L(p)$ and $U(p)$ are respectively strictly lower and upper triangular matrices. We can write (1) as

$$D(p) \cdot x = b(p) - (L(p) + U(p)) \cdot x.$$

Then

$$x = D^{-1}(p) \cdot (b(p) - (L(p) + U(p) \cdot x), \tag{4}$$

where

$$L(p) = \begin{pmatrix} 0 & 0 & \cdots & 0 \\ a_{21}(p) & 0 & \cdots & 0 \\ \vdots & \ddots & \ddots & \vdots \\ a_{n1}(p) & \cdots & a_{nn-1}(p) & 0 \end{pmatrix} \tag{5}$$

$$U(p) = \begin{pmatrix} 0 & a_{12}(p) & \cdots & a_{1n}(p) \\ \vdots & 0 & \ddots & \vdots \\ \vdots & \ddots & \ddots & a_{n-1n}(p) \\ 0 & 0 & \dots & 0 \end{pmatrix} \tag{6}$$

$$D^{-1}([p]) = \begin{pmatrix} \frac{1}{\text{dual}\, a_{11}([p])} & 0 & \cdots & 0 \\ 0 & \frac{1}{\text{dual}\, a_{22}([p])} & \cdots & 0 \\ \vdots & \ddots & \ddots & \vdots \\ 0 & 0 & \cdots & \frac{1}{\text{dual}\, a_{nn}([p])} \end{pmatrix} \tag{7}$$

Since the diagonal entries $a_{ii}(p)$ of $A(p)$ are nonzero, we can carry out the following iterative method derived from (4):

$$\left[x_i^{(l+1/2)}\right] = \begin{pmatrix} b_i([p]) - \sum_{j=1}^{i-1} \text{dual}\left(a_{ij}([p])\left[x_i^{(l+1/2)}\right]\right) \\ -\sum_{j=i+1}^{n} \text{dual}\left(a_{ij}([p])\left[x_i^{(l)}\right]\right) \end{pmatrix} \Big/ \text{dual}(a_{ii}([p])),$$

$$1 \leq i \leq n, \tag{8}$$

$$\left[x_i^{(l+1)}\right] = \begin{pmatrix} b_i([p]) - \sum_{j=1}^{i-1} \text{dual}\left(a_{ij}([p])\left[x_i^{(l+1/2)}\right]\right) \\ -\sum_{j=i+1}^{n} \text{dual}\left(a_{ij}([p])[x_i^{(l+1)}]\right) \end{pmatrix} \Big/ \text{dual}(a_{ii}([p])), \tag{9}$$

where the $[x^{(0)}]'s$ initial interval vector. We call this iteration procedure the **generalized symmetric single step method**.

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Theorem 2. *Consider interval linear system (1), we define* $L(p)$, $U(p)$ *and* $D^{-1}([p])$ *as in (5), (6) and (7), respectively. Then, the sequence* $\left[x^{(l)}\right]_{l=0}^{\infty}$ *calculated according to the iteration method (generalized symmetric single step method defined as in (8) and (9)), converges for all interval vectors* $[x^{(0)}] \in \mathbb{IR}^n$ *to* $[x^*]$*, where* $[x^*]$ *is the unique fixed point of the equation (4).*

5 GENERALIZED SYMMETRIC SINGLE STEP METHOD WITH INTERECTION

In this section we consider modifications of the preceding iterative methods which are based on the fact that if for any of these methods one is starting with an interval vector containing the limit, then all iterates contain the limit. Therefore the enclosure of the limit might be improved by forming intersections after each iteration step.

Theorem 3. *Let* $A(p) \in \mathbb{R}^{n\times n}$ and $b(p) \in \mathbb{R}^n$ *be given.* $L(p)$, $U(p)$ *and* $D^{-1}([p])$ *as in (5), (6) and (7), respectively. Let* $[x^*]$ *is the unique fixed point of the equation (4). We assume that we have an interval vector* $[\text{initial}] \in \mathbb{IR}^n$ *satisfying* $[x^*] \subseteq [\text{initial}]$. *We consider the generalized symmetric single step method with intersection.*

$$\begin{cases} [x^{(0)}] := [\text{initial}] \\ \textbf{for } i = 1 \textbf{ to } n \textbf{ do} \\ \left[x_i^{(l+1/2)}\right] := \left[x_i^{(l)}\right] \cap \left\{ \left(b_i([p]) - \sum_{j=1}^{i-1} \text{dual}\left(a_{ij}([p])\left[x_i^{(l+1/2)}\right]\right) - \sum_{j=i+1}^{n} \text{dual}\left(a_{ij}([p])\left[x_i^{(l)}\right]\right) \right) \Big/ \text{dual}(a_{ii}([p])) \right\} \\ \textbf{for } i = 1 \textbf{ to } n \textbf{ do} \\ \left[x_i^{(l+1)}\right] := \left[x_i^{(l+1/2)}\right] \cap \left\{ \left(b_i([p]) - \sum_{j=1}^{i-1} \text{dual}\left(a_{ij}([p])\left[x_i^{(l+1/2)}\right]\right) - \sum_{j=i+1}^{n} \text{dual}\left(a_{ij}([p])\left[x_i^{(l+1)}\right]\right) \right) \Big/ \text{dual}(a_{ii}([p])) \right\} \end{cases} \quad (10)$$

Then $\lim_{l\to\infty}[x^{(l)}] = [x^*]$

To get an interval vector [initial]. We assume that $A(p) \in \mathbb{R}^{n\times n}$ is an H-matrix. Let $L(p)$, $U(p)$ *and* $D^{-1}([p])$ and $[x^*]$ defined as in Theorem 2. Then we consider symmetric single step method with arbitrary $[x^{(0)}]$. We assume that $P := \prec [D([p])] \succ^{-1} |[L([p])] + [U([p])]|$, where $\rho(P) < 1$ (see Theorem 11.4 in [1]). For $m > l$ we get [2]:

$$\begin{aligned} q\left(\left[x^{(m)}\right],\left[x^{(l)}\right]\right) &\le q\left(\left[x^{(m)}\right],\left[x^{(m-1)}\right]\right) + \cdots + q\left(\left[x^{(l+1)}\right],\left[x^{(l)}\right]\right) \\ &\le P^{m-1} \cdot q\left(\left[x^{(m)}\right],\left[x^{(0)}\right]\right) + \cdots + P^{l} \cdot q\left(\left[x^{(1)}\right],\left[x^{(0)}\right]\right) \\ &= P^{l} \cdot \left(I + P + \cdots + P^{m-l-1}\right) \cdot P^{m-1} \cdot q\left(\left[x^{(1)}\right],\left[x^{(0)}\right]\right) \\ &\le P^{l} \cdot \left(\sum_{i=0}^{\infty} P^{i}\right) \cdot P^{m-1} \cdot q\left(\left[x^{(1)}\right],\left[x^{(0)}\right]\right) \\ &= P^{l} \cdot (I-P)^{-1} \cdot P^{m-1} \cdot q\left(\left[x^{(1)}\right],\left[x^{(0)}\right]\right). \end{aligned}$$

Since $\lim_{l\to\infty}[x^{(l)}] = [x^*]$, it holds that (set $m := 1$)

$$q\left(\left[x^*\right],\left[x^{(l)}\right]\right) \le P \cdot (I-P)^{-1} \cdot q\left(\left[x^{(1)}\right],\left[x^{(0)}\right]\right) =: u,$$

Then

$$\underline{x}^{(l)} - u \le \underline{x}^*, \quad \overline{x}^* \le \overline{x}^{(l)} + u$$

Hence, we get $\left[x^*\right] \subseteq \left[\underline{x}^{(l)} - u \ , \ \overline{x}^{(l)} + u\right] =: [\text{initial}]$

Algorithm 1. Interval linear systems (H-Matrix)

1. Computation of an initial interval vector

$P := \prec [D([p])] \succ^{-1} |[L([p])] + [U([p])]|,$

$[\text{initial}] := \left[\underline{x}^{(l)} - u, \overline{x}^{(l)} + u\right], \qquad l \ge 0$

2. Verification step

$[x^{(1)}]$:= [initial]

repeat

if intersection = 0 **then**

Using equations (8) and (9)

else Using equation (10)

until $[x^{(l+1)}]$ *and* $[x^{(l)}]$ *are equals*

3.

if $[x^{(l+1)}]$ *and* $[x^{(l)}]$ *are equals* **then**

$\hat{x} \in [x^{(l+1)}]$ ($\hat{x}$ the exact solution)

else *no inclusion can be computed*

6 NUMERICAL AND PRACTICAL EXAMPLES

Example(1):

$$\begin{pmatrix} 1 & p_1 \\ p_1 & p_2 \end{pmatrix} \cdot x = \begin{pmatrix} 2+p_2 \\ 2+p_2 \end{pmatrix},$$

where: $p_1 \in [-0.6, -0.4]$, $p_2 \in [1.8, 2.2]$

iteration =9,

Proposed method	Elaraby [4]	Popova [20]
[5.09756097,6.39130434]	[4.843137,7.0000]	[4.877162,6.551409]
[3.24390243,3.65217391]	[2.607843,4.6667]	[2.598498,4.258645]

Example(2):

$$\begin{pmatrix} 3 & p & p \\ p & 3 & p \\ p & p & 3 \end{pmatrix} \cdot x = \begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix}$$

where: $p \in [0, 1]$

iteration =2,

Proposed method	Elaraby [4]	Popova [20]
[0.111111,0.333334]	[0.111111,0.333334]	[0.177533,0.772466]
[0.111111,0.333334]	[0.111111,0.333334]	[0.080561,0.469439]
[0.111111,0.333334]	[0.111111,0.333334]	[-0.382168,0.132168]

N.B . We got the same result as Elaraby, because there is no dependency will be happened between the parameters.

Example(3):

$$\begin{pmatrix} 1+p_1+p_2 & p_1 & p_2 \\ 0 & p_1+p_2 & p_2 \\ 0.1 & 0 & 3p_1+p_2 \end{pmatrix} \cdot x = \begin{pmatrix} p_1+5p_2 \\ 2+p_1+3p_2 \\ 1+2p_1+p_2 \end{pmatrix}$$

where: $p_1 \in [0.4, 0.5]$, $p_2 \in [0.2, 0.3]$

iteration =10,

Proposed method	Elaraby [4]	Popova [20]
[-0.498863958,0.001988635]	[-0.776142,-0.132614]	[-0.468005,-0.022753]
[3.7604403341,4.523865872]	[3.325426,5.114795]	[3.631414,5.540667]
[1.305492442,1.428429383]	[1.103743,1.69829]	[1.256884,1.461288]

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Application:

we consider a linear resistive network, presented in [10,13]. The resistive network consists of two current sources J1 and J2 and nine resistors. The problem of finding the voltages $v_1,\cdots,v_5$, when the voltage of each conductance $g_i, i=1,2,\cdots,9$ varies independently in prescribed bounds $[g]_i, i=1,2,\cdots,9$, leads to the following parameterized linear system

$$\begin{pmatrix} g_1+g_6 & -g_6 & 0 & 0 & 0 \\ -g_6 & g_2+g_6+g_7 & -g_7 & 0 & 0 \\ 0 & -g_7 & g_3+g_7+g_8 & -g_8 & 0 \\ 0 & 0 & -g_8 & g_4+g_8+g_9 & -g_9 \\ 0 & 0 & 0 & -g_9 & g_5+g_9 \end{pmatrix} \cdot v = J$$

where $J=(10,0,10,0,0)^T$ and the parameters are subject to tolerances $[g]_i=[1-\delta,1+\delta], i=1,2,\cdots,9$.

We solve the system for different values of the tolerances δ varying from 0.1% to 10% of the nominal value.

iteration =24, tolerance=0.1%

Proposed method	HBR method [10]	Elaraby [4]	Popova [20]
[7.03075,7.06131]	[6.8693, 7.2950]	[6.89898,7.29765]	[7.01337,7.16844]
[4.14748,4.16051]	[4.0689, 4.4971]	[3.97569,4.40530]	[4.11566,4.24797]
[5.41170,5.42022]	[5.2501, 5.6612]	[5.26906,5.65655]	[5.39177,5.51732]
[2.16468,2.16809]	[2.0183, 2.3568]	[2.04981,2.32733]	[2.13647,2.22716]
[1.08234,1.08404]	[1.0397, 1.1931]	[1.00461,1.18717]	[1.05937,1.2244]

7 CONCLUSION

The problem of solving parametric linear systems of equations is very important in practical applications. A simple method for determining an outer solution to the linear system considered has been suggested in section 5 by using the method presented in section 4. Some numerical and practical examples are solved. The methods that presented can be applied to big real life problems such as structural engineering [12,19] without any problems.

REFERENCES

[1] Alefeld, G., Herzberger,J. : Introduction to interval computations, Academic Press, London, 1983.

[2] G. Alefeld and U. Schaefer: Iterative Methods for Linear Complementarity Problem with Interval Data, Computing 70, pp.235-259 (2003).

[3] Dessombz O., et al.: Analysis of mechanical systems using interval computations applied to finite element methods, Journal of Sound and Vibration 239 (2001) 5, 949-968.

[4] Elaraby, A. A.; El-Owny, H.: Iterative Method for solving parametric interval linear systems with affine-linear dependency. ICCI Conference, Helwan ,Egypt, Dec. 10-13, 2012.

[5] Hofschuster, W.; Kraemer, W., C-XSC 2.0 – A C++ Class library for Extended Scientific Computing. In: Numerical Software with Result Verification, R. Alt, A. Frommer, B. Kearfott, W. Luther (eds), Springer Lecture Notes in Computer Science 2991, pp. 15-35, 2004.

[6] Kaucher, E.: Ueber metrische und algebraische Eigenschaften einiger beim numerischen Rechnen auftretender Raume. PhD thesis, Karlsruhe, 1973.

[7] Kaucher, E.: Interval analysis in the Extended Interval Space $\mathbb{IR}$. Computing, Suppl. 2, pp. 33-49 , 1980.

[8] Kolev, L.: A Method for Outer Interval Solution of Linear Parametric Systems. Reliable Computing, vol. 10, nr. 3, pp. 227-239, 2004.

[9] Kraemer, W.; Popova, E. D.: Zur Berechnung von verlaesslichen Außen- und Inneneinschließungen bei parameterabhaengigen linearen Gleichungssystemen. PAMM - Proceedings in Applied Mathematics and Mechanics, vol. 4, issue 1, pp. 670-671, 2004.

[10] Milan Hladik: Enclosures to the solution sets of parametric interval linear systems. SCAN2010, Lyon, France, September 27-30, 2010.

[11] Moore, R.: Interval analysis, Prentice-Hall, Inc. Englewood Cliffs, N. J. , 1966.

[12] Neumaier, A.; Pownuk, A.: Linear Systems with Large Uncertainties with Applications to Truss Structures. Reliable Computing, vol. 13, issue 2, pp. 149-171, 2007.

[13] OKumura, K.:An Application of Interval Operations to Electric Network Analysis, Bull. of the Japan Soc. for Industrial & Applied Mathematics, 2 (1993), pp. 115-127.

[14] Ortolf, H.J.: Eine Verallgemeinerung der Itervallarithmetik. Geselschaft fuer Mathematik und Datenverarbeitung, Bonn 11, pp. 1-71, 1969.

[15] Popova, E., Kraemer, W.: Parametric Fixed- Point Iteration Implemented in C-XSC, Preprint 2003/3, Universitaet Wuppertal, 2003.

[16] Popova, E.: Improved Parametric Fixed-Point Iteration, Preprint Inst. of Mathematics & Informatics, BAS, Sofia, 2003.

[17] Popova, E.: Generalizing the Parametric Fixed- Point Iteration. Proceeding in Applied Mathematics and Mechanics (PAMM) 4, issue 1, pp. 680-681, 2004.

[18] Popova, E.; Kraemer,W.: Inner and Outer Bounds for the Solution Set of Parametric Linear Systems. Journal of Computational and Applied Mathematics, vol. 199, issue 2, pp. 310-316, 2007.

[19] Popova, E.; Datcheva, M.; Iankov, R.; Schanz, T.: Mechanical Models with Interval Parameters. In: K. Guerlebeck, L. Hempel, C. Koenke (Eds.) IKM2003: Digital Proceedings of 16th International Conference on the Applications of Computer Science and Mathematics in Architecture and Civil Engineering, ISSN 1611-4086, Bauhaus University Weimar, 2003.

[20] Popova, E.D.: Parametric interval linear solver. Numer. Algorithms, vol. 37, nr. 1-4, pp. 345-356, 2004.

[21] Rump, S.: Verification methods for dense and sparse systems of equations, Topic in Validated Computations, Oldenburg, North-Holland, 1994.

[22] S.P. Shary: On optimal solution of interval linear equations. SIAM J. Numer. Anal. 32 (1995) 610–630.

[23] Varga, R. S.: Matrix iterative analysis. Prentice-Hall, Englewood Cliffs, New Jersey, 1962.

