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Heat Transfer Boundary Layer Flow Past an inclined Stretching Sheet in Presence of Magnetic field

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ABSTRACT

In the present study the problem of a steady hydro magnetic flow of an incompressible electrically conducting fluid over an inclined stretching sheet, where the flow is generated due to a linear stretching sheet. Using suitable similarity transformations the governing dimensional partial differential equations of the problem are reduced to dimensionless couple nonlinear ordinary differential equations and are solved numerically by Runge-Kutta fourth-fifth (RK-45) order method along shooting technique. The effects of various parameters on the flow fields are investigated and presented graphically. The results illustrate that velocity profile decrease due to increase of magnetic parameter, Angle of inclination, Prandtl number, Eckert number and Chandrasekhar number while velocity increases for increasing values of Grashof number . On the other hand temperature profile increases in the presence of magnetic parameter, angle of inclination, Prandtl number, Eckert number and Chandrasekhar number while temperature decreases for increasing Grashof number. The results have possible technological applications in liquid-based systems involving stretchable materials. The skin friction coefficient and the local Nusselt number are proportional to rate of change of velocity and temperature gradient respectively which are presented in Tables 1-2.

Keywords : MHD; Heat and Mass Ttransfer; Angle of Inclination; Stretching Sheet.

1 INTRODUCTION

THE study of boundary layer flow and heat transfer over an inclined stretching plate has generated much interest. In recent years its significant applications in industrial manufacturing processes such as paper production , hot rolling, wire drawing, drawing of plastic films, glass-fiber, aerodynamic, extrusion of plastic sheets, cooling of metallic sheets in a cooling bath, which would be in the form of an electrolyte and polymer sheet extruded continuously from a die are few practical applications of moving surfaces. During its manufacturing process a stretched sheet interacts with the ambient fluid thermally and mechanically. Both the kinematics of stretching and the simultaneous heating or cooling during such processes has a decisive influence on the quality of the final products (Magyari& Keller [1]).In recent years; MHD flow problems have become more important industrially. Indeed, MHD laminar boundary layer behavior over a stretching surface is a significant type of flow having considerable practical applications in chemical engineering, electrochemistry and polymer processing. In this pioneering work, Sakiadis [2] developed the flow field due to a flat surface, which is moving with a constant velocity in a quiescent fluid. Crane [3] extended the work of Sakiadis [2] for the two-dimensional problem where the surface velocity is proportional to the distance from the flat surface. Crane [3] investigated the flow due to a stretching sheet with linear surface velocity and obtained the similarity solution to the problem. Later this problem has been extended to various aspects by considering non-Newtonian fluids, more general stretching velocity, magneto-hydrodynamic (MHD) effects, porous sheets, porous media and heat or mass transfer. Andreson et al. [4] extended the work of Crane [3] to non-Newtonian power law fluid over a linear stretching sheet. Grubka and Bobba [5] analyzed heat transfer studies by considering the power-law variation of surface temperature. Chakrabarti and Gupta [6] have studied the hydro magnetic flow and heat transfer in a fluid initially at rest and at uniform temperature over a stretching sheet at a different uniform temperature. Cortell [7] studied the magneto hydrodynamics flow of a power-law fluid over a stretching sheet. Abel and Mahesh [8] presented an analytical and numerical solution for heat transfer in a steady laminar flow of an incompressible viscoelastic fluid over a stretching sheet with power-law surface temperature, including the effects of variable thermal conductivity and non-uniform heat source and radiation. Samad and Mohebujjaman [9] investigated the case along a vertical stretching sheet in presence of magnetic field and heat generation. Jhankal and Kumar studied MHD Boundary Layer Flow Past a Stretching Plate with Heat Transfer [10]. As many natural phenomena and engineering problems are worth being subjected to MHD analysis, the effect of transverse magnetic field on the laminar flow over a stretching surface was studied by a number of

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researchers [11-14].Singh, P.K.[15] studied heat and mass transfer in MHD boundary layer flow past an inclined plate with viscous dissipation in porous medium. Since the study of heat transfer is important in some cases, in the present paper we studied the hydro-magnetic flow and heat transfer of fluid over an inclined stretching plate with the effect of heat. The resulting governing equations are transformed into a system of non-linear ordinary differential equations by applying a suitable similarity transformation. These equations are solved numerically by RK- 45 method using shooting technique and discussed the results from the physical point of view.

2 MATHEMATICAL FORMULATION OF THE PROBLEM

Consider a two dimensional steady laminar MHD viscous incompressible electrically conducting fluid along an inclined stretching sheet. X- direction is taken along the leading edge of the inclined stretching sheet and Y is normal to it and extends parallel to X-axis. A magnetic field of strength B0 is introduced to the normal to the direction to the flow. The plate temperature Tw(>T∞), where T∞ is the temperature of the fluid far away from the plate. Let u and v be the velocity components along the X and Y axis respectively in the boundary layer region. Under the above assumptions and usual boundary layer approximation, the dimensional governing equations of continuity, momentum and energy under the influence of externally imposed magnetic field are:

Equation of continuity:

$$\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}=0 \qquad (1)$$

Momentum equation:

$$u\frac{\partial u}{\partial x}+v\frac{\partial u}{\partial y}=\nu\frac{\partial^2 u}{\partial y^2}+g\beta(T-T_\infty)\cos\gamma-\frac{\sigma B_0^2 u}{\lambda\rho} \qquad (2)$$

Energy Equation:

$$u\frac{\partial T}{\partial x}+v\frac{\partial T}{\partial y}=\alpha\frac{\partial^2 T}{\partial y^2}+\frac{\mu}{\rho c_p}\left(\frac{\partial u}{\partial y}\right)^2+\frac{\sigma B_0^2 u^2}{\rho c_p} \qquad (3)$$

Boundary conditions are:

$$u=ax,\ v=0,\ T=T_w,+\lambda x^m \ \text{ at }\ y=0,$$

$$u=0,\ T=T_\infty \ \text{ as }\ y\to\infty$$

To convert the governing equations into a set of similarity equations, we introduce the following similarity transformation:

$$u=axf'(\eta),\ v=-\sqrt{\frac{\mu a}{\rho}}f(\eta),\ \eta=y\sqrt{\frac{\rho a}{\mu}},\ \theta(\eta)=\frac{T-T_\infty}{T_w-T_\infty}$$

From the above transformations, the non-dimensional, nonlinear and coupled ordinary differential equations are obtained as

$$f'''+ff''-f'^2+G_r\theta\cos\gamma-Qf'=0 \qquad (4)$$

$$\theta''+P_r f\theta'-P_r m f'\theta+P_r M f'^2+P_r E_c f''^2=0 \qquad (5)$$

The transform boundary conditions:

$$f=0,\ f'=1,\ \ \theta=1 \text{ at } \eta=0 \text{ and } f'=\theta=0 \ \text{ as } \eta\to\infty$$

3 RESULTS AND DISCUSSION

Using the similarity transformation the governing equations of the problem are reduced to coupled third and second order non-linear ordinary differential equations subject to boundary conditions and are solved numerically by Runge-Kutta fourth-fifth order method using well known software along with shooting iteration technique. In the present study the result of velocity profile and temperature profile for the effect of various parameters which are shown in the following figures.

Effect of magnetic parameter: The effects of M on the velocity and Temperature profiles are shown in Fig. 2 and Fig.8. From Fig.2 it is seen that an increase in the magnetic parameter decreases the primary velocity. This result agrees with the expectations, since the magnetic field exerts a retarding effect on the free convective flow. This field may control the flow characteristics, an increase in M results in thinning of the boundary layer. But reverse trend arises for temperature profile. Similar results are found for the effects of Chandrasekhar number, Q which is shown in Fig.7 and Fig.13 respectively.

Effect of angle of inclination: Fig.3 and Fig.9 show the velocity and Temperature profiles obtained by the numerical simulation for various values of γ . It is observed that the velocity is decreased for increasing values of γ . But it is interesting to note that there is no variation on the temperature profile for $30^0 \le \gamma \le 45^0$ whereas noticeable increasing effects for $60^0 \le \gamma \le 80^0$.

Effect of Grashof number: Fig.4 and Fig.10 exhibit the velocity and Temperature profiles obtained by the numerical simulation for various values of G_r . Since the flow is accelerated due to the enhancement in buoyancy force corresponding to an increasing in the G_r , as a result the thermal G_r influences the velocity within the boundary layer, so velocity increases which is shown in Fig.4. For increasing values of G_r, the temperature profile is decreased, as we have considered cooling plate, so the thermal boundary layer is thinner which is expected.

Effect of Prandtl number: Fig.5 and Fig.11 show the variation of velocity and Temperature profiles under the influence of P_r. For increasing values of P_r the velocity is decreased, i.e. there appears a thin boundary which indicates the decrease of free convection. From Fig.5 it also seen that the momentum boundary layer thickness is thicker in case of air and salt water than fresh water, whereas reverse trend arises for temperature profile which is expected.

Effect of Eckert number: Fig.6 and Fig.12 show the velocity and Temperature profiles obtained by the numerical simulation for various values of E_c. For increasing values of E_c the velocity is decreased and temperature profile is increased.

Tables 1–2 exhibit the behavior of $f''(0)$, and $-\theta'(0)$, for various values of magnetic parameter, Chandrasekhar number, Prandtl number, **Eckert number, Grashof number** and angle of inclination. From Table-1, it is observed that $f''(0)$ is decreased for various values of M, Q, γ, E_c and P_r and increased for increasing values of G_r. From Table- 2, it is observed that $\theta'(0)$, is decreased with the increasing values of G_r but increased for increasing values of M, Q, γ, E_c and P_r.

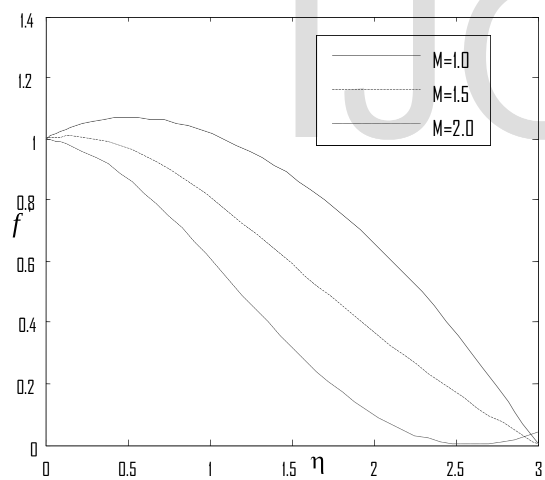

Fig.2 Velocity profile for various values of
M and $\gamma = 60^0, m = 1.0, Q = 0.8, P_r = 1.0, G_r = 5.0, E_c = 1.0$

4 Conclusions

In this paper we investigated the steady MHD boundary layer flow and heat transfer over on an inclined stretching sheet. Numerical solutions are obtained through shooting technique. The results illustrate that velocity profile decrease due to increase of magnetic parameter, Angle of inclination, Prandtl number, Eckert number and Chandrasekhar number while velocity increases for increasing values of Grashof number . On the other hand temperature profile increases in the presence of magnetic parameter, angle of inclination, Prandtl number, Eckert number and Chandrasekhar number while temperature decreases for increasing Grashof number.

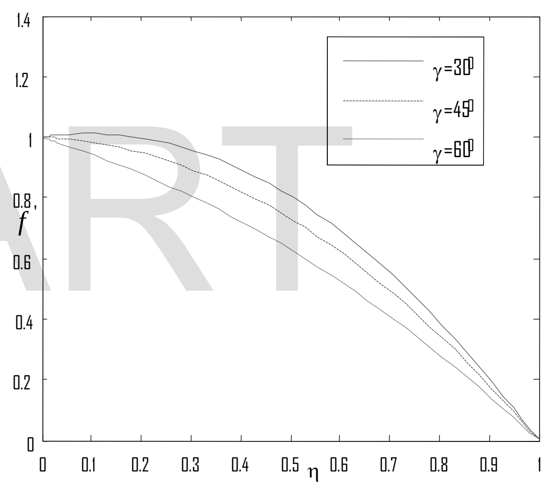

Fig.3 Velocity profile for various values of
γ and $M = 1.0, m = 1.0, Q = 0.8, P_r = 1.0, G_r = 5.0, E_c = 1.0$

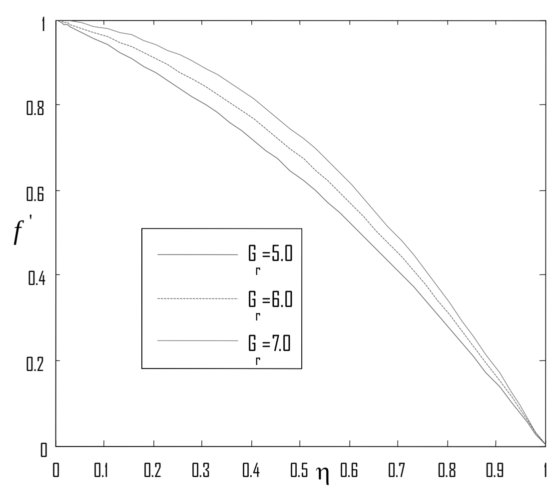

Fig.4 Velocity profile for various values of

G_r and $M = 1.0, m = 1.0, Q = 0.8, P_r = 1.0, \gamma = 60^0, E_c = 1.0$

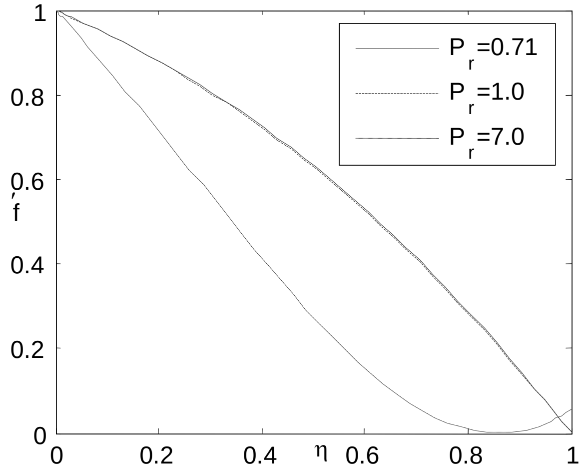

Fig.5 Velocity profile for various values of
P_r and $\gamma = 60^0, m = 1.0, Q = 0.8, M = 1.0, G_r = 5.0, E_c = 1.0$

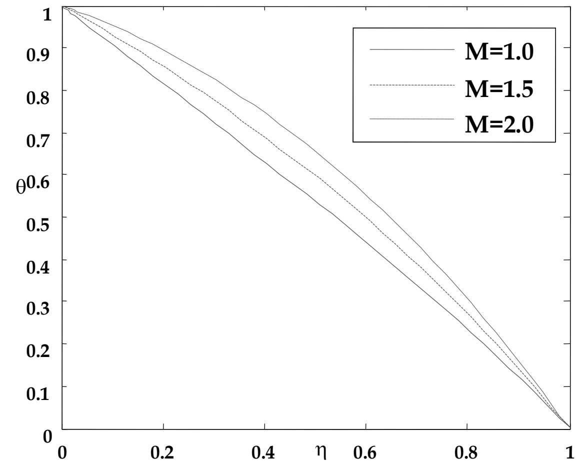

Fig.8 Temperature profile for various values of
M and $\gamma = 60^0, m = 1.0, Q = 0.8, P_r = 1.0, G_r = 5.0, E_c = 1.0$

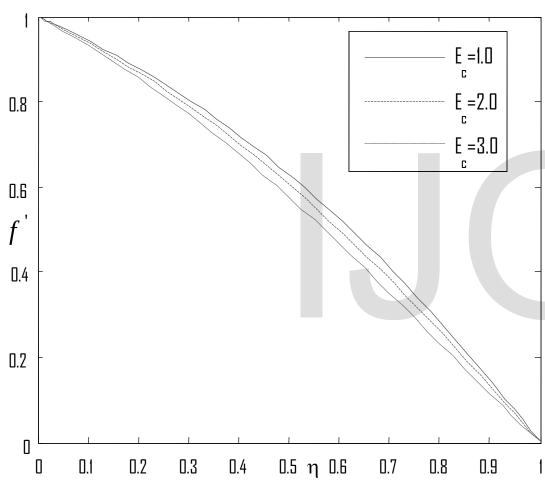

Fig.6 Velocity profile for various values of
E_c and $\gamma = 60^0, m = 1.0, Q = 0.8, M = 1.0, G_r = 5.0, P_r = 1.0$

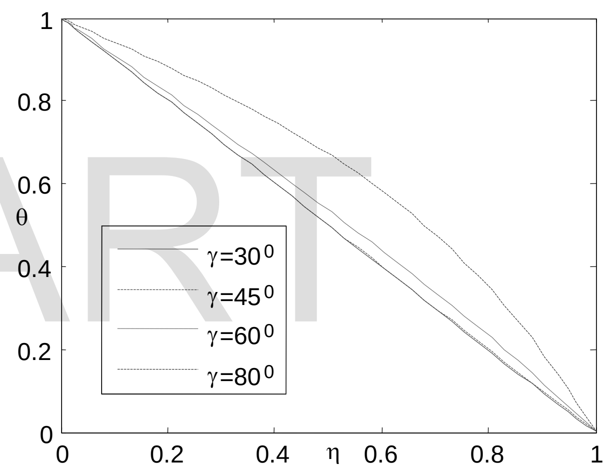

Fig.9 Temperature profile for various values of
γ and $M = 1.0, m = 1.0, Q = 0.8, P_r = 1.0, G_r = 5.0, E_c = 1.0$

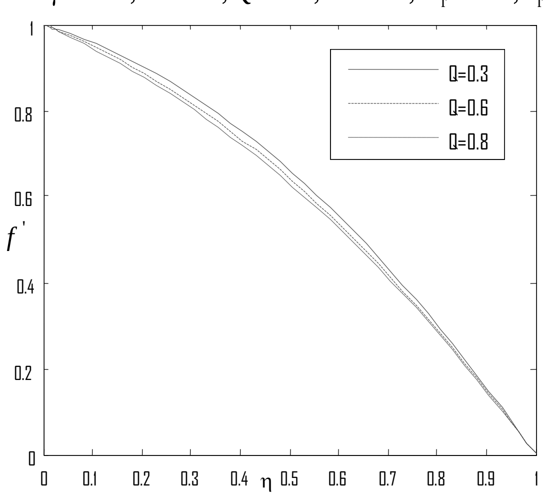

Fig.7 Velocity profile for various values of
Q and $\gamma = 60^0, m = 1.0, P_r = 1.0, M = 1.0, G_r = 5.0, E_c = 1.0$

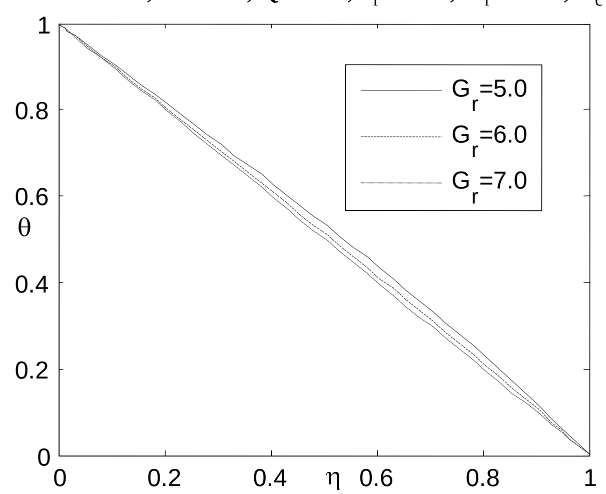

Fig.10 Temperature profile for various values of
G_r and $M = 1.0, m = 1.0, Q = 0.8, P_r = 1.0, \gamma = 60^0, E_c = 1.0$

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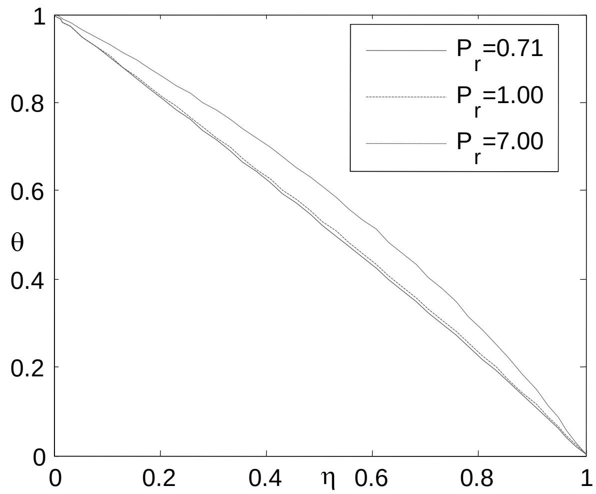

Fig.11 Temperature profile for various values of P_r and $\gamma = 60^0, m = 1.0, Q = 0.8, M = 1.0, G_r = 5.0, E_c = 1.0$

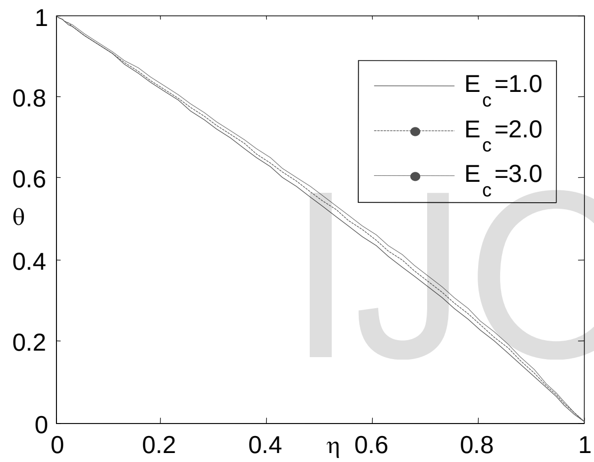

Fig.12 Temperature profile for various values of E_c and $\gamma = 60^0, m = 1.0, Q = 0.8, M = 1.0, G_r = 5.0, P_r = 1.0$

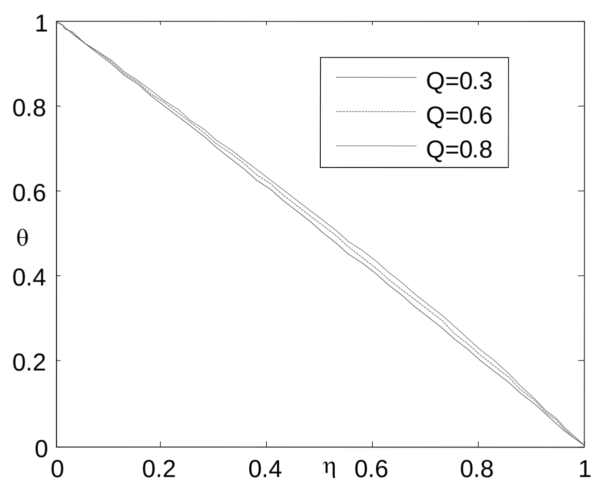

Fig.13 Temperature profile for various values of Q and $\gamma = 60^0, m = 1.0, P_r = 1.0, M = 1.0, G_r = 5.0, E_c = 1.0$

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Table-1: The skin friction coefficient at the wall, for various values of M, E_c, γ, P_r, G_r and Q

M	Q	G_r	γ	P_r	E_c	$f''(0)$
1.0	0.8	5.0	60^0	1.0	1.0	0.29521
1.5	0.8	5.0	60^0	1.0	1.0	0.10417
2.0	0.8	5.0	60^0	1.0	1.0	-0.086097
1.0	0.3	5.0	60^0	1.0	1.0	-0.358494
1.0	0.6	5.0	60^0	1.0	1.0	-0.462076
1.0	0.8	5.0	60^0	1.0	1.0	-0.52966
1.0	0.8	**5.0**	60^0	1.0	1.0	-0.52966
1.0	0.8	**6.0**	60^0	1.0	1.0	-0.31664
1.0	0.8	**7.0**	60^0	1.0	1.0	-0.10606
1.0	0.8	5.0	**30^0**	1.0	1.0	0.24153
1.0	0.8	5.0	**45^0**	1.0	1.0	-0.085208
1.0	0.8	5.0	**60^0**	1.0	1.0	-0.52966
1.0	0.8	5.0	60^0	**0.71**	1.0	-0.52002
1.0	0.8	5.0	60^0	**1.0**	1.0	-0.52966
1.0	0.8	5.0	60^0	**7.0**	1.0	-1.34502
1.0	0.8	5.0	60^0	1.0	**1.0**	-0.52966
1.0	0.8	5.0	60^0	1.0	**2.0**	-0.57893
1.0	0.8	5.0	60^0	1.0	**3.0**	-0.644907

Table-2: The local Nusselt number, $-\theta'(0)$, for various values of M, E_c, γ, P_r, G_r and Q.

M	Q	G_r	γ	P_r	E_c	$-\theta'(0)$
1.0	0.8	5.0	60^0	1.0	1.0	0.901009
1.5	0.8	5.0	60^0	1.0	1.0	0.649491
2.0	0.8	5.0	60^0	1.0	1.0	0.402410
1.0	**0.3**	5.0	60^0	1.0	1.0	0.965803
1.0	**0.6**	5.0	60^0	1.0	1.0	0.93194
1.0	**0.8**	5.0	60^0	1.0	1.0	0.901009
1.0	0.8	**5.0**	60^0	1.0	1.0	0.901009
1.0	0.8	**6.0**	60^0	1.0	1.0	0.95277
1.0	0.8	**7.0**	60^0	1.0	1.0	0.98432
1.0	0.8	5.0	**30^0**	1.0	1.0	0.98712
1.0	0.8	5.0	**45^0**	1.0	1.0	0.98633
1.0	0.8	5.0	**60^0**	1.0	1.0	0.901009
1.0	0.8	5.0	**80^0**	1.0	1.0	0.57498
1.0	0.8	5.0	60^0	**0.71**	1.0	0.92617
1.0	0.8	5.0	60^0	**1.0**	1.0	0.901009
1.0	0.8	5.0	60^0	**7.0**	1.0	0.62654
1.0	0.8	5.0	60^0	1.0	**1.0**	0.901009
1.0	0.8	5.0	60^0	1.0	**2.0**	0.86173
1.0	0.8	5.0	60^0	1.0	**3.0**	0.82561

Nomenclature

MHD	Magnetohydrodynamics
c_p	Specific heat of with constant pressure
g	Gravitational acceleration
f'	Velocity Profile
M	Magnetic parameter, $M = \frac{a\,\sigma B_0^2}{\rho\, c_p \lambda} x^{2-m};\quad 0 \le m \le 2$
ν	Kinematic viscosity
η	Similarity variable
α	Thermal diffusivity, $\alpha = \frac{k}{\rho\, c_p}$
β	Thermal Expansion Coefficient
ρ	Density
σ	Fluid electrical conductivity
θ	Dimensionless temperature
u	Velocity component in x-direction
v	Velocity component in y-direction
T	Temperature
P_r	Prandle number, $P_r = \frac{\mu c_p}{k}$
G_r	Local thermal Grashof number, $G_r = \frac{g\beta(T_w - T_\infty)}{a^2 x}$
E_c	Eckert number, $E_c = \frac{a^2 x^{2-m}}{\lambda c_p}$
Q	Chandrasekhar number, $Q = \frac{\sigma B_0^2}{a\lambda\rho}$
λ	Magnetic diffusivity
γ	Angle of inclination
B_0	Constant magnetic field intensity
a	Constant velocity
T_w	Temperature at the Plate
U_w	Stretching velocity at the wall
T_∞	Temperature of the fluid outside the boundary layer
Subscript	
w	Quantities at wall
∞	Quantities at the free stream