HEAT AND MASS TRANSFER EFFECTS ON FLOW PAST PARABOLIC STARTED VERTICAL PLATE WITH CONSTANT HEAT FLUX

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ABSTRACT

An exact solution of unsteady flow past a parabolic starting motion of the infinite vertical plate with constant heat flux, in the presence uniform mass diffusion has been studied. The plate temperature as well as concentration level near the plate are raised uniformly. The dimensionless governing equations are solved using Laplace-transform technique. The effect of velocity profiles are studied for different physical parameters like Prandt number, thermal Grashof number, mass Grashof number, Schmidt number and time. It is observed that the velocity increases with increasing values the thermal Grashof number or mass Grashof number. The trend is just reversed with respect to the chemical reaction parameter.

Key words: parabolic, heat flux, vertical plate.

INTRODUCTION

Natural convection on flow past an linearly accelerated vertical plate in the presence of viscous dissipative heat using perturbation method by Gupta *et al* (1979). Kafousias and Raptis (1981) extended this problem to include mass transfer effects subjected to variable suction or injection. Soundalgekar (1982) studied the mass transfer effects on flow past a uniformly accelerated vertical plate. Mass transfer effects on flow past an accelerated vertical plate with uniform heat flux was analyzed by Singh and Singh (1983). Free convection effects on flow past an exponentially accelerated vertical plate was studied by Singh and Naveen Kumar (1984). The skin friction for accelerated vertical plate has been studied analytically by Hossain and Shayo (1986). Mass transfer effects on exponentially accelerated infinite vertical plate with constant heat flux and uniform mass diffusion was studied by Basant Kumar Jha *et al* (1991). Agrawal et al (1998) studied free convection due to thermal and mass diffusion in laminar flow of an accelerated infinite vertical plate in the presence of magnetic filed. Agrawal et al (1999) further extended the problem of unsteady free convective flow and mass diffusion of an electrically conducting elasto-viscous fluid past a parabolic starting motion of the infinite vertical plate with transverse magnetic plate. The governing equations are tackled using Laplace transform technique.

The object of the present paper is to study the effects of flow past an infinite vertical plate subjected to parabolic motion under the action of constant heat flux with uniform mass diffusion. The dimensionless governing equations are solved using the Laplace-transform technique. The solutions are in terms of exponential and complementary error function.

MATHEMATICAL ANALYSIS

The unsteady flow of a viscous incompressible fluid past an infinite vertical plate with uniform diffusion, in the presence of constant heat flux has been considered. The x'-axis is taken along the plate in the vertically upward direction and the y-axis is taken normal to the plate. At time $t' \le 0$, the plate and fluid are at the same temperature T_∞ and concentration C'_∞ .At time $t' > 0$, the plate is started with a velocity $u = u_0.t'^2$ in its own plane against gravitational field and the temperature from the plate is raised to T_w and the concentration level near the plate are also raised to C'_w. Then under usual Boussinesq's approximation for unsteady parabolic starting motion is governed by the following equations:

$$\frac{\partial u}{\partial t'} = g\beta(T - T_\infty) + g\beta^*(C' - C'_\infty) + \nu \frac{\partial^2 u}{\partial y^2} \tag{1}$$

$$\rho C_p \frac{\partial T}{\partial t'} = k \frac{\partial^2 T}{\partial y^2} \tag{2}$$

$$\frac{\partial C'}{\partial t'} = D \frac{\partial^2 C'}{\partial y^2} \tag{3}$$

With the following initial and boundary conditions:

$$\begin{array}{lllll} & u = 0, & T = T_\infty, & C' = C'_\infty & \text{for all } y, t' \le 0 \\ t' > 0: & u = u_0.t'^2, & \dfrac{\partial T}{\partial y} = -\dfrac{q}{k}, & C' = C'_w & \text{at } y = 0 \\ & u \to 0 & T \to T_\infty, & C' \to C'_\infty & \text{as } y \to \infty \end{array} \tag{4}$$

On introducing the following non-dimensional quantities:

$$U = u\left(\frac{u_0}{\nu^2}\right)^{1/3}, \quad t = \left(\frac{u_0^{\,2}}{\nu}\right)^{1/3} t', \quad Y = y\left(\frac{u_0}{\nu^2}\right)^{1/3}, \quad \theta = \frac{T - T_\infty}{\frac{q}{k}\left(\frac{\nu}{u_0}\right)^{1/3}}, \; C = \frac{C' - C'_\infty}{C'_w - C'_\infty}$$

$$Gr = \frac{\nu^{1/3}\, g\beta q}{k\, u_0^{\,2/3}}, \; Gc = \frac{g\beta^*(C'_w - C'_\infty)}{(\nu.u_0)^{1/3}}, \; \Pr = \frac{\mu C_p}{k}, \; Sc = \frac{\nu}{D} \tag{5}$$

The equations (1) to (3) reduces to the following dimensionless form:

$$\frac{\partial U}{\partial t} = Gr\theta + GcC + \frac{\partial^2 U}{\partial Y^2} \tag{6}$$

$$\frac{\partial \theta}{\partial t} = \frac{1}{\Pr}\frac{\partial^2 \theta}{\partial Y^2} \tag{7}$$

$$\frac{\partial C}{\partial t} = \frac{1}{Sc}\frac{\partial^2 C}{\partial Y^2} \tag{8}$$

The corresponding initial and boundary conditions in dimensionless form are as follows:

$$U = 0, \quad \theta = 0, \quad C = 0 \quad \text{for all } Y, t \le 0$$

$$t > 0: \quad U = t^2, \quad \frac{\partial \theta}{\partial Y} = -1, \quad C = 1 \quad \text{at} \quad Y = 0 \tag{9}$$

$$U \to 0, \quad \theta \to 0, \quad C \to 0 \quad \text{as} \quad Y \to \infty$$

The dimensionless governing equations (6) to (8) and the corresponding initial and boundary conditions (9) are tackled using Laplace transform technique.

$$\theta = 2\sqrt{t}\left[\frac{\exp(-\eta^2 \Pr)}{\sqrt{\pi}\sqrt{\Pr}} - \eta\, erfc(\eta\sqrt{\Pr})\right] \tag{10}$$

$$C = erfc(\eta\sqrt{Sc}) \tag{11}$$

$$U = \frac{t^2}{3}\left[(3 + 12\eta^2 + 4\eta^4)erfc(\eta) - \frac{\eta}{\sqrt{\pi}}(10 + 4\eta^2)\exp(-\eta^2)\right] +$$
$$+d\,\frac{t^{3/2}}{3}\left[\begin{array}{l}\eta(6 + 4\eta^2)erfc(\eta) - \frac{4}{\sqrt{\pi}}(1 + \eta^2)\exp(-\eta^2) \\ + \frac{4}{\sqrt{\pi}}(1 + \eta^2 \Pr)\exp(-\eta^2 \Pr) - \eta\sqrt{\Pr}(6 + 4\eta^2 \Pr)erfc(\eta\sqrt{\Pr})\end{array}\right] +$$
$$+e\,t\left[\begin{array}{l}(1 + 2\eta^2 Sc)erfc(\eta\sqrt{Sc}) - \frac{2\eta\sqrt{Sc}}{\sqrt{\pi}}\exp(-\eta^2 Sc) \\ -(1 + 2\eta^2)erfc(\eta) + \frac{2\eta}{\sqrt{\pi}}\exp(-\eta^2)\end{array}\right] \tag{12}$$

where, $d=\dfrac{Gr}{\sqrt{\Pr}(1-\Pr)}$ $e=\dfrac{Gc}{(1-Sc)}$ and $\eta=\dfrac{y}{2\sqrt{t}}$

By the expression (12) the skin-friction at the plate is given by

$$\begin{aligned}\tau &= -\left(\frac{dU}{dy}\right)_{y=0}\\ &= -\frac{1}{2\sqrt{t}}\left(\frac{dU}{d\eta}\right)_{\eta=0}\\ &= \frac{8t^{3/2}}{3\sqrt{\pi}}-\frac{Gr\,t}{\sqrt{\Pr}(1+\sqrt{\Pr})}-\frac{2Gc\sqrt{t}}{\sqrt{\pi}(1+\sqrt{Sc})}\end{aligned} \tag{13}$$

By the expression (10), the rate of heat transfer in terms of Nusselt number in non dimensional form is given by

$$\begin{aligned}Nu &= -\left(\frac{d\theta}{dy}\right)_{y=0}\\ &= -\frac{1}{2\sqrt{t}}\left(\frac{d\theta}{d\eta}\right)_{\eta=0}\\ &=1\end{aligned} \tag{14}$$

By the expression (11), the rate of mass transfer in terms of Sherwood number in non dimensional form is given by

$$\begin{aligned}Sh &= -\left(\frac{dC}{dy}\right)_{y=0}\\ &= -\frac{1}{2\sqrt{t}}\left(\frac{dC}{d\eta}\right)_{\eta=0}\\ &= \frac{\sqrt{Sc}}{\sqrt{\pi t}}\end{aligned}$$

$$Sh \,\alpha\, \frac{\sqrt{Sc}}{\sqrt{t}} \tag{15}$$

From the above expression, it is observed that the Sherwood number is inversely proportional to square root of time and directly proportional to square root of Schmidt number.

TABLE I
Numerical values of skin-friction

Sc	Pr	t	Gr	Gc	τ
0.16	7.0	0.2	5	5	-0.841844
0.16	0.71	0.2	5	5	-3.453797
0.3	0.71	0.2	5	5	-3.826521
0.6	0.71	0.2	5	5	-4.398954
2.01	0.71	0.2	5	5	-6.021702
0.16	0.71	0.4	5	5	-4.488149
0.16	0.71	0.6	5	5	-4.954072
0.16	0.71	0.8	5	5	-5.059398
0.16	0.71	0.2	5	2	-1.334366
0.16	0.71	0.2	2	5	-3.247297
0.16	0.71	0.2	2	2	-1.127866

RESULTS AND DISCUSSION

For physical understanding of the problem numerical computations are carried out for different physical parameters *Gr*, *Gc*, *Sc* and *t* upon the nature of the flow and transport. The value of the Schmidt number Sc is taken to be 0.6 which corresponds to water-vapor. Also, the values of Prandtl number Pr are chosen such that they represent air (Pr = 0.71). The numerical values of the velocity are computed for different physical parameters like Prandtl number, thermal Grashof number, mass Grashof number, Schmidt number and time.

Figure 1. represents the effect of concentration profiles for different Schmidt number (Sc =0.16, 0.3, 0.6, 2.01). The effect of concentration is important in concentration field. The profiles have the common feature that the concentration decreases in a monotone fashion from the surface to a zero value far away in the free stream. It is observed that the wall concentration increases with decreasing values of the Schmidt number.

The temperature profiles are calculated for different values of Prandtl number (Pr=0.71, 7.0) and at time (t=0.2) are shown in figure 2. The effect of Prandtl number is important in temperature profiles. It is observed that the temperature increases with decreasing Prandtl number.

Figure 3 represents temperature profiles for different values of time (t=0.2, 0.4, 0.6, 0 .8) when Prandtl number (Pr=0.71). It is observed that temperature profiles decreases with increasing values of time.

The velocity profiles for different values of the time (t=0.2, 0.3, 0.4, 0.5) Pr=0.71, Sc=0.16, Gr=2, Gc=5 are shown in figure 4. It is observed that the velocity increases with decreasing values of time. Figure 5 demonstrates the effects of different Schmidt number (Sc=0.16, 0.24, 0.30, 0.60), Grashof number (Gr = 2), mass Grashof number (Gc = 5), and Pr=0.71 on the velocity at t = 0.2. It is observed that the velocity increases with decreasing values of the Schmidt number.

From Figures 6 and 7 we see that velocity increases with increase in Gc either heated or cooled (Gr < 0 or Gr > 0), Pr=0.71, Sc=0.16 and time at t=0.2.

The skin friction is tabulated in Table I. It is clear that skin-friction decreases with increase of Schmidt number. It is also observed that skin-friction increases with increase of Prandtl number. As time advances skin-friction decreases. Moreover the value of the skin friction decreases with increasing thermal Grashof number or mass Grashof number.

CONCLUSION

An exact solution of flow past a parabolic starting motion of the infinite vertical plate with uniform mass diffusion, in the presence of constant heat flux has been studied. The dimensionless governing equations are solved by the usual Laplace transform technique. The effect of the temperature, the concentration and the velocity fields for different physical parameters like Prandtl number, Schmidt number, thermal Grashof number and mass Grashof number are studied graphically. The conclusions of the study are as follows:

(i) The velocity increases with increasing mass Grashof number for both heating and cooling of the plate (Gr<0 or Gr>0), but the trend is just reversed with respect to the Schmidt number and time.

(ii) The temperature of the plate increases with decreasing values of the Prandtl number, but the trend is just reserved with respect to time.

(iii) The plate concentration increases with decreasing values of the Schmidt number.

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NOMENCLATURE

A Constants
C' species concentration in the fluid $kg\,m^{-3}$
C dimensionless concentration
C_p specific heat at constant pressure $J.kg^{-1}.k$
D mass diffusion coefficient $m^2.s^{-1}$
Gc mass Grashof number
Gr thermal Grashof number
g acceleration due to gravity $m.s^{-2}$
k thermal conductivity $W.m^{-1}.K^{-1}$

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Pr Prandtl number
Sc Schmidt number
T temperature of the fluid near the plate K
t' time s
u velocity of the fluid in the x'-direction $m.s^{-1}$
u_0 velocity of the plate $m.s^{-1}$
u dimensionless velocity
y coordinate axis normal to the plate m
Y dimensionless coordinate axis normal to the plate

Greek symbols
β volumetric coefficient of thermal expansion K^{-1}
β^* volumetric coefficient of expansion with concentration K^{-1}
μ coefficient of viscosity $Ra.s$
ν kinematic viscosity $m^2.s^{-1}$
ρ density of the fluid $kg.m^{-3}$
τ dimensionless skin-friction $kg.m^{-1}.s^2$
θ dimensionless temperature
η similarity parameter
$erfc$ complementary error function

Subscripts
w conditions at the wall
∞ free stream conditions

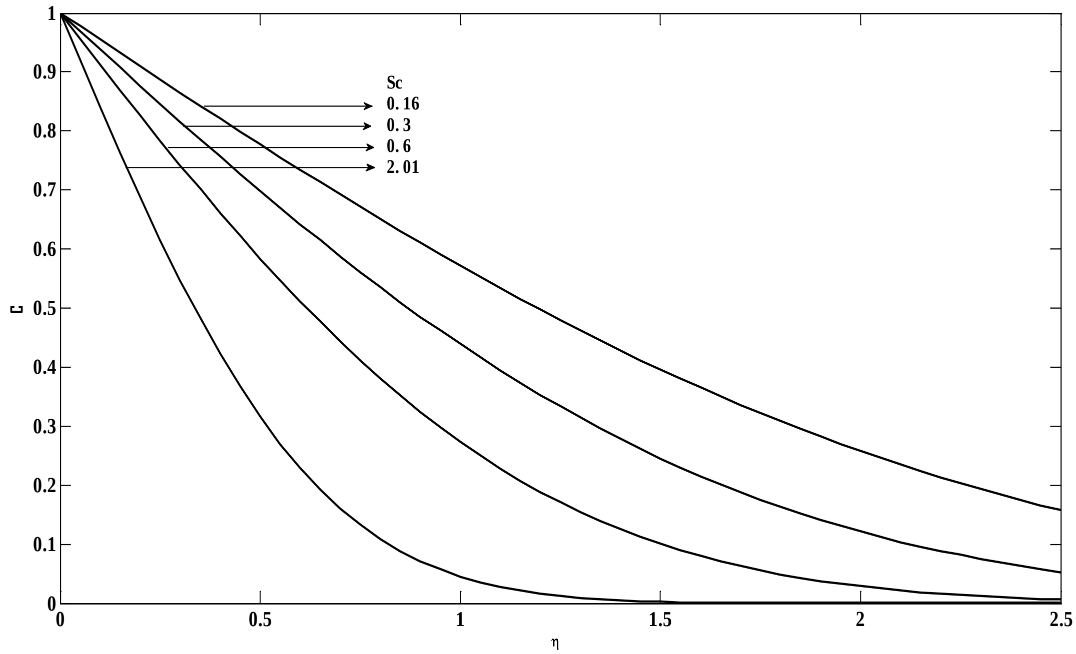

Figure 1: Concentration profiles for different values of Sc

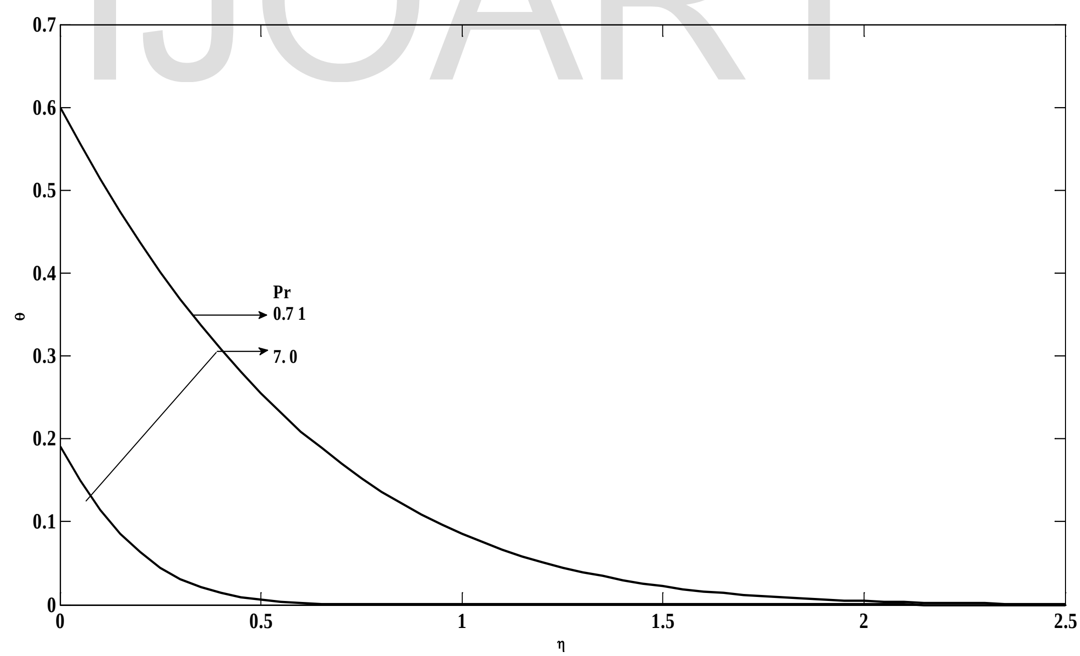

Figure 2: Temperature profiles for different values of Pr

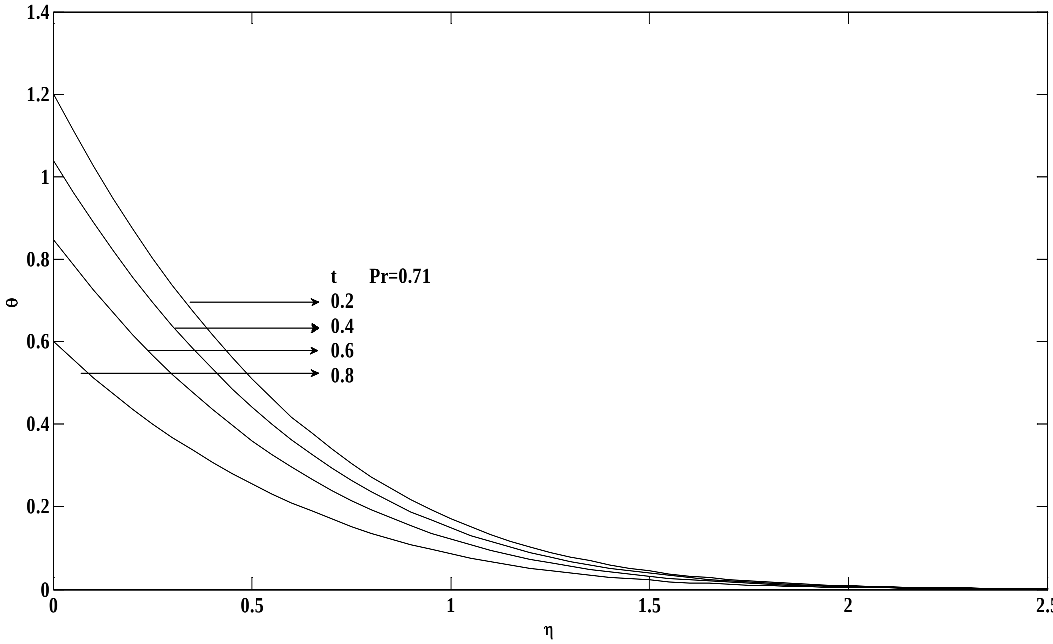

Figure 3: Temperature profiles for different values of t

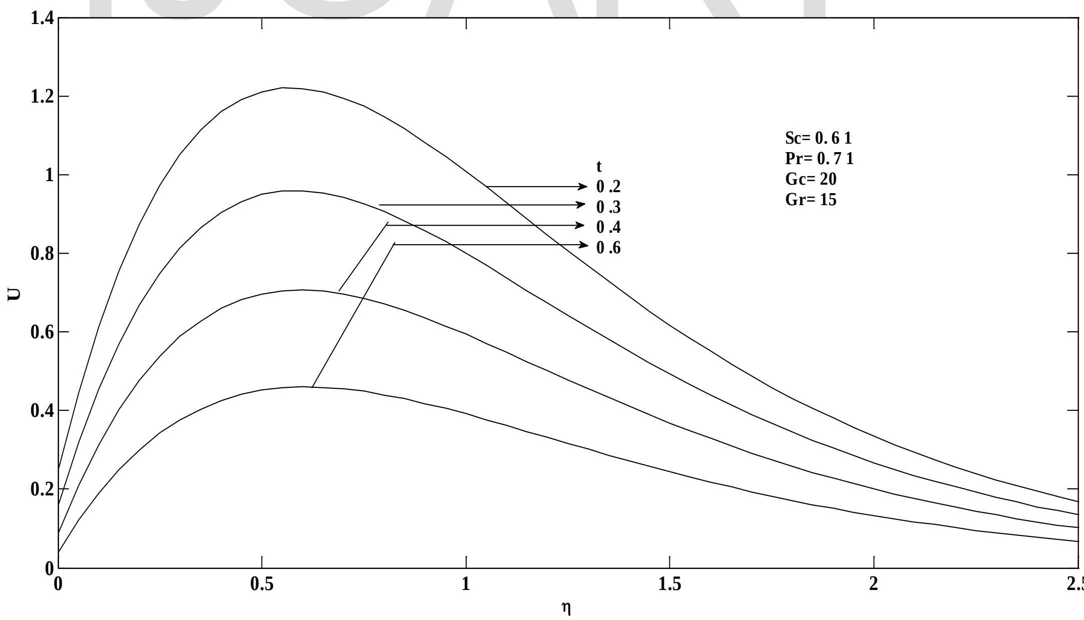

Figure 4: Velocity Profiles for different values of t

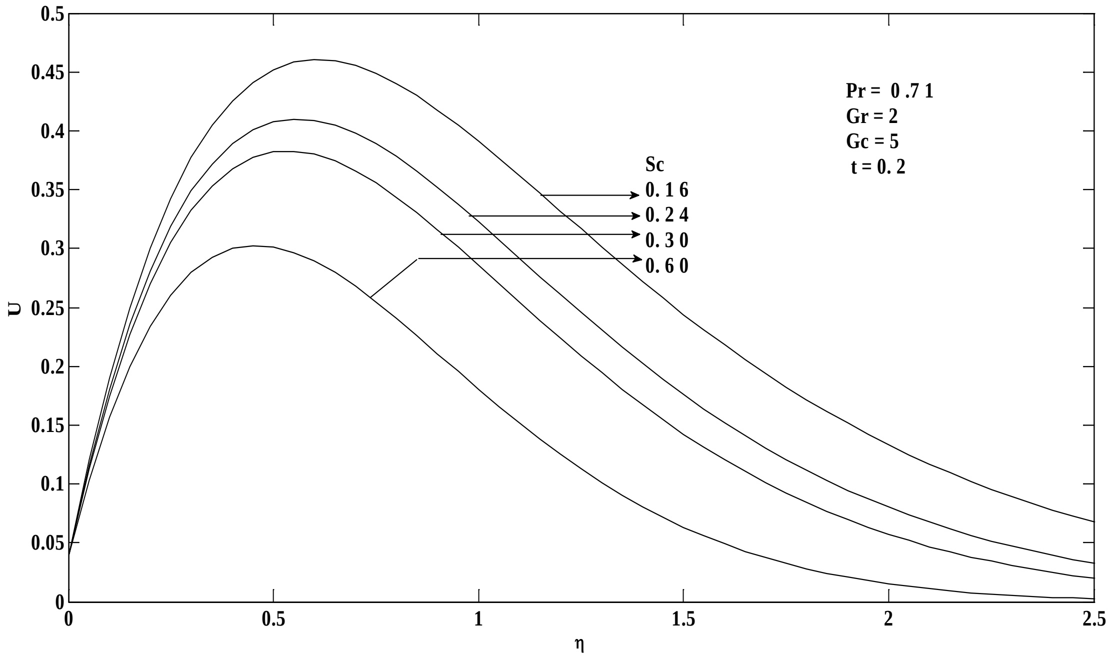

Figure 5: Velocity Profiles for different values of Sc

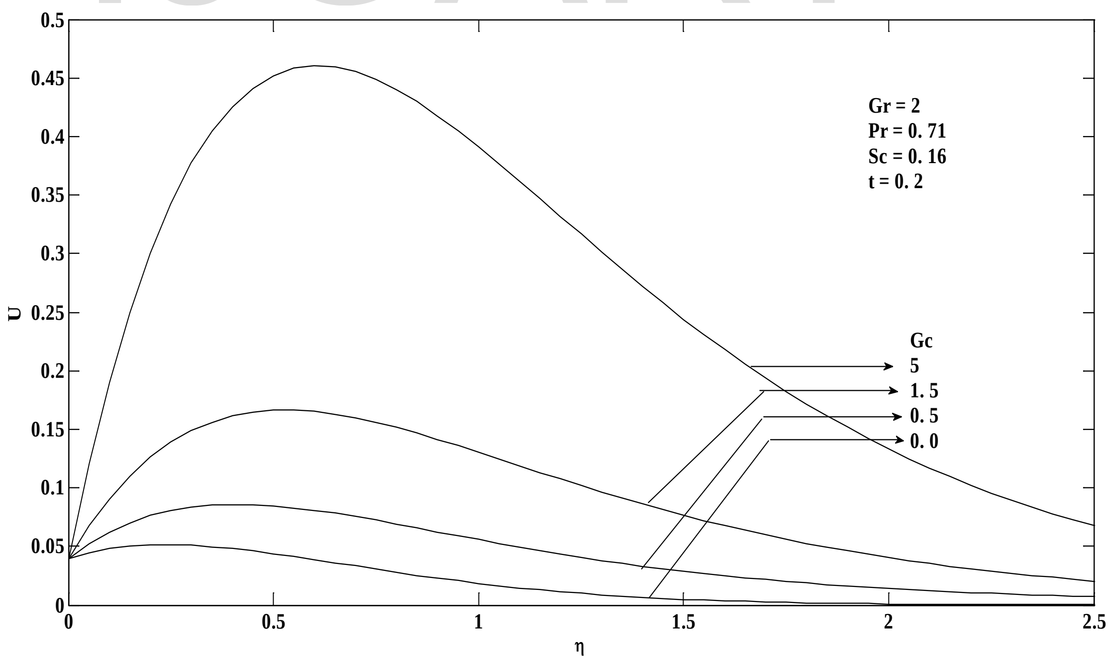

Figure 6: Velocity Profiles for different values of Gc

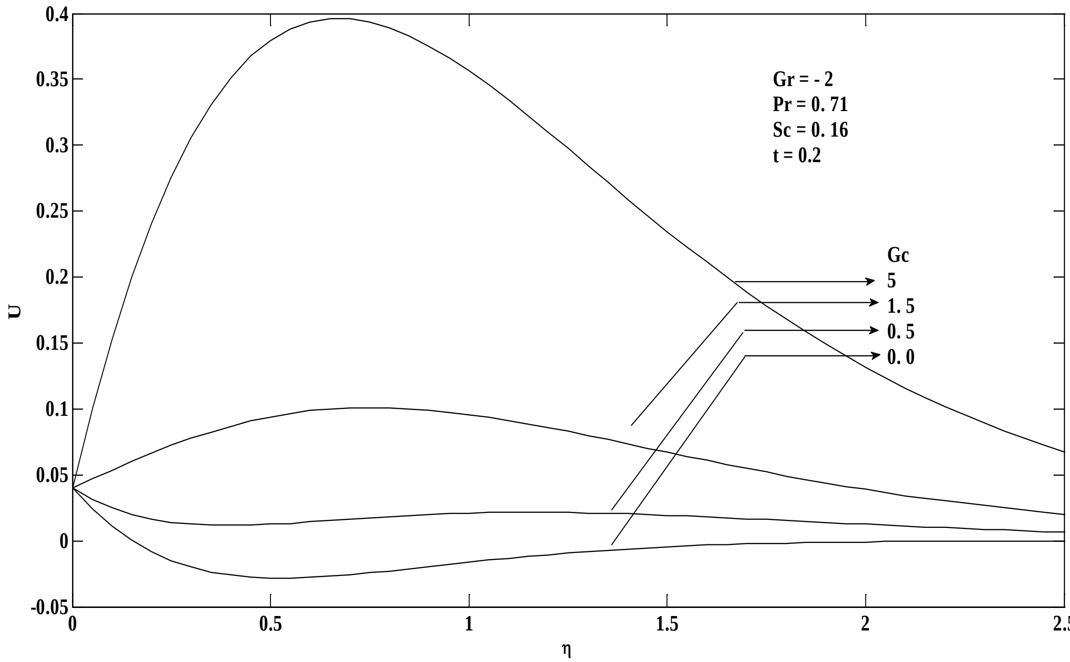

Figure 7: Velocity Profiles for different values of Gc