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Global output convergence for delayed recurrent neural networks under impulsive effects *

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ABSTRACT

In this paper, we investigate convergence of state output for a class of delayed recurrent neural networks with impulsive effects. Based on properties of time-varying inputs and monotonicity of activation function, we establish some sufficient conditions to guarantee output convergence of the networks in which state variable subjected to impulsive displacements at fixed moments of time.

Keywords : Output convergence; Recurrent neural networks; Delays; Impulse

1 INTRODUCTION

THE model of a class of recurrent neural networks (RNNs) without linear decay term is described by

$$\frac{dx_i(t)}{dt}=\sum_{j=1}^{n} w_{ij}g_j(x_j(t))+u_i(t) \quad , \quad x_i(0)=x_{i0} \quad ,$$

$i \in \mathrm{N} := \{1,2,\cdots,n\}$

The above RNNs is quite different from well-known Hopfield-type networks which show important applications in optimization [7,8]. However, in other optimization applications, convergence behavior of output space would be required, e.g., [5,6]. Under the time-varying thresholds, Liu et al. [1] and Hu et al.[2] discussed some sufficient conditions to guarantee global output convergence of this class of networks. With monotonicity and globally Lipschitz assumptions posed on activation functions, Zhang et al. [3] derived global output convergence criteria for the following RNNs with time-varying delays

$$\frac{dx_i(t)}{dt}=\sum_{j=1}^{n}\Big[w_{ij}g_j(x_j(t))+w_{ij}^{\tau}g_j(x_j(t-\tau_{ij}(t)))\Big]+u_i(t) \quad ,$$

$i \in \mathrm{N}$.

Along this direction, Liang et al. [4] continued to consider a class of RNNs described by integro-different equations

$$\frac{dx_i(t)}{dt}=\sum_{j=1}^{n}\Big[w_{ij}g_j(x_j(t))+w_{ij}^{\tau}\int_{-\infty}^{t}k_{ij}(t-s)g_j(x_j(s))ds\Big]+u_i(t),$$

$i \in \mathrm{N}$.

To the best of authors' knowledge, few authors reported output convergent dynamics of neural networks with impulses. In this paper ,we consider a class of RNNs with delays and impulses described by

$$\begin{cases}\dfrac{dx_i(t)}{dt}=\sum_{j=1}^{n}\Big[w_{ij}g_j(x_j(t))+w_{ij}^{\tau}g_j(x_j(t-\tau))\Big]+u_i(t), t\neq t_k, \quad t\geq t_0\\ \Delta x_i(t)=I_k(t,x_i(t^-)),\ t\in \mathrm{P}:=\{t_k \mid k\in \mathbf{Z}^+\}\end{cases} \tag{1}$$

where $i \in \mathrm{N}$, each $x_i(t)$ denotes the state of neuron i , $W=(w_{ij})_{n\times n}$ and $W^{\tau}=(w_{ij}^{\tau})_{n\times n}$ are constant connection weights, τ is the time-varying delays with upper bound by τ , the inputs $u_i(t)$ ($i\in \mathrm{N}$) are some continuous functions defined on $[t_0,+\infty)$, $g_i(\cdot)$ ($i\in \mathrm{N}$) are networks' output activation functions. We call $x(t)=(x_1(t),\cdots,x_n(t))^T$ the state of the networks at time t and call $G(x(t))=(\mathrm{g}_1(x_1(t)),\cdots,\mathrm{g}_n(x_n(t)))^T$ the output of the networks. Usually, the fixed impulsive instants satisfy with $t_1<t_2<\cdots$ and $\lim_{k\to+\infty}t_k=+\infty$. This paper studies the output convergence of recurrent neural networks with impulses and time-varying inputs. Several sufficient conditions will be established for RNNs (1) to be globally output convergent.

2 IMPULSIVE OUTPUT CONVERGENCE

Without otherwise statement, in this paper, we assume that

(A_1) There exist positive constant $\ell_j^{\perp}$ and $\ell_j^{ú}$ such that $\ell_j^{\perp} \le \frac{g_j(u)-g_j(v)}{u-v} \le \ell_j^{ú}$ for any $u,\ v \in$ and $u \ne v$, $j \in \mathrm{N}$.

(A_2) There exists a constant vector $u \in{}^n$ such that $\lim_{t\to+\infty} u(t) = u$ where $u(t) = (u_1(t),\cdots,u_n(t))^T$.

Let $C = C([-\tau,0],\mathbf{R}^n)$ be the Banach space of bounded and continuous functions. For simplicity, let $\Xi := \{x \in{}^n \,|(W+W^T)g(x)+u=0\}$. We assume that $\Xi \ne \phi$, i.e., there exists at least one constant vector $x^* \in{}^n$ such that $(W+W^T)g(x^*)+u=0$. Let $z(t)=x(t)-x^*$, $\hat{u}(t)=u(t)-u$ and $f(z(t)) = g(z(t)+x^*)-g(x^*)$, then RNNs(1) can be written as

$$\begin{cases} \frac{dz_i(t)}{dt} = \sum_{j=1}^{n}\left[w_{ij} f_j(z_j(t)) + w_{ij}^{\tau} f_j(z_j(t-\tau))\right] + \hat{u}_i(t),\ t \notin \mathrm{P}, t \ge t_0 \\ \Delta z_i(t) = H_k(t, z_i(t^-)),\ t \in \mathrm{P} \end{cases} \tag{2}$$

where $H_k(t,z_i(t^-)) = I_k(t,z_i(t^-)+x_i^*)$, $t \in \mathrm{P}$ and $i \in \mathrm{N}$.

Definition 1 The RNNs (1) is globally output convergent if ,for any $\psi \notin C$, there exists a $x^* \in \Xi$ such that $\lim_{t\to+\infty} g(x(t,\psi)) = g(x^*)$.

Remark 1 Due to the boundedness and squashing effects of activation function $g(\cdot)$, output convergence of RNNs (1) does not imply the state convergence [3]. With the time-varying input $\hat{u}(t)$ and the lack of a linear decay term, output convergence of RNNs (1) will be complete different from equilibrium-type stability [10].

In this section, we will show that, under impulsive effects, RNNs[1] is globally output convergent. Firstly, we introduce the following assumption:

(A_3) There exists a sequence $\{\beta_k\}_1^{+\infty}$ with $\beta_k > 0$ and $\sum_{k=1}^{\infty}\beta_k < +\infty$ such that

$$\sup_{t\in\mathbf{R}}\left|1+\frac{H_k(t,s)}{s}\right| \le \min_{i\in\mathrm{N}}\left\{\frac{\sqrt{\ell_i^{\perp}(1+\beta_k)}}{\ell_i^{\perp}}\right\}$$

hold for all $k \in \mathbf{Z}^+$ and $\forall s \in \mathbf{R}$.

Remark 2 Consider special case for (A_3). As $H_k(t,s) = H_k(s)$ and $\min_{i\in\mathrm{N}}\left\{\sqrt{\ell_i^{\perp}(1+\beta_k)}/\ell_i^{\perp}\right\} \le 1$. Let $\tilde{H}_k(s) := s + H_k(s)$. Then, impulsive jumps characterized by $\tilde{H}_k(s)$ satisfy sublinear estimation $\sum_{i=1}^{n}\tilde{H}_k(|z_i(\cdot)|) \le \tilde{H}_k\sum_{i=1}^{n}(|z_i(\cdot)|)$ which should be satisfied in [9]. In fact, state jumps $|z_i(t_k)| = \tilde{H}_k(z_i(t_k^-)) \le e_k\,|z_i(t_k^-)|$ with small magnitudes $0 < e_k < 1$ have been widely employed in [10,11,12].

Lemma 1 Under assumption (A_3), for each $i \in \mathrm{N}$, the following inequality

$$f_i(s+H_k(t,s))^2 \le (1+\beta_k)\, f_i(s)^2$$

holds for $k \in \mathbf{Z}^+$, $t \in \mathrm{P}$ and $\forall s \in \mathbf{R}$.

Proof: It follows from (A_1) that

$$\forall u \in \mathbf{R},\ \begin{cases} uf_i(u)\ell_i^{\perp} \le f_i(u)^2 \le uf_i(u)\ell_i^{ú} \\ u^2\ell_i^{\perp} \le uf_i(u) \le u^2\ell_i^{ú} \end{cases}$$

holds for each $i \in \mathrm{N}$. Hence, we have

$$f_i(s+H_k(t,s))^2 \le \ell_i^{ú}(s+H_k(t,s))\, f_i(s+H_k(t,s) \le (\ell_i^{ú})^2(s+H_k(t,s))^2$$

Assumption (A_3) implies that

$$\sup_{t\in\mathbf{R}}|s+H_k(t,s)| \le \min_{i\in\mathrm{N}}\left\{\frac{\sqrt{\ell_i^{\perp}(1+\beta_k)}}{\ell_i^{ú}}\right\}|s|$$

which leads to

$$f_i(s+H_k(t,s))^2 \le (\ell_i^{ú})^2\left(\frac{\sqrt{\ell_i^{\perp}(1+\beta_k)}}{\ell_i^{ú}}\right)^2 s^2 \le (1+\beta_k)\, f_i(s)^2.$$

The proof is complete.

Theorem 1 Assume (A_1)-(A_3) hold. If there exist positive constants L_i, L_i^{τ}, L_i^{v}, $\breve{\theta}_i^1$ and $\breve{\theta}_i^2$ such that $\breve{\theta}_i^1 + \breve{\theta}_i^2 = 1$, $g(t) = diag(G(t)W^{(0)}, G(t)W^{(\tau)})$ is positive definite and

$$\int_{t_0}^{\infty}\hat{u}_i(s)^2 ds < +\infty$$

for each $i \in \mathrm{N}$, where

$$\begin{cases} W^{(0)} = diag(w_{ij}^0),\ \ W^{(\tau)} = (\frac{1}{2L_i^{\tau}}\left|w_{ij}^{(\tau)}\right|\ ),\ \ G(t) = diag(D^+ g_i(z_i(t))), \\ w_{ii}^0 = w_{ii}\breve{\theta}_i^1 + \sum_{j\ne i}\frac{L_i}{2}|w_{ij}| + \sum_{j=1}^{n}\frac{L_i^{\tau}}{2}\left|w_{ij}^{\tau}\right| + \frac{L_i^{v}}{2},\ \ w_{ii}^0 = \frac{1}{2L_i}|w_{ij}|,\ i \ne j, \end{cases}$$

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Then RNNs (1) is globally output convergent.

Proof: Consider a candidate Lyapunov function $V(t)=\frac{1}{2}\sum_{i=1}^{n} f_i(z_i(t))^2$. For any $k\in\mathbf{Z}^+$, by calculating the derivatives of V along the trajectory of (2) for $[t_k, t_{k+1})$, we have

$$\begin{aligned}
D^+V(t) &\le \sum_{i=1}^{n} \left|f_i(z_i(t))\right| N_i(z(t)) \left[w_{ii}\left|f_i(z_i(t))\right| + \sum_{j\ne i}\left|w_{ij}\right|\left|f_j(z_j(t))\right| + \sum_{j=1}^{n}\left|w_{ij}^{\tau}\right|\left|f_j(z_j(t-\tau))\right| + \left|\hat{u}_i(t)\right| \right] \\
&\le \sum_{i=1}^{n} N_i(z(t)) \left[w_{ii}\breve{\theta}_i^{1} f_i(z_i(t))^2 + w_{ii}\breve{\theta}_i^{2} f_i(z_i(t))^2 + \sum_{j\ne i}\left|w_{ij}\right| \left(L_i \left|f_i(z_i(t))\right| \times \frac{1}{L_i}\left|f_j(z_j(t))\right| \right) \right. \\
&\quad \left. + \sum_{j=1}^{n}\left|w_{ij}^{\tau}\right| \left(L_i^{\tau}\left|f_i(z_i(t))\right| \times \frac{1}{L_i^{\tau}}\left|f_j(z_j(t-\tau))\right| \right) + L_i^{v}\left|f_i(z_i(t))\right| \times \frac{1}{L_i^{v}}\left|\hat{u}_i(t)\right| \right] \\
&\le \sum_{i=1}^{n} N_i(z(t)) \left[w_{ii}\breve{\theta}_i^{1} f_i(z_i(t))^2 + w_{ii}\breve{\theta}_i^{2} f_i(z_i(t))^2 + \sum_{j\ne i}\left|w_{ij}\right| \left(\frac{L_i}{2} f_i(z_i(t))^2 + \frac{1}{2L_i} f_j(z_j(t))^2 \right) \right. \\
&\quad \left. + \sum_{j=1}^{n}\left|w_{ij}^{\tau}\right| \left(\frac{L_i^{\tau}}{2} f_i(z_i(t))^2 + \frac{1}{2L_i^{\tau}} f_j(z_j(t-\tau))^2 \right) + \frac{L_i^{v}}{2} f_i(z_i(t))^2 + \frac{1}{2L_i^{v}}\hat{u}_i(t)^2 \right] \\
&\le \sum_{i=1}^{n} N_i(z(t)) \left\{ \left[w_{ii}\breve{\theta}_i^{1} + \sum_{j\ne i}\frac{L_i}{2}\left|w_{ij}\right| + \sum_{j=1}^{n}\frac{L_i^{\tau}}{2}\left|w_{ij}^{\tau}\right| + \frac{L_i^{v}}{2} \right] f_i(z_i(t))^2 \right. \\
&\quad \left. + \sum_{j\ne i}\frac{1}{2L_i}\left|w_{ij}\right| f_j(z_j(t))^2 + \sum_{j=1}^{n}\frac{1}{2L_i^{\tau}}\left|w_{ij}^{\tau}\right| f_j(z_j(t-\tau))^2 + \frac{1}{2L_i^{v}}\hat{u}_i(t)^2 + w_{ii}\breve{\theta}_i^{2} f_i(z_i(t))^2 \right\}
\end{aligned}$$

where $N_i(z(t)) := D^+ g_i(z_i(t))$, $i\in\mathrm{N}$. Therefore, we have

$$\begin{aligned}
D^+V(t) &\le \left|f(z(t))\right|^T G(t)W^{(0)}\left|f(z(t))\right| + \left|f(z(t-\tau))\right|^T G(t)W^{(\tau)}\left|f(z(t-\tau))\right| \\
&\quad + \sum_{i=1}^{n} N_i(z(t)) \left[\frac{1}{2L_i^{v}}\hat{u}_i(t)^2 + w_{ii}\breve{\theta}_i^{2} f_i(z_i(t))^2 \right] \\
&\le \left(\left|f(z(t))\right|^T, \left|f(z(t-\tau))\right|^T\right) g(t) \left(\left|f(z(t))\right|^T, \left|f(z(t-\tau))\right|^T\right)^T \\
&\quad + \sum_{i=1}^{n} N_i(z(t)) \left[\frac{1}{2L_i^{v}}\hat{u}_i(t)^2 + w_{ii}\breve{\theta}_i^{2} f_i(z_i(t))^2 \right] \\
&\le \sum_{i=1}^{n} N_i(z(t)) w_{ii}\breve{\theta}_i^{2} f_i(z_i(t))^2 + \sum_{i=1}^{n}\frac{\ell_i^{ú}}{2L_i^{v}}\hat{u}_i(t)^2.
\end{aligned} \tag{3}$$

It follows from that Lemma 1 that

$$\begin{aligned}
\left|V(t_k)-V(t_k^-)\right| &= \frac{1}{2}\sum_{i=1}^{n}\left| f_i(z_i(t_k))^2 - f_i(z_i(t_k^-))^2 \right| \\
&\le \frac{1}{2}\sum_{i=1}^{n}\left| f_i(z_i(t_k^-)) + H_k(t_k, z_i(t_k))^2 - f_i(z_i(t_k^-))^2 \right| \\
&\le \frac{\beta_k}{2}\sum_{i=1}^{n} f_i(z_i(t_k^-))^2 \\
&= \beta_k V(t_k^-).
\end{aligned} \tag{4}$$

Integrating both sides of (3) from t_0 to t and using (4), we get

$$\begin{aligned}
V(t) &\le V(t_0) + \int_{t_0}^{t}\sum_{i=1}^{n} N_i(z(s)) w_{ii}\breve{\theta}_i^{2} f_i(z_i(s))^2 \\
&\quad + \sum_{i=1}^{n}\frac{\ell_i^{ú}}{2L_i^{v}}\int_{t_0}^{t}\hat{u}_i(s)^2 ds + \sum_{t_0<t_k\le t}\left[V(t_k)-V(t_k^-)\right] \\
&\le V(t_0) + \sum_{i=1}^{n}\frac{\ell_i^{ú}}{2L_i^{v}}\int_{t_0}^{t}\hat{u}_i(s)^2 ds + \sum_{t_0<t_k\le t}\beta_k V(t_k^-).
\end{aligned} \tag{5}$$

Suppose $\mathrm{P}\cap(t_0,t] = \{t_{m1}, t_{m2}\cdots, t_{ml}\}$ and $m_z = m_{z-1}+1,\ \ z=2,3,\cdots,l$. We assert that

$$V(t) \le \beta\left[V(t_0) + \sum_{i=1}^{n}\frac{\ell_i^{ú}}{2L_i^{v}}\int_{t_0}^{t}\hat{u}_i(s)^2 ds \right],$$

where $\beta = \prod_{k=1}^{+\infty}(1+\beta_k) \in [1,+\infty).$ (6)

In fact, as $t_0 < t < t_{m1}$, it follows from (5) that

$$\begin{cases}
V(t) \le V(t_0) + \sum_{i=1}^{n}\frac{\ell_i^{ú}}{2L_i^{v}}\int_{t_0}^{t}\hat{u}_i(s)^2 ds, \\
V(t_{m1}^-) \le V(t_0) + \sum_{i=1}^{n}\frac{\ell_i^{ú}}{2L_i^{v}}\int_{t_0}^{t_{m1}}\hat{u}_i(s)^2 ds
\end{cases} \tag{7}$$

As $t_{m1} \le t < t_{m2}$, it follows from (5) and (7) that

$$\begin{cases}
V(t) \le V(t_0) + \sum_{i=1}^{n}\frac{\ell_i^{ú}}{2L_i^{v}}\int_{t_0}^{t}\hat{u}_i(s)^2 ds + \beta_{t_{m1}}V(t_{m1}^-) \\
\qquad \le (1+\beta_{t_{m1}})V(t_0) + (1+\beta_{t_{m1}})\sum_{i=1}^{n}\frac{\ell_i^{ú}}{2L_i^{v}}\int_{t_0}^{t}\hat{u}_i(s)^2 ds, \\
V(t_{m1}^-) \le (1+\beta_{t_{m1}})V(t_0) + (1+\beta_{t_{m1}})\sum_{i=1}^{n}\frac{\ell_i^{ú}}{2L_i^{v}}\int_{t_0}^{t_{m2}}\hat{u}_i(s)^2 ds
\end{cases} \tag{8}$$

As $t_{m2} \le t < t_{m3}$, it follows from (5), (7) and (8) that

$$\begin{cases}
V(t) \le V(t_0) + \sum_{i=1}^{n}\frac{\ell_i^{ú}}{2L_i^{v}}\int_{t_0}^{t}\hat{u}_i(s)^2 ds + \beta_{t_{m1}}V(t_{m1}^-) + \beta_{t_{m2}}V(t_{m2}^-) \\
\qquad \le (1+\beta_{t_{m1}})(1+\beta_{t_{m2}})V(t_0) + (1+\beta_{t_{m1}})(1+\beta_{t_{m2}})\sum_{i=1}^{n}\frac{\ell_i^{ú}}{2L_i^{v}}\int_{t_0}^{t}\hat{u}_i(s)^2 ds, \\
V(t_{m2}^-) \le (1+\beta_{t_{m1}})(1+\beta_{t_{m2}})V(t_0) + (1+\beta_{t_{m1}})(1+\beta_{t_{m2}})\sum_{i=1}^{n}\frac{\ell_i^{ú}}{2L_i^{v}}\int_{t_0}^{t_{m3}}\hat{u}_i(s)^2 ds
\end{cases} \tag{9}$$

From mathematical induction argument, we can derive that

$$V(t) \le V(t_0)\prod_{t_0<t_k\le t}(1+\beta_k)+\prod_{t_0<t_k\le t}(1+\beta_k)\sum_{i=1}^{n}\frac{\ell_i^{ú}}{2L_i^{v}}\int_{t_0}^{t}\hat{u}_i(s)^2ds$$
$$\le \beta\left[V(t_0)+\sum_{i=1}^{n}\frac{\ell_i^{ú}}{2L_i^{v}}\int_{t_0}^{t}\hat{u}_i(s)^2ds\right].$$

Since $\int_{t_0}^{\infty}\hat{u}_i(s)^2ds<+\infty$ for each $i\in N$, $V(t)$ is bounded for $[t_0,+\infty)$. Hence, there exists a $\varsigma_i>0$ such that $\hat{u}_i(t)<\sqrt{\frac{\ell_i^{\perp}}{\ell_i^{ú}}L_i^{v}\breve{\theta}_i^2|w_{ii}|\varsigma_i}$ for all $t\ge s_1^*$. It follows from (3) that

$$D^+V(t)\le\sum_{i=1}^{n}\ell_i^{\perp}w_{ii}\breve{\theta}_i^2\varsigma_i+\sum_{i=1}^{n}\frac{\ell_i^{ú}}{2L_i^{v}}\hat{u}_i(t)^2$$
$$<\frac{1}{2}\sum_{i=1}^{n}\ell_i^{\perp}w_{ii}\breve{\theta}_i^2\varsigma_i<0,\ \forall t\ge s_1^* \tag{10}$$

Hence, $V(t)$ is nonincreasing on $[s_1^*,t_m)$ and $[t_k,t_{k+1})$ $(k\ge m)$. So, $V(t)\le V(s_1^*)$ for $s_1^*\le t\le t_m$ and $V(t)\le V(t_k)$ for $t_k\le t\le t_{k+1}$ $(k\ge m)$. The boundedness of $V(t)$ and (4) leads to the basic fact $\sum_{k=1}^{+\infty}\left|V(t_k)-V(t_k^-)\right|<+\infty$.

Let $\xi_k=\sum_{v=m}^{+\infty}\left|V(t_v)-V(t_v^-)\right|$ $(k\ge m)$ and define $H:[s_1^*,+\infty)\to\mathbf{R}$ by

$$H(t):=\begin{cases}V(t),\ s_1^*\le t<t_m\\-\xi_k+V(t),\ t_k\le t<t_{k+1},\ k\ge m\end{cases} \tag{11}$$

Obviously, $\lim_{k\to+\infty}\xi_k$ exists and $H(t)$ is nonincreasing on both $[s_1^*,t_m)$ and $[t_k,t_{k+1})$ $(k\ge m)$. We claim that $H(t_k)\ge H(t_{k+1})$ for $k\ge m$. In fact, for $k\ge m$, we have

$$H(t_k)=-\xi_k+V(t_k)\ge\xi_k+V(t_{k+1}^-)$$
$$\ge-\sum_{v=m}^{k}\left|V(t_v)-V(t_v^-)\right|-\left[V(t_{k+1})-V(t_{k+1}^-)\right]+V(t_{k+1})$$
$$=-\sum_{v=m}^{k+1}\left|V(t_v)-V(t_v^-)\right|+V(t_{k+1})$$
$$=-\xi_{k+1}+V(t_{k+1})=H(t_{k+1})$$

which leads to the monotonicity of $H(t)$ on $[s_1^*,+\infty)$. It is easily to see that $H(t)$ is boundedness and so $\lim_{t\to+\infty}H(t)$ exists. By (11), we see that $\lim_{t\to\infty}V(t)=\alpha$ exists and $\alpha\ge0$. We claim that $\alpha=0$. Otherwise, there exist constants $\alpha_i\ge0$ $(i\in N)$ and a $i^*\in N$ such that $\sum_{k=1}^{n}\alpha_k=\alpha$, $\lim_{t\to+\infty}\frac{f_i(z_i(t))^2}{2}=\alpha_i$ and $\alpha_{i^*}\ge0$.

Therefore, we can see that there exists a $s_2^*>s_1^*$ such that $\forall t>s_2^*$

$$f_i(z_i(t))^2\ge\frac{\alpha_i}{2}\ \text{ and } \hat{u}_i(t)^2\le\frac{\alpha}{2}\breve{\theta}_i^2|w_{ii}|L_i^{v}$$

It follows from (3) that $\forall t>s_2^*$

$$D^+V(t)\le\sum_{k=1}^{n}N_i(z(t))\frac{\breve{\theta}_i^2}{4}w_{ii}\alpha_i$$

Integrate above inequality from s_2^* to t, we get

$$V(t)\le V(s_2^*)+\sum_{k=1}^{n}\frac{\breve{\theta}_i^2}{4}w_{ii}\alpha_i\int_{s_2^*}^{t}N_i(z(s))ds+\sum_{s_2^*\le t_k\le t}\left[V(t_k)-V(t_k^-)\right]$$
$$\le V(s_2^*)+\sum_{k=1}^{n}\frac{\breve{\theta}_i^2}{4}w_{ii}\alpha_i\int_{s_2^*}^{t}N_i(z(s))ds+\sum_{s_2^*\le t_k\le t}\beta_kV(t_k^-)$$
$$\le\prod_{s_2^*\le t_k<t}(1+\beta_k)\left[V(s_2^*)+\sum_{k=1}^{n}\frac{\breve{\theta}_i^2}{4}w_{ii}\alpha_i\int_{s_2^*}^{t}N_i(z(s))ds\right]$$
$$\le\beta\left[V(s_2^*)+\sum_{k=1}^{n}\frac{\breve{\theta}_i^2}{4}w_{ii}\alpha_i\int_{s_2^*}^{t}N_i(z(s))ds\right]$$

which leads to

$$V(t)\le\beta\left[V(s_2^*)+\sum_{k=1}^{n}\frac{\breve{\theta}_i^2}{4}w_{ii}\alpha_i\ell_i^{\perp}(t-s_2^*)\right]\to-\infty$$

as $t\to+\infty$.

This contradicts to $V(t)\ge0$. Based on above analysis, we have show that $\alpha=0$. Hence $\lim_{t\to+\infty}f_i(z_i(t))=0$ and the output vector of (2) is globally output convergent.

For any given constant $r>1$ and each $k\in\mathbf{Z}^+$, define $S_r:=\{x\in\mathbf{R}:|x|<r\}$ and $H_k:\mathbf{R}\times S_r\to S_r$. We make the following assumption :

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$(\hat{A}_3)$ $H_k(t,s) \in C^1(\mathbf{R} \times S_r, S_r)$ and there exists a $\rho_k \in (0,1)$ such that $\sup_{t\in\rho} \frac{\partial H_k(t,s)}{\partial s} \le \rho_k - 1$, $s \in S_r$ holds for $k \in \mathbf{R}^+$.

For each $k \in \mathbf{Z}^+$, the function $F_k(s) := s + H_k(t_k, s)$ has the following property, which will be used when we prove output convergence of RNNs (2).

Lemma2 For each $k \in \mathbf{Z}^+$, $F_k(s) := s + H_k(t_k, s)$ maps bounded sets of S_r into bounded sets of S_r. Moreover, $|F_k(\hat{r})| \le \rho_k |r|$ holds for any $\hat{r} \in S_r$ and $k \in \mathbf{Z}^+$.

Proof: It follows from $(\hat{A}_3)$ that

$$\frac{dF_k(s)}{ds} - 1 \le \sup_{t\in\rho} \frac{\partial H_k(t,s)}{\partial s} \le \rho_k - 1.$$

Together with $F_k(0) = 0$ $(k \in \mathbf{Z}^+)$, for any $\hat{r} \in [-r, r]$, we have

$$\int_0^{\hat{r}} \frac{dF_k(s)}{ds} \le \rho_k |\hat{r}| \Rightarrow |F_k(\hat{r})| \le \rho_k |r| < r$$

Thus, for any bounded set $E \in S_r$, $F_k(E) \in S_r$. This complete the proof.

Theorem 2 Assume that (A_1) - (A_2) and $(\hat{A}_3)$ hold. If

$$\ell_i^{\perp} > 0 \text{ and } \sum_{i=1, i\neq j}^{n} \ell_i^{\text{ú}} |w_{ii}| > w_{ii}\ell_i^{\perp} + \sum_{i=1}^{n} |w_{ii}^{\tau}| \ell_j^{\text{ú}} \qquad (12)$$

holds for all $j \in \mathrm{N}$ and there exist constants $\delta_i > 0$, $v > 0$ such that

$$\ln \rho_{k+1} + \int_{t_i}^{t_{i+1}} \left[\frac{m^{\tau}(\frac{v}{2} + \varepsilon) + m(\varepsilon - \frac{v}{2})}{v - \varepsilon} \right] ds \le -\delta_{i+1} \qquad (13)$$

where $\sum_{i=1}^{+\infty} \delta_i = +\infty$ and

$$m = \min_{j\in\mathrm{N}} \left\{ -w_{ii}\ell_i^{\perp} + \sum_{i=1, i\neq j}^{n} \ell_i^{\text{ú}} |w_{ii}| \right\},$$

$$m^{\tau} = \max_{j\in\mathrm{N}} \left\{ \sum_{i=1}^{n} |w_{ii}^{\tau}| \ell_j^{\text{ú}} \right\}$$

Then RNNs (2) is globally output convergent.

Proof: Define a candidate Lyapunov function $V(t) = \sum_{i=1}^{n} |z_i(t)|$ for all $t \ge t_0$. Then we get

$$\begin{aligned} D^+V(t) &\le \sum_{i=1}^{n} \left[w_{ii}\ell_i^{\perp} |z_i(t)| + \sum_{j\neq i} |w_{ii}| \ell_j |z_j(t)| + \sum_{j=1}^{n} |w_{ij}^{\tau}| \ell_j |z_j(t-\tau)| + |\hat{u}_i(t)| \right] \\ &\le \sum_{i=1}^{n} \left[\sum_{j=1}^{n} m_{ij} |z_j(t)| + \sum_{j=1}^{n} m_{ij}^{\tau} |z_j(t-\tau)| + |\hat{u}_i(t)| \right] \\ &\le \sum_{j=1}^{n} \left(\sum_{j=1}^{n} m_{ij} \right) |z_j(t)| + \sum_{j=1}^{n} \left(\sum_{j=1}^{n} m_{ij}^{\tau} \right) |z_j(t-\tau)| + \sum_{i=1}^{n} |\hat{u}_i(t)| \\ &\le -mV(t) + m^{\tau}V(t-\tau) + \hat{u}(t),\ t \in [t_k, t_{k+1}) \end{aligned} \qquad (14)$$

where $\hat{u}(t) = \sum_{i=1}^{n} |u_i(t)|$. It follows from (12) that $m > m^{\tau}$. Since $\hat{u}(t) \to 0$ as $t \to +\infty$, there exists a constant $\delta > 0$ such that $\hat{u}(t) < \delta$ for all $t \ge t_0$. For any given $H > \max\left\{1, \frac{\delta}{m - m^{\tau}}\right\}$, we will prove $\sup_{t_0-\tau\le\theta\le t_0} V(\theta) \le H$ imply $V(t) \le H$ for all $t \ge t_0$. Otherwise, there is a $s_1^* > t_0$ with $s_1^* \in [t_0, t_1)$ or $s_1^* \in [t_k, t_{k+1})$, $k \ge 1$ such that $D^+V(s_1^*) > 0$, $V(s_1^*) > H$, $V(t) \le H$ for $t_0 - \tau \le t < s_1^*$. As $s_1^* \in [t_0, t_1)$, it follows from (14) that $D^+V(t) \le -mH + m^{\tau}H + \delta \le 0$. This is a contradiction and it shows that $V(t) \le H$ for all $t \in [t_0 - \tau, t_1)$. As $s_1^* \in [t_1, t_2)$, due to $V(t_1) \le \rho_1 \sup_{t_0-\tau\le\theta\le t_1^-} V(\theta) \le H$, it follows from (14) that $D^+V(t) \le -mH + m^{\tau}H + \delta \le 0$. This is a contradiction. Thus, $V(t) \le H$ for all $t \in [t_0 - \tau, t_2)$. By similar argument, we can show that $V(t) \le H$ for all $t \in [t_0 - \tau, t_k)$ for $k \in \mathbf{Z}^+$. Therefore, $V(t) \le H$ for all $t \ge t_0 - \tau$. Let $v := \lim_{t\to+\infty} \sup V(t) \ge 0$.

Next, we will prove that $v = 0$. If it is not true, i.e., $v > 0$, we will prove it leads to a contradiction. By $\lim_{t\to+\infty} \hat{u}(t) = 0$, there must exists a $s_2^* > t_0$ such that $\hat{u}(t) \le \frac{v(m-m^\tau)}{2}$ for all $t \ge s_2^*$. Choose a $\varepsilon > 0$ such that $0 < \varepsilon < \frac{m-m^\tau}{2(m+m^\tau)}$. By $\lim_{t\to+\infty} \sup V(t) = v$, there exists a $s_3^* \ge s_2^*$ such that $\sup_{t-\tau\le\theta\le t} V(\theta) \le v+\varepsilon$ for all $t > s_3^*$. We assert that there exists a $s_4^* \ge s_3^*$ such that

$$D^+V(t) \le 0, \quad \forall t \ge s_4^*. \tag{15}$$

Otherwise, there must exist a $s_5^* \ge s_3^*$ such that $D^+V(s_5^*) > 0$ and $V(s_5^*) \ge v-\varepsilon$. However, from (14), we have

$$D^+V(t) \le -m(v-\varepsilon) + m^\tau(v+\varepsilon) + \frac{v(m-m^\tau)}{2}$$
$$=(m^\tau - m)v + (m^\tau + m)\varepsilon \le 0$$

which is a contradiction. This proves that (15) is true, i.e., $V(t)$ is nonincreasing on $[s_4^*, t_m)$ and $[t_k, t_{k+1})$, $k \ge m$. Together with (15), we get that $V(t_k) \ge V(t_{k+1}^-)$ for $k \in \mathbf{Z}^+$. Hence, $\{V(t_k)\}$ is bounded and nonincreasing, $\lim_{k\to\infty} V(t_k) = v$. There exist a $s_6^* \ge s_4^*$ such that $V(t) > v - \varepsilon$ and hence $D^+V(t) \le -mV(t) + m^\tau(v+\varepsilon) + \frac{v}{2}(m-m^\tau)$ for all $t \ge s_6^*$ which leads to

$$\int_{V(t_k)}^{V(t_{k+1}^-)} \frac{ds}{s} \le \int_{t_k}^{t_{k+1}} \left[-m + \frac{m^\tau(v+\varepsilon) + \frac{v}{2}(m-m^\tau)}{V(s)} \right] ds$$
$$\le \int_{t_k}^{t_{k+1}} \left[-m + \frac{m^\tau(v+\varepsilon) + \frac{v}{2}(m-m^\tau)}{v-\varepsilon} \right] ds. \tag{16}$$

For any $s \in S_H$, it follows from Lemma 2 that $F_k(s) \in [-\rho_k |s|, \rho_k |s|]$. So, we get

$$V(t_{k+1}) = \sum_{i=1}^n \left| z_i(t_{k+1}^-) + H_{k+1}(t_{k+1}, z_i(t_{k+1}^-)) \right|$$
$$\le \sum_{i=1}^n \rho_{k+1} \left| z_i(t_{k+1}^-) \right|$$
$$= \rho_{k+1} V(t_{k+1}^-)$$

Together with (16), we can get that

$$\int_{V(t_k)}^{V(t_{k+1})} \frac{ds}{s} \le \int_{V(t_{k+1}^-)}^{V(t_{k+1})} \frac{ds}{s} + \int_{t_k}^{t_{k+1}} \left[-m + \frac{m^\tau(v+\varepsilon) + \frac{v}{2}(m-m^\tau)}{v-\varepsilon} \right] ds$$
$$\le \int_{V(t_{k+1}^-)}^{\rho_{k+1}V(t_{k+1}^-)} \frac{ds}{s} + \int_{t_k}^{t_{k+1}} \left[-m + \frac{m^\tau(v+\varepsilon) + \frac{v}{2}(m-m^\tau)}{v-\varepsilon} \right] ds$$
$$\le \ln \rho_{k+1} + \int_{t_k}^{t_{k+1}} \left[\frac{m^\tau(\frac{v}{2}+\varepsilon) + m(\varepsilon - \frac{v}{2})}{v-\varepsilon} \right] ds$$
$$\le -\delta_{k+1}.$$

We get $V(t_{k+1}) \le V(t_k) - (v+\varepsilon)\delta_{k+1}$ which implies that $\lim_{k\to+\infty} V(t_k) = -\infty$, a contradiction and so $v = 0$. Since $V(t) \le V(t_k)$ for all $t \in [t_k, t_{k+1})$, it follows that $\lim_{t\to\infty} V(t) = 0$ which implies that $\lim_{t\to\infty} x(t) = 0$. The proof is complete.

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3 Conclusions

In this paper, we investigate the global output convergence of a class of delayed recurrent neural networks under impulsive perturbation. Based on convergence of time-varying inputs and monotonicity of activation functions, we establish some sufficient conditions to guarantee output convergence of the networks subjected to impulsive state displacements at fixed moments of time.

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