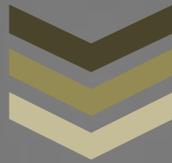


π



Geometric Estimation of Value of Pi

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Theorem to measure area of a Circle & to estimate the
value of "Pi"

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06-Jun-12

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1. π An irrational Number:

Archimedes was probably the first mathematician in modern history of mankind to attempt to measure the circumference of a circle by breaking it in to small segments and arrive at the value of Pi.

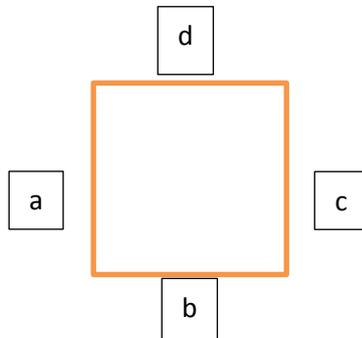
Pi is an irrational number, because it grows perpetually without a set pattern of repetition of its fractions like in '1/3'. To call a number irrational, it is to be proved that it fails the test of ratios.

Indeed Pi is as mysterious as square, square root and inverse of number ONE. Pythagoras developed a relative method to find out the square root of Number 1 by way of his famous theorem. One is not only a rational number but it is the root of all numeric systems and foundation of ratios. A square of one unit can be drawn, its area is surly 1 square unit and its root can be visually seen to be one only. Pythagoras though solved the riddle, failed to see it in Pi.

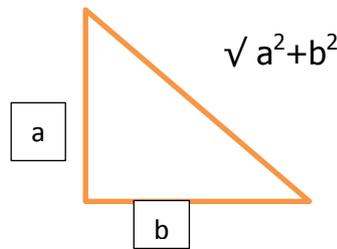
Geometric Estimation of Value of Pi

2. Relationship of geometric figures: [Square, Triangle, & Circle]

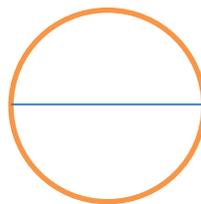
- The square is a four sided symmetric figure represented by lines a, b, c & d (assumption : one unit side)



- The triangle (right angle) is a three sided figure with two sides of unit length represented by sides 'a', 'b', and a hypotenuse (diagonal of the square) :



- A circle of one side (diameter) symmetric figure with line 'a' in the middle with **two arcs (hypotenuse *i.e. straight lines compressed between two points on either side of line 'a'*)**.

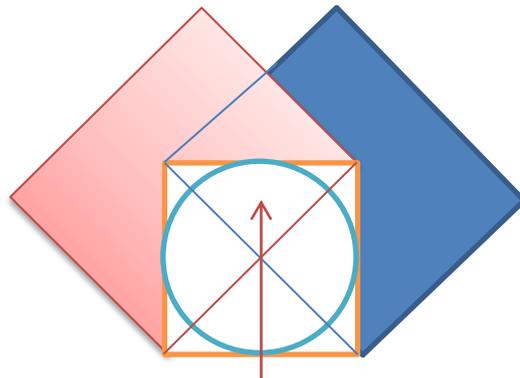


? $\sqrt{1}$ or \sqrt{a}

- All the above figures are interrelated to one another by way of a side of the square.
- \sqrt{a} is the mysterious number 1, **which is responsible in forcing additional length to the hypotenuse to form arcs.**

3. The Theorem:

Area of an inscribed circle in a square is equal to square of $1/4^{\text{th}}$ of sums of square roots of the squares on the opposite diagonals and the square root of the area of Secondary Square formed by overlapping of squares on the opposite diagonals.



(Square formed by overlapping area of squares on the opposite diagonals)

OR

Area of an inscribed circle in a square is equal to square of $1/2$ of 1.25 times the length of a diagonal.

4. Dissection of a circle of one unit diameter:

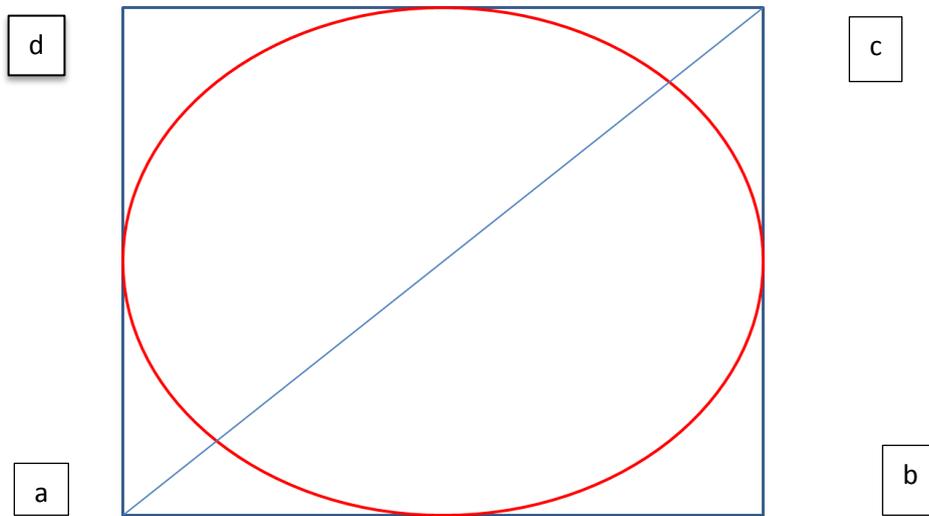
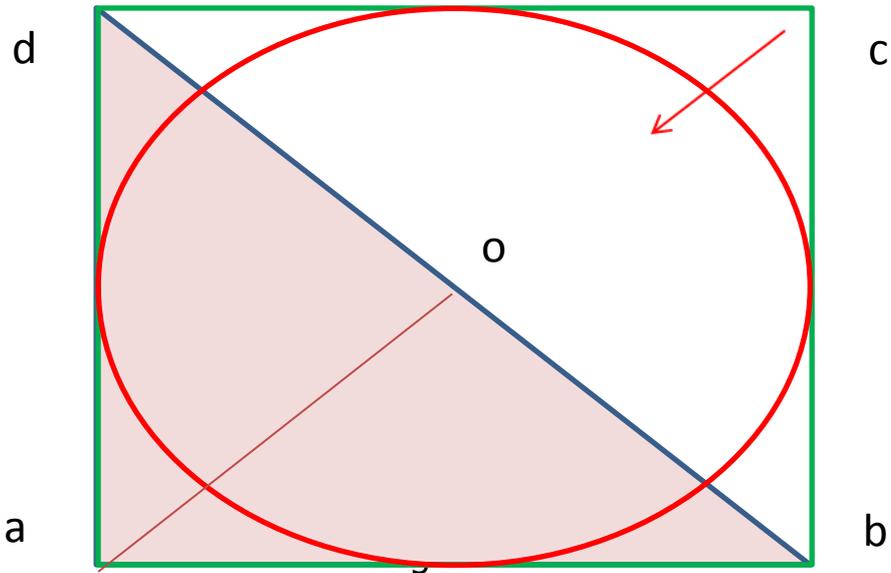


Figure 1

The above figure is a circle of one unit diameter. As the circle is inscribed within the square, the area of the circle is less than the area of the square. The area of the square is one square unit.

Now draw a straight line joining points 'a' and 'c'. As 'ac' is the diagonal of the square of one unit or hypotenuse of the triangle, its length is $= \sqrt{(ab)^2 + (bc)^2} = 1.4142135623730950488016887242097$ units

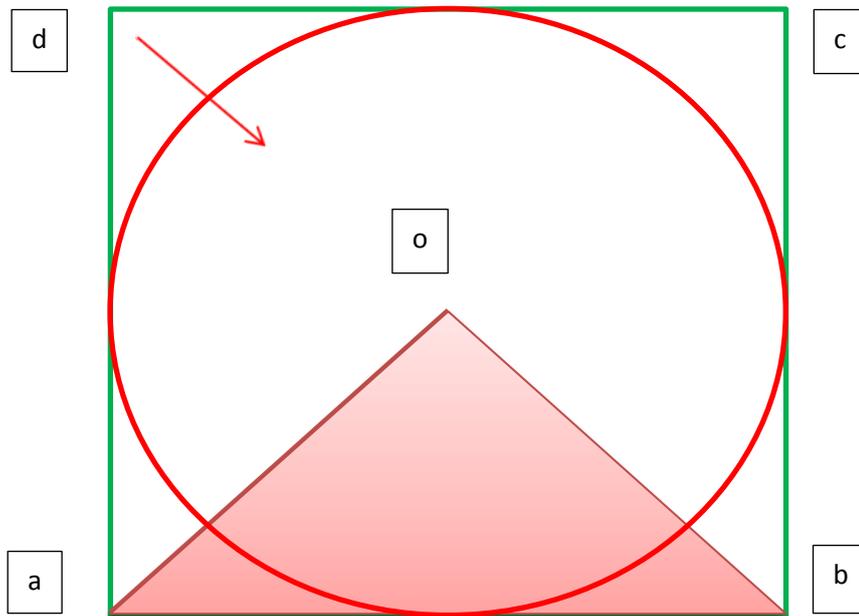
Figure 2



If the diagonal 'ac' is pulled out by half its length through point 'a', the perimeter of the square is reduced by half; with formation of a second diagonal/hypotenuse 'db' and the area reduced by half

The diagonal 'ac' has changed its position (by 90 degrees rotation) to 'db', while retaining its length. This is the combination of primary and secondary hypotenuse representing half of every property of the primary diagonal/hypotenuse and secondary hypotenuse.

Figure – 3



If the hypotenuse 'db' is drawn out through point 'b' by half its length, the area further gets reduced by half of the **residual** area of the resultant triangle in the previous figure. The combined length of lines 'ao' and 'bo' = the length of hypotenuse. Thus, average length of primary and secondary hypotenuse is equal to 1.25 times of diagonal. (Note *Line a:o is created twice which represents the square root of number 1 or side 'ad' and hypotenuse rotated again by 90 degrees i.e. 180 degree rotation creating two half circles on either end of diagonal/hypotenuse but carving out only 3/4th of the square*).

The pink triangle 'a:o:b' in figure-3; having an area of 0.25 square units is addition of secondary area to the area of the triangle created by primary and secondary diagonals, which may be opened like a flap of an envelope (to visualize the effect) OR overlapping area of two identical triangles placed on one another on a common base & facing hypotenuse of each other. (Folding the square along diagonal and again folding it to form another triangle)

It may be observed from Figure – 2 that, when half of the secondary hypotenuse is drawn from point 'b', half of secondary hypotenuse is added at 'ao' on the existing $\frac{1}{2}$ hypotenuse.

Besides, while primary diagonal/hypotenuse represents 2 unit square area the secondary hypotenuse represents 0.50 square unit of the square.

The area & diagonals/hypotenuse has a ratio of 1: (1.00 +0.25) or 1:1.25

Hence, hypotenuse x 1.25 represent the combined area of the area relating to primary and secondary hypotenuse.

Geometric Estimation of Value of Pi

Based on the above observations, the area and circumference of a circle may be estimated by using the following equations:

$$[(\text{Hypotenuse} \times 1.25) / 2]^2 = \text{Area}$$

and

$$[(\text{hypotenuse} \times 1.25)^2] / \text{Diameter} = \text{circumference}$$

This value may also be found out by using the following equation.

$$(\text{Diameter} \times 1.25)^2 / 2 = \text{Area}$$

and

$$[(\text{Diameter} \times 1.25)^2] / [\text{Diameter} \times 2] = \text{Circumference}$$

The Area/Circumference may be also be found out by using the following equation:

$(25/8) \times (r^2)$ for Area and $(25/8) \times \text{Diameter}$ for circumference where $25/8$ is revised the value of π

Geometric Estimation of Value of Pi

5. Calculations: (One unit diameter Circle inscribed in a square)

Diagonal (Primary)	1.414214	x 1.25	1.767767
Hypotenuse (Secondary)	1.414214		
Additional ½ Hypotenuse	0.707107	(Line 'a:o')	
Total	3.535534	/ 2 =	1.767767

Hypotenuse = 1.4142135623730950488016887242097

- $1.4142135623730950488016887242097 \times 1.25$
 $= 1.7677669529663688110021109052621$
- $(1.7677669529663688110021109052621)^2$
 $= 3.125 = \frac{25}{8} = [(1/1.28) \times 4]$

Now let us take the value of Pi and process it backwards to find out the relationship of ratios.

- $\pi = 3.1415926535897932384626433832795$
- $\sqrt{\pi} = 1.7724538509055160272981674833411$
- $1.7724538509055160272981674833411 / 1.4142135623730950488016887242097$
 $= 1.2533141373155002512078826424055$

(Ratio is greater than 1.25)

- $1.7724538509055160272981674833411 / 1.25$
 $=$ value of hypotenuse
 $= 1.4179630807244128218385339866729$

Whereas, the correct value of hypotenuse

$$= 1.4142135623730950488016887242097$$

This figure exceeds value of hypotenuse by , 0.00374951835131777303684526246322. This excess value has no proportional relation to any fractional value of hypotenuse or to the overlapping area and hence has

no rational explanation available for presence of its value in computing the value of π .

If this figures were to be $(\sqrt{2}/400)$ 0.00353553390593273762200422181052, the value of π should have been 3.14064453125.

It is obvious that π is not only an irrational number but also fails the test of ratios in division of 3.14064453125 by 0.14064453125 results in fractional number.

Therefore, the rational value of Pi is 3.125 as derived from the theorem. This is the value of Pi in the clay tablet from period of Babylon civilization.

(Note While folding the square across two points, if the other half is held perpendicular to it and movement of the tip/edge of the triangle or a point = to radius is traced on the vertical plane, half circle is created

6. π is irrational because:

- Circle is a symmetric figure and successive sub division of it by its rational fractions should result in a whole number. But 3.1415926535897932384626433832795 divided by 0.1415926535897932384626433832795 do not result in a whole number, but in a fraction as 22.187539917793137309379015457712
- Radios/(Radian) is a successive sub divisional fraction of a ray-disk (of a three dimensional objects like a concave lance) starting from a single point and ending at the outer boundary like pyramid/prism. Hence, it has volume, surface area as well a mean linear length/distance from the starting point. A circle created by a radius should be divisible by itself (area of cross section of the pyramid/prism) without residual value.
- From the tables annexed at the end of this paper, it may be reasonably assumed that the area of a radius may be measured as a fraction of a circle (0.00195312500000). **A circle is therefore made up of identical equilateral triangles**

7. Conclusion:

As the circumference derived by using 3.125 as value of π , is rationally divisible in halves successively and the area of the circle is also divisible in successive halves with rational values, this revised value of Pi is rational.

v.i.z : $0.125/4 = 0.03125$ and $3.00 / 0.03125 = 96$ OR $3.125/0.125 = 25$

Thus, one unit diameter circle is made up of 100 triangles of half unit height and base of 0.03125 units with an area of 0.0078125 square units each and hence the area of a circle is 0.78125 units.

A circle encompasses more area than a square with perimeter equal to the circumference of the circle. The overlapping areas of the squares on the two diagonals represent the limitation of the excess area covered by the circle inscribed in a square.

Correctness of this value of Pi (3.125) may be established by actual precision measuring the circumference of a solid circular object where the thickness of the measuring material/ tape is carved out of the solid circular object as otherwise the measurement will result in error of measuring a larger circle (diameter + twice the thickness of the measuring tape) than the object whose circumference is being measured. (Circumference of a circle with 8 unit diameter is 25 units with Pi = 3.125)

(Calculations and ratios used for this paper are attached as Appendices)

Article/paper by

Mohankumar Shetty

Geometric Estimation of Value of Pi

Appendix 1

“Half and Half of Half”, the relationship

Half & Half of Half				
(Values for one unit figures)				
Figure	Area	Diagonal 1	diagonal 2	Radius
		180 degrees		360Degrees
Square	1.00000	1.00	0.25000	0.03125000000000
Triangle	0.50000	0.50	0.12500	0.01562500000000
Circle	0.78125	0.25	0.06250	0.00781250000000
Radius	?	0.75	0.03125	0.00390625000000
			0.03125	0.00195312500000

Appendix 2

THEROM Pi = 3.125							Pi = 3.14159265358979		
Diameter	hypotenuse	Overlapping Area	Sums of Sqrts	1/4th Sqrt	Area of Circle	Circumference	Radius	Pi*r^2 (Area)	2Pir (Circumference)
1	1.41421356237310	0.70710678118655	3.53553390593274	0.88388347648318	0.78125000000000	3.12500000	0.500000	0.785398163397448000	3.141592653589790000
1.00531	1.42172303638930	0.71086151819465	3.55430759097324	0.88857689774331	0.78956890320313	3.14159375	0.502655	0.793761237057883000	3.158274510580350000
1.28	1.81019335983756	0.90509667991878	4.52548339959390	1.13137084989848	1.28000000000000	4.00000000	0.640000	1.286796350910380000	4.021238596594930000
2	2.82842712474619	1.41421356237310	7.07106781186548	1.76776695296637	3.12500000000000	6.25000000	1.000000	3.141592653589790000	6.283185307179580000
3	4.24264068711928	2.12132034355964	10.60660171779820	2.6516504294955	7.03125000000000	9.37500000	1.500000	7.068583470577030000	9.424777960769370000
4	5.65685424949238	2.82842712474619	14.14213562373100	3.53553390593274	12.50000000000000	12.50000000	2.000000	12.566370614359200000	12.566370614359200000
5	7.07106781186548	3.53553390593274	17.67766952966370	4.41941738241592	19.53125000000000	15.62500000	2.500000	19.634954084936200000	15.707963267948900000
6	8.48528137423857	4.24264068711928	21.21320343559640	5.30330085889911	28.12500000000000	18.75000000	3.000000	28.274333882308100000	18.849555921538700000
7	9.89949493661167	4.94974746830583	24.74873734152920	6.18718433538229	38.28125000000000	21.87500000	3.500000	38.484510064749000000	21.991148575128500000
8	11.31370849898480	5.65685424949238	28.28427124746190	7.07106781186548	50.00000000000000	25.00000000	4.000000	50.265482457436600000	25.132741228718300000
9	12.72792206135790	6.36396103067893	31.81980515339460	7.95495128834866	63.28125000000000	28.12500000	4.500000	63.617251235193300000	28.274333882308100000
10	14.14213562373100	7.07106781186548	35.35533905932740	8.83883476483184	78.12500000000000	31.25000000	5.000000	78.539816339744700000	31.415926535897900000

Appendix 3

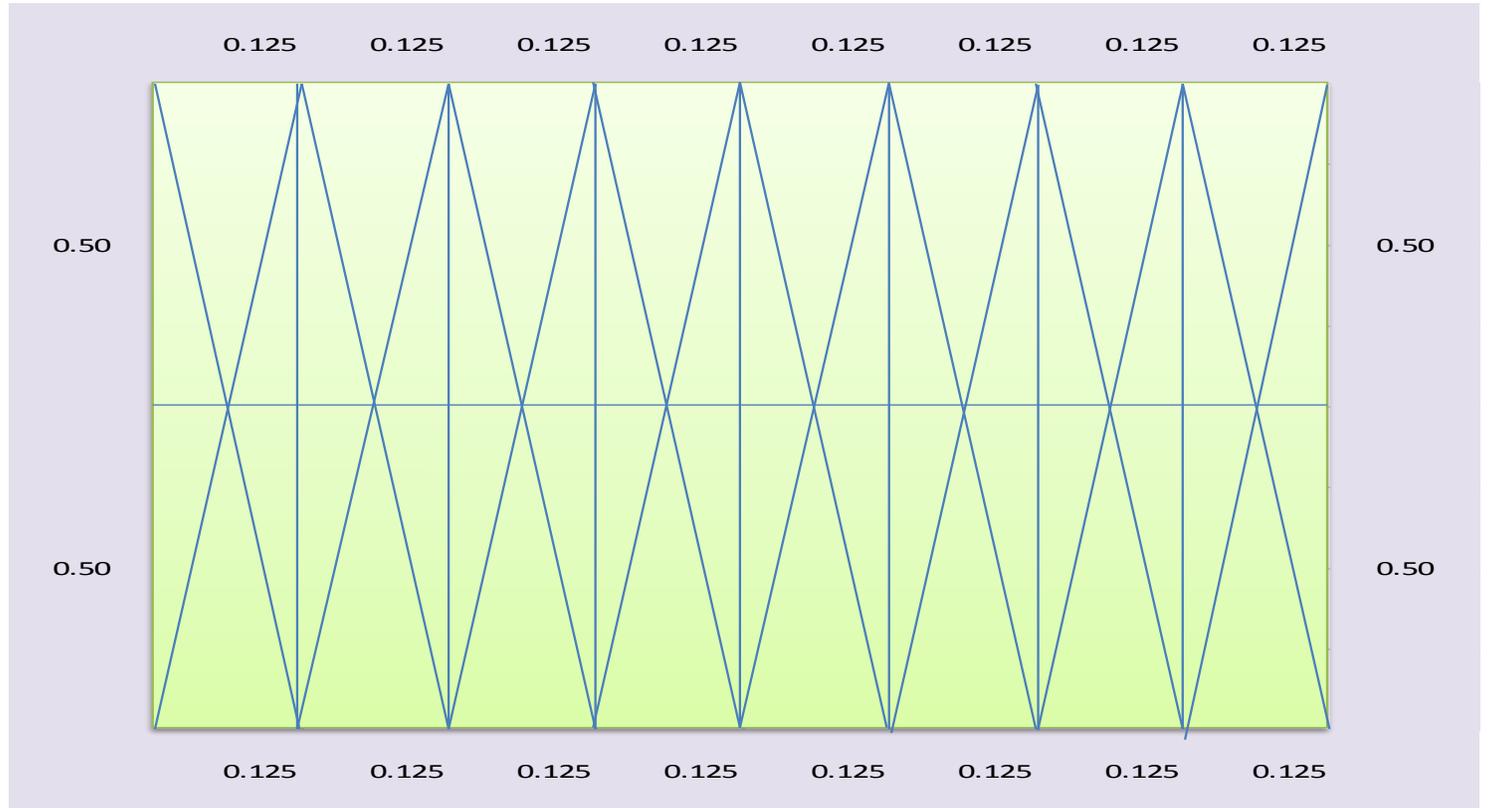
Side/ Diameter	Perimeter	Area of the square	Perimeter * 1.25	Area * 1.25	Circumference	Area of the Circle	Ratio of Area (Square/Circle)	Ratio of Perimeter/Circumference
a	b = a*4	c = a^2	d = c * 1.25	e = c * 1.25	f = d^2 / 2b	g = e^2 / 2c	h = c/g	i = b/f
1	4	1	5	1.25000	3.12500	0.78125	1.28	1.28
2	8	4	10	5.00000	6.25000	3.12500	1.28	1.28
3	12	9	15	11.25000	9.37500	7.03125	1.28	1.28
4	16	16	20	20.00000	12.50000	12.50000	1.28	1.28
5	20	25	25	31.25000	15.62500	19.53125	1.28	1.28
6	24	36	30	45.00000	18.75000	28.12500	1.28	1.28
7	28	49	35	61.25000	21.87500	38.28125	1.28	1.28
8	32	64	40	80.00000	25.00000	50.00000	1.28	1.28
9	36	81	45	101.25000	28.12500	63.28125	1.28	1.28

Geometric Estimation of Value of Pi

Appendix 4

HALF & HALF OF HALF								
	<u>1</u>	<u>2</u>	<u>3</u>	<u>4</u>	<u>5</u>	<u>6</u>	<u>7</u>	<u>8</u>
<u>1</u>	1.0000000000000000	0.5000000000000000	0.2500000000000000	0.1250000000000000	0.0625000000000000	0.0312500000000000	0.0156250000000000	0.0078125000000000
<u>2</u>	0.5000000000000000	0.2500000000000000	0.1250000000000000	0.0625000000000000	0.0312500000000000	0.0156250000000000	0.0078125000000000	0.0039062500000000
<u>3</u>	0.2500000000000000	0.1250000000000000	0.0625000000000000	0.0312500000000000	0.0156250000000000	0.0078125000000000	0.0039062500000000	0.0019531250000000
<u>4</u>	0.1250000000000000	0.0625000000000000	0.0312500000000000	0.0156250000000000	0.0078125000000000	0.0039062500000000	0.0019531250000000	0.0009765625000000
<u>5</u>	0.0625000000000000	0.0312500000000000	0.0156250000000000	0.0078125000000000	0.0039062500000000	0.0019531250000000	0.0009765625000000	0.0004882812500000
<u>6</u>	0.0312500000000000	0.0156250000000000	0.0078125000000000	0.0039062500000000	0.0019531250000000	0.0009765625000000	0.0004882812500000	0.0002441406250000
<u>7</u>	0.0156250000000000	0.0078125000000000	0.0039062500000000	0.0019531250000000	0.0009765625000000	0.0004882812500000	0.0002441406250000	0.0001220703125000
<u>8</u>	0.0078125000000000	0.0039062500000000	0.0019531250000000	0.0009765625000000	0.0004882812500000	0.0002441406250000	0.0001220703125000	0.0000610351562500

Appendix 5



The above figure is a one unit side square divided in to 32 triangles of 0.125 base & 0.50 hight.
 Each triangle has a area of 0.03125 units and 25 of them will form a circle with 0.78125 square area
 $25/32 = 0.78125$

Geometric Estimation of Value of Pi

Appendix 61

ALTERNATE method using hypotenuse						3.141592654		
Diameter	hypotenuse	Hypotenuse*1.25	1/2 of C	Area of Circle	Circumfrence	Radiou	Pir ² (Area)	2Pir (Circumfrence)
1	1.414213562373100	1.767766952966370	0.883883476483184	0.7812500	3.1250000	0.5000000	0.785398163397448	3.141592653589790
1.00531	1.421723036389300	1.777153795486620	0.888576897743310	0.7895689	3.1415938	0.502655	0.793761237057883	3.158274510580350
1.28	1.810193359837560	2.262741699796950	1.131370849898480	1.2800000	4.0000000	0.6400000	1.286796350910380	4.021238596594930
2	2.828427124746190	3.535533905932740	1.767766952966370	3.1250000	6.2500000	1.0000000	3.141592653589790	6.283185307179580
3	4.242640687119280	5.303300858899110	2.651650429449550	7.0312500	9.3750000	1.5000000	7.068583470577030	9.424777960769370
4	5.656854249492380	7.071067811865480	3.535533905932740	12.5000000	12.5000000	2.0000000	12.566370614359200	12.566370614359200
5	7.071067811865480	8.838834764831840	4.419417382415920	19.5312500	15.6250000	2.5000000	19.634954084936200	15.707963267948900
6	8.485281374238570	10.606601717798200	5.303300858899110	28.1250000	18.7500000	3.0000000	28.274333882308100	18.849555921538700
7	9.899494936611670	12.374368670764600	6.187184335382290	38.2812500	21.8750000	3.5000000	38.484510006474900	21.991148575128500
8	11.313708498984800	14.142135623731000	7.071067811865480	50.0000000	25.0000000	4.0000000	50.265482457436600	25.132741228718300
9	12.727922061357900	15.909902576697300	7.954951288348660	63.2812500	28.1250000	4.5000000	63.617251235193300	28.274333882308100
10	14.142135623731000	17.677669529663700	8.838834764831840	78.1250000	31.2500000	5.0000000	78.539816339744700	31.415926535897900