Five Dimensional Kaluza-Klein Type of Cosmological Model of Universe with Dynamical Cosmological term Λ

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ABSTRACT

For the five dimensional Kaluza-Klein type model, the age of the universe is proportional to Hubble parameter. They have established a relationship between Cosmic matter and vacuum energy density parameters for flat universe.

Keywords

1 INTRODUCTION

In the original theory of Kaluza and Klein space-time is extended by the addition of an extra dimension from the four used in Einstein's general relativity. Kaluza-Klein theory has been extensively investigated and has some interesting properties.

One of the most important and outstanding problems in cosmology is of cosmological constant problem. The recent observations indicate that $\Lambda \sim 10^{55}\,\mathrm{cm}^{-2}$ while particle physics prediction for Λ is grater than this value by a factor of order 10^{120}. This discrepancy is known as cosmological constant problems. Some of the recent discussions on the cosmological constant "Problem" and consequence on cosmology with a time - varying cosmological constant are investigated by Ratro and Peebles (1988), Dolgov et al. (1990), Dolgov (1983, 1997), Sahni and Starobinsky (2000), Padmanabhan (2003) and Peebles (2003).

2. Einstein Field Equation

Consider the line element

$$ds^2 = -dt^2 + a(t)^2\left[\frac{dr^2}{1-kr^2} + r^2\left(d\theta^2 + \sin^2\theta\, d\phi^2\right) + \left(1-kr^2\right) d\varphi^2\right], \qquad (2.1)$$

where $a(t)$ is the scale factor and k = 0, ± 1 is the curvature factor.

The usual energy momentum tensor is modified by addition of a term

$$T_{ij}^{vac} = -\Lambda(t)\, g_{ij}\ , \qquad (2.2)$$

where $\Lambda(t)$ is the cosmological term and g_{ij} is the metric tensor. For the perfect fluid distribution Einstein field equations with the cosmological constant Λ and gravitational constant G may be written as:

$$G_{ij} = R_{ij} - \frac{1}{2}Rg_{ij} = -8\,\pi\, G\, T_{ij} + \Lambda(t)\, g_{ij}\ . \qquad (2.3)$$

The energy momentum tensor T_{ij} in the presence of a perfect fluid has the form

$$T_{ij} = (p+\rho)u_i u_j + pg_{ij} \qquad (2.4)$$

where p and ρ are respectively, the energy and pressure of the cosmic fluid and u_i is the fluid five-velocity such that $u^i u_i = -1$.

Therefore, from equation (2.4) the energy momentum tensor T_j^i can be expressed as

$$T_o^o = -\rho, \quad T_1^1 = T_2^2 = T_3^3 = T_4^4 = p\ . \qquad (2.5)$$

The Einstein's tensor gives

$$G_0^0 = \frac{6}{a^2\left(k+\dot{a}^2\right)}\ , \qquad (2.6)$$

(2.2)

$$G_1^1 = G_2^2 = G_3^3 = G_4^4 = \frac{1}{a^2}\left[3\ddot{a}a + 3k + 3\dot{a}^2\right], \qquad (2.7)$$

where over dot indicates a derivative w. r. to 't'.

From equation (2.6) and equation (2.7) with the help of equations (2.5) the field equations (2.3) can be expressed for (k = 0) as

$$\frac{6\dot{a}^2}{a^2} = 8\pi\, G\rho + \Lambda\,, \qquad (2.8)$$

$$\frac{3\ddot{a}}{a} + \frac{3\dot{a}^2}{a^2} = -8\pi\, G\;\; p + \Lambda\,. \qquad (2.9)$$

From equation (2.8) and equation (2.9) we get

$$\frac{6\dot{a}^2}{a^2} = -\,8\pi\, G\,(\rho + 2\,p) + \Lambda\,. \qquad (2.10)$$

Here numbers of equations are less than number of unknown therefore for the complete determinacy of the system, we consider two equations

$$\text{(i) } \Lambda = \mu\dot{H} \qquad (2.11)$$

$$\text{(ii) } p = \omega\rho \qquad (2.12)$$

where ω, the equation of state parameter, for the dust, radiation, vacuum fluid and stiff fluid can take the constant values 0, 1/3, -1 and +1 respectively and μ is a free parameter and $H = \frac{\dot{a}}{a}$.

From the equation (2.8) and equation (2.11) we get

$$6H^2 - \mu\dot{H} = 8\pi\, G\rho\,. \qquad (2.13)$$

By using equation (2.10), equation (2.11) and (2.12) becomes

$$6(H^2 + \dot{H}) = 8\pi\, G\rho\,(1+2\omega) + \mu\dot{H}\,. \qquad (2.14)$$

Equation (2.13) and (2.14), on simplification, yield the differential equation of the form

$$6H^2\,(1+\omega) = \dot{H}\,(\mu + \mu\omega - 3). \qquad (2.15)$$

3. The Models

Now integrating equation (2.15) gives

$$H = \frac{-(\mu + \mu\omega - 3)}{6\,(1+\omega)\,t}\,. \qquad (3.1)$$

Now putting $H = \frac{\dot{a}}{a}$ in equation (3.1) and integrating it further we get our general solution as

$$a(t) = Ct^{\left(\frac{(3-\mu-\mu\omega)}{6\,(1+\omega)}\right)}, \qquad (3.2)$$

$$\rho(t) = \frac{-(\mu + \mu\omega - 3)}{16\pi\, G(1+\omega)^2}\, t^{-2}, \qquad (3.3)$$

$$\Lambda(t) = \frac{\mu(\mu + \mu\omega - 3)}{6(1+\omega)}\, t^{-2}, \qquad (3.4)$$

where C is integration constant .

Case (i) Dust Case (ω =0)

For dust case equations (3.2), (3.3), (3.4) and (3.1) respectively takes the form

$$a(t) = Ct^{\frac{(3-\mu)}{6}}, \qquad (3.5)$$

$$\rho(t) = \frac{-(\mu - 3)}{16\pi\, G}\, t^{-2}, \qquad (3.6)$$

$$\Lambda(t) = \frac{\mu\,(\mu - 3)}{6}\, t^{-2}\,, \qquad (3.7)$$

$$H = \frac{-(\mu - 3)}{6\,t},$$

$$t = \frac{-(\mu - 3)}{6H}. \qquad (3.8)$$

Equation (3.6) suggests that for physically valid ρ (i.e. ρ > 0), we should get for μ < 3. So, μ can be negative as well. Again, from equation (3.7) we find that for a repulsive Λ, the constraint on μ is that it must be negative. Thus equations (3.6), (3.7) and (3.8) all point towards a negative μ.

Case (ii) Radiation Case (ω =1/3)

$$a(t) = Ct^{\frac{(9-3\mu-\mu)}{24}}, \tag{3.9}$$

$$\rho(t) = \frac{-3\mu\,(4\mu-9)}{256\pi\, G}\,t^{-2}, \tag{3.10}$$

$$\Lambda(t) = \frac{\mu\,(4\mu-9)}{24}\,t^{-2}, \tag{3.11}$$

$$H = \frac{-(4\mu-9)}{24t},$$

$$t = \frac{-(4\mu-9)}{24H}. \tag{3.12}$$

Equation (3.10) suggests that for physically valid ρ, μ < 9/4 whereas equation (4.1) demands negative μ for repulsive Λ. Thus in this case also a negative μ is necessary for equation (3.10) to equation (4.2).

4. Equivalent Relationship Between Λ–Dependent Models

Using the value of t from equation (3.8) in equation (3.6) we get

$$\Omega_m = \frac{8\pi\, G\rho}{6H^2} = \frac{-3}{\mu-3}. \tag{4.1}$$

Again, using equation (3.8) in equation (3.7) we get

$$\Omega_\Lambda = \frac{\Lambda}{6H^2} = \frac{\mu}{\mu-3}. \tag{4.2}$$

Adding equation (4.1) and equation (4.2) we get

$$\Omega_m + \Omega_\Lambda = 1. \tag{4.3}$$

This is another form of equation (2.8) for flat universe.

Also, using the value μ from equation (4.1), we get from equation (3.8)

$$t = \frac{1}{2H\Omega_m}. \tag{4.4}$$

Thus, if t_0 and H_0 be the values of t and H at the present epoch,

$$t_0 = \frac{1}{2H_0\Omega_{m0}} \tag{4.5}$$

Equation (4.5) is the same expression for t_0 as obtained by (Ray and Mukhopadhya (2004)) for $\Lambda \sim \left(\frac{\dot{a}}{a}\right)^2$, $\Lambda \sim \frac{\ddot{a}}{a}$ and $\Lambda \sim \rho$. Again using equation (4.2) in equation (3.5) to equation (3.7), it can easily be shown that the expression for $a(t)$, $\rho(t)$, $\Lambda(t)$ become identical with the expressions for those quantities as obtained by (Ray and Mukhopadhyay (2004)) in the context of General theory of relativity. This means that $\Lambda \sim \dot{H}$ model is equivalent to $\Lambda \sim \left(\frac{\dot{a}}{a}\right)^2$, $\Lambda \sim \frac{\ddot{a}}{a}$ and $\Lambda \sim \rho$.

Again from equation (4.1) and equation (4.2) we get

$$\mu = -3\left(\frac{\Omega_\Lambda}{\Omega_m}\right) \tag{4.6}$$

But we (Ray and Mukhopadhyay (2004)) already have shown that

$$\gamma = \frac{\Omega_\Lambda}{\Omega_m} \tag{4.7}$$

Thus, equation (4.8) and equation (4.9) yield

$$\mu = -3\gamma, \tag{4.8}$$

$$\gamma = \frac{-\mu}{3}. \tag{4.9}$$

Also, we (Khadekar and Patki (2005)) have shown that α, β and γ respectively the free parameter of $\Lambda \sim \left(\frac{\dot{a}}{a}\right)^2$, $\Lambda \sim \frac{\ddot{a}}{a}$ and $\Lambda \sim \rho$ model can be interconnected by the relation

$$\alpha = \frac{3\beta\,(1+2\omega)}{\beta+\omega\beta-3} = \frac{6\gamma}{\gamma+1}.$$

For dust particle

$$\alpha = \frac{3\beta}{\beta-3} = \frac{6\gamma}{\gamma+1} \tag{4.10}$$

Thus combining equation (4.9) with equation (4.10) we have

$$\alpha = \frac{3\beta}{\beta-3} = \frac{6\mu}{\mu-3} \tag{4.11}$$

Similarly for radiation filled universe.

Using the value of t from equation (3.12) in equation (3.10) we get

$$\Omega_m = \frac{-9}{4\mu-9} \tag{4.12}$$

Again, using equation (3.12) in equation (3.11) we get

$$\Omega_\Lambda = \frac{4\mu}{4\mu-9} \tag{4.13}$$

Adding equation (4.12) and (4.13) we get

$$\Omega_m + \Omega_\Lambda = 1. \tag{4.14}$$

Using the value of μ from equation (4.12) we get equation (3.12)

$$t = \frac{9}{24H\Omega_m}. \tag{4.15}$$

For present universe

$$t_0 = \frac{9}{24H_0\Omega_{m_0}}. \tag{4.16}$$

In this case μ is related to γ of our previous investigation (Ray and Mukhopadhyay (2004)).

From equation (4.12) and equation (4.13) we get

$$\mu = \frac{-9}{4}\frac{\Omega_\Lambda}{\Omega_m} \tag{4.17}$$

$$\gamma = \frac{\Omega_\Lambda}{\Omega_m} \tag{4.18}$$

$$\gamma = \frac{-4\mu}{9}. \tag{4.19}$$

Again equation (4.16) is the some expression for t_0 as obtained by (Ray and Mukhopadhayay (2004)) for $\Lambda \sim \left(\frac{\dot{a}}{a}\right)^2$, $\Lambda \sim \frac{\ddot{a}}{a}$ and $\Lambda \sim \rho$ model in radiation case. Also, we (Khadekar and Patki (2005)) have shown that for α, β and γ respectively, the three parameters model can be connected by

$$\alpha = \frac{3\beta(1+2\omega)}{\beta+\omega\beta-3} = \frac{6\gamma}{\gamma+1}.$$

For radiation ω = 1/3

$$\alpha = \frac{15\beta}{4\beta-9} = \frac{24\mu}{4\mu-9} \tag{4.20}$$

5. Physical Feature of the Models

The deceleration parameter q is defined as

$$q = \frac{-\ddot{a}a}{\dot{a}^2} = -\left(1+\frac{\dot{H}}{H^2}\right) \tag{5.1}$$

Thus using equation (3.1) we have

$$q = -\left(1+\frac{6(1+\omega)}{(\mu+\mu\omega-3)}\right) \tag{5.2}$$

For an accelerating universe, q < 0 and have

$$\left(1-\frac{6(1+\omega)}{3-\mu-\mu\omega}\right)>0,$$

$$1>\frac{6(1+\omega)}{3-\mu-\mu\omega},$$

$$3-\mu-\mu\omega>6(1+\omega),$$

$$\mu<\frac{-3(1+2\omega)}{(1+\omega)}. \tag{5.3}$$

For an accelerating Universe q < 0 and hence

$\mu<\frac{-3(1+2\omega)}{(1+\omega)}$ equation (5.2) tells us that for a dust field accelerating universe, μ should be less than -3. We have already shown that for physically valid ρ, Λ and t, μ must be negative. Thus for μ < -3 we get an accelerating universe with repulsive Λ through our $\Lambda \sim \dot{H}$ model.

Now the equation (5.1) can be written as

$$\dot{H}=-H^2(q+1). \tag{5.4}$$

For variable cosmological constant as,

$$\Lambda=-\mu\, H^2(q+1). \tag{5.5}$$

From equation (5.5) we observed that Λ remains a repulsive force for μ < 0, so that q > -1. This means that if the acceleration of the universe exceeds a certain limit then Λ will become an attractive force. Since, for our model both $\dot{H}$ and H^2 are proportional to t^{-2} then it is clear from equation (5.1) that q is a constant quantity. But through our model it has been possible to exhibit indirectly via equation (5.3) that in future Λ may become an attractive force provided q becomes less -1.

6. Conclusion

In the present work by choosing a phenomenological model Λ *viz.* $\Lambda \sim \dot{H}$, we investigate that this model of Λ is equivalent of other three types of $\Lambda=\alpha\left(\frac{\dot{a}}{a}\right)^2$, $\Lambda=\beta\frac{\ddot{a}}{a}$ and $\Lambda=\gamma\,\rho$. We have also established a relationship between α, β and γ the three parameter of the three form of Λ which ultimately yield $\Omega_m+\Omega_\Lambda=1$, the relation between cosmic matter and vacuum energy density parameters for flat universe. For this model the age of the universe is proportional to Hubble parameter.

It has been possible through our model to put a limit on q, the declaration parameter of Universe. If one is content with the idea of a repulsive Λ only, then we find the q cannot exceed a certain limit. But, if one is bold enough to think of an attractive Λ in future, then q can go on decreasing indefinitely, i.e. the rate of acceleration can increase indefinitely.

Finally, it should also mentioned that any linear combination of $\Lambda \sim \left(\frac{\dot{a}}{a}\right)^2$, $\Lambda \sim \frac{\ddot{a}}{a}$ and $\Lambda \sim \rho$ is also equivalent in the context of higher dimensional space time.

Acknowledgement. The authors are thankful to Professor G. S. Khadekar for suggesting the problem.

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