

EFFECTS OF THERMAL STRESSES ON THICK CIRCULAR PLATE DUE TO HEAT GENERATION

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ABSTRACT

In this paper, an attempt has been made to study thermoelastic response of a thick circular plate occupying the space due to heat generation with radiation type boundary conditions. Here we apply integral transform techniques to find the thermoelastic solution.

Keywords: Thermo elastic Response, Circular Plate, Thermal Stresses, integral transform

1. INTRODUCTION

Most materials tend to expand if their temperature rises and, to a first approximation, the expansion and compression is proportional to the temperature change. This temperature changes induced by expansion and compression is based on Thermoelasticity, which is a branch of applied Mathematics, which specially deals with the study of temperature changes and coupling between mechanical deformation and thermal energy calculated in terms of stress. Therefore, a number of theoretical studies concerning them have

been reported so far. However, to simplify the analyses, almost all the studies were conducted on the assumption that the upper and lower surfaces of the thin discs or circular are insulated or heat is dissipated with uniform heat transfer coefficients throughout the surfaces. For example, Nowacki, W. [42] has determined steady-state thermal stresses in a thick circular plate subjected to an axis symmetric temperature distribution on the upper face with zero temperature on the lower face and circular edge. Ishihara et al. [23] has considered a circular plate and discussed the transient thermoelastic-plastic bending problem, making use of the strain increment theorem. In all afore mentioned investigations an axis

symmetrically heated plate has been considered. Similar studies were also conducted for thick objects. For Example, Nasser, M.El-Maghraby [39-40] investigated problems due to heat sources in generalized thermoelastic body. Kulkarni, V.S. and Deshmukh K.C. [28] has investigated their research on disc for determining quasi-static thermal stresses in a thick annular disc and circular plates subjected to arbitrary initial temperature on the upper face with lower face at zero.

2 STATEMENT OF THE PROBLEM

Consider thick circular plate of thickness $2h$ occupying the space $D: 0 \leq r \leq a, -h \leq z \leq h$, the material is homogenous and isotropic. The differential equation governing the displacement potential function $\phi(r, z, t)$ as Nowacki [47] is

$$\frac{\partial^2 \phi}{\partial r^2} + \frac{1}{r} \frac{\partial \phi}{\partial r} + \frac{\partial^2 \phi}{\partial z^2} = \left(\frac{1+\nu}{1-\nu} \right) \alpha_t T \quad (1)$$

where ν and α_t are Poisson's ratio and linear coefficient of thermal expansion of the material of the plate and T is the temperature of the plate

satisfying the differential equation as Noda [41] is

$$\frac{\partial^2 T}{\partial r^2} + \frac{1}{r} \frac{\partial T}{\partial r} + \frac{\partial^2 T}{\partial z^2} + \chi(r, z, t) = \frac{1}{k} \frac{\partial T}{\partial t} \quad (2)$$

Subject to initial condition and boundary conditions

$$T(r, z, t) = F(r, z) \text{ at } t = 0 \text{ for all } 0 \leq r \leq a, -h \leq z \leq h \quad (3)$$

$$\frac{\partial T}{\partial z} = g_1(z, t) \text{ at } r = 0 \text{ for all } -h \leq z \leq h, t > 0 \quad (4)$$

$$\frac{\partial T}{\partial z} = g_2(z, t) \text{ at } r = a \text{ for all } -h \leq z \leq h, t > 0 \quad (5)$$

$$T + k_1 \frac{\partial T}{\partial z} = f_1(r, t) \text{ at } z = h \text{ for all } 0 \leq r \leq a, t > 0 \quad (6)$$

$$T + k_2 \frac{\partial T}{\partial z} = f_2(r, t) \text{ at } z = -h \text{ for all } 0 \leq r \leq a, t > 0 \quad (7)$$

where k is thermal diffusivity of material of the plate.

The displacement function in the cylindrical coordinate system are represented

by Love's function as

$$u_r = \frac{\partial \phi}{\partial r} - \frac{\partial^2 L}{\partial r \partial z} \quad (8)$$

$$u_z = \frac{\partial \phi}{\partial z} + 2(1-\nu)\nabla^2 L - \frac{\partial^2 L}{\partial z^2} \quad (9)$$

The Love's function must satisfy

$$\nabla^2 \nabla^2 L = 0 \quad (10)$$

where

$$\nabla^2 = \frac{\partial^2}{\partial r^2} + \frac{1}{r} \frac{\partial}{\partial r} + \frac{\partial^2}{\partial z^2} \quad (11)$$

The component of stresses are represented by the thermoelastic displacement

potential ϕ and Love's function L as Noda [41] are

$$\sigma_{rr} = 2G \left\{ \left(\frac{\partial^2 \phi}{\partial r^2} - \nabla^2 \phi \right) + \frac{\partial}{\partial z} \left(\nu \nabla^2 L - \frac{\partial^2 L}{\partial r^2} \right) \right\} \quad (12)$$

$$\sigma_{\theta\theta} = 2G \left\{ \left(\frac{1}{r} \frac{\partial \phi}{\partial r} - \nabla^2 \phi \right) + \frac{\partial}{\partial z} \left(\nu \nabla^2 L - \frac{1}{r} \frac{\partial^2 L}{\partial r^2} \right) \right\} \quad (13)$$

(8)

$$\sigma_{zz} = 2G \left\{ \left(\frac{\partial^2 \phi}{\partial z^2} - \nabla^2 \phi \right) + \frac{\partial}{\partial z} \left\{ \left((z-\nu) \nabla^2 L - \frac{\partial^2 L}{\partial z^2} \right) \right\} \right\} \quad (14)$$

(9)

$$\sigma_{rz} = 2G \left\{ \frac{\partial^2 \phi}{\partial r \partial z} + \frac{\partial}{\partial r} \left\{ \left((1-\nu) \nabla^2 L - \frac{\partial^2 L}{\partial z^2} \right) \right\} \right\} \quad (15)$$

(10)

For traction free surface stress function

$$\sigma_{\theta z} = \sigma_{r\theta} = 0 \text{ at } z = \pm h \text{ for thick plate.} \quad (16)$$

Equations to constitute the mathematical formulation of the problem under

(11)

consideration.

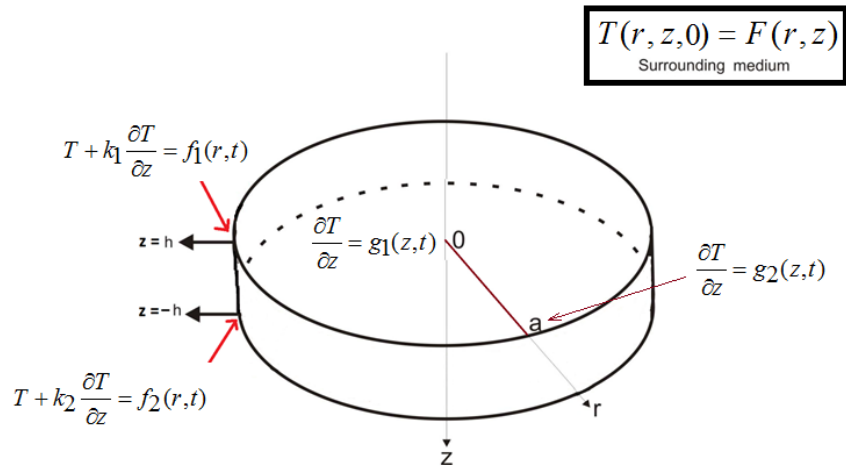


Figure 1: Shows the geometry of the problem

3 SOLUTION OF THE PROBLEM

Applying Hankel transform defined to the equation, we get

$$-\xi_m^2 T^*(\xi_m, z, t) + \frac{d^2 T^*}{dz^2}(\xi_m, z, t) + \chi^*(\xi_m, z, t) = \frac{1}{k} \frac{dT^*}{dt} \quad (17)$$

Again applying Marchi-Fasulo transform defined in to above equation, we obtain

$$\frac{d\bar{T}^*}{dt} + kP^2 \bar{T}^* = \Psi \quad (18)$$

where

$$P^2 = \xi_m^2 + a_n^2 \quad (19)$$

Equation is a linear equation whose solution is given by

$$\bar{T}^*(\xi_m, n, t) = e^{-kP^2 t} \int_0^t \Psi e^{kP^2 t'} dt' + C e^{-kP^2 t} \quad (20)$$

Using (3), we get

$$C = F^*(m, n) \quad (21)$$

Thus we have

$$\bar{T}^*(\xi_m, n, t) = e^{-kP^2 t} \left[\int_0^t \Psi e^{kP^2 t'} dt' + \bar{F}^*(m, n) \right]$$

Applying inversion of Marchi-Fasulo transform and Hankel transform to the differential equation, we get

$$T(r, z, t) = \frac{2}{a^2} \sum_m \sum_n \frac{J_0(r\xi_m)}{[J_1(a\xi_m)]^2} \frac{P_n(z)}{\lambda_n} e^{-kP^2 t}$$

$$\times \left[\int_0^t \Psi e^{kP^2 t'} dt' + \overline{F}^* (m, n) \right] \quad (22)$$

This is the desired solution of the given problem.

Let us assume Love's function L , which satisfy condition as

$$L(r, z) = \frac{2}{a^2} \sum_m \sum_n \frac{J_0(r\xi_m)}{[J_1(a\xi_m)]^2} \frac{P_n(z)}{\lambda_n} \Omega \quad (23)$$

where

$$\Omega = e^{-kP^2 t} \left[\int_0^t \Psi e^{kP^2 t'} dt' + \overline{F}^* (m, n) \right] \quad (24)$$

we get displacement potential ϕ as

$$\phi = A \sum_m \sum_n \frac{J_0(r\xi_m)}{[J_1(a\xi_m)]^2} \left[\frac{P_n'(z)}{\lambda_n} \Omega + B(t) \right] \quad (25)$$

where

$$A = \left(\frac{1 + \nu}{1 - \nu} \right) \frac{2\alpha_t}{a^2} \quad (26)$$

$$B(t) = \int e^{-kP^2 t} \left(\int_0^t \Psi e^{-kP^2 t'} dt' + \overline{F}^* (m, n) \right) dt \quad (27)$$

4 DETERMINATION OF DISPLACEMENT FUNCTION

Substituting, we get

$$u_r = A \sum_m \sum_n \frac{\xi_m J_1(r\xi_m)}{[J_1(a\xi_m)]^2} \left[\frac{P_n'(z)}{\lambda_n} \psi + B(t) \right] \quad (21)$$

$$- \frac{2}{a^2} \sum_m \sum_n \frac{\xi_m J_1(r\xi_m)}{[J_1(a\xi_m)]^2} \frac{P_n'(z)}{\lambda_n} \Omega$$

and

$$u_z = A \sum_m \sum_n \frac{J_0(r\xi_m)}{[J_1(a\xi_m)]^2} \left[\frac{P_n''(z)}{\lambda_n} \Omega + B(t) \right]$$

$$+ \frac{2(1-\nu)}{a^2} \sum_m \sum_n \frac{\xi_m^2 [J_1'(r\xi_m) + J_1(r\xi_m)] P_n(z)}{[J_1(a\xi_m)]^2 \lambda_n} \Omega$$

$$+ \frac{2(1-2\nu)}{a^2} \sum_m \sum_n \frac{J_0(r\xi_m)}{J_1(a\xi_m)^2} \frac{P_n''(z)}{\lambda_n} \Omega$$

Substituting in above equations, we obtain

$$\begin{aligned}
 \sigma_{rr} = & 2G \left\{ \frac{2(\nu-1)}{a^2} \sum_m \sum_n \frac{\xi_m^2 J_1'(r\xi_m) P_n'(z)}{[J_1(a\xi_m)]^2 \lambda_n} \Omega \right. \\
 & + \frac{2}{a^2 r} \sum_m \sum_n \frac{\xi_m J_1(r\xi_m) P_n'(z)}{[J_1(a\xi_m)]^2 \lambda_n} \Omega \\
 & + \frac{2}{a^2} \sum_m \sum_n \frac{J_0(r\xi_m) P_n'(z)}{[J_1(a\xi_m)]^2 \lambda_n} \Omega \\
 & - \frac{A}{r} \sum_m \sum_n \frac{\xi_m J_1(r\xi_m)}{J_1(a\xi_m)^2} \left[\frac{P_n'(z)}{\lambda_n} \Omega + B(t) \right] \\
 & \left. - A \sum_m \sum_n \frac{J_0(r\xi_m)}{[J_1(a\xi_m)]^2} \left[\frac{P_n'(z)}{\lambda_n} \Omega + B(t) \right] \right\} \\
 & + \frac{2\nu}{a^2} \sum_m \sum_n \frac{J_0(r\xi_m) P_n^{111}(z)}{[J_1(a\xi_m)]^2 \lambda_n} \Omega \\
 & - A \sum_m \sum_n \frac{J_0(r\xi_m)}{J_1(a\xi_m)^2} \left[\frac{P_n^1(z)}{\lambda_n} \Omega + B(t) \right] \\
 & - A \sum_m \sum_n \xi_m^2 \frac{J_1^1(r\xi_m)}{[J_1(a\xi_m)]^2} \left[\frac{P_n^1(z)}{\lambda_n} \Omega + B(t) \right] \Bigg\} \\
 \sigma_{\theta\theta} = & 2G \left\{ \frac{2\nu}{a^2} \sum_m \sum_n \frac{\xi_m^2 J_1^1(r\xi_m) P_n^1(z)}{[J_1(a\xi_m)]^2 \lambda_n} \Omega \right. \\
 & + \frac{2(\nu-1)}{a^2} \sum_m \sum_n \frac{\xi_m J_1(r\xi_m) P_n^1(z)}{[J_1(a\xi_m)]^2 \lambda_n} \Omega \\
 & + \frac{2(1-\nu)}{a^2} \sum_m \sum_n \frac{J_0(r\xi_m) P_n^{111}(z)}{[J_1(a\xi_m)]^2 \lambda_n} \Omega \\
 & - A \sum_m \sum_n \frac{\xi_m^2 J_1^1(r\xi_m)}{[J_1(a\xi_m)]^2} \left[\frac{P_n^1(z)}{\lambda_n} \Omega + B(t) \right] \\
 & \left. + \frac{2(\nu-1)}{a^2} \sum_m \sum_n \frac{\xi_m J_1(r\xi_m) P_n^1(z)}{[J_1(a\xi_m)]^2 \lambda_n} \Omega \right\} \\
 \sigma_{zz} = & 2G \left\{ \frac{(2-\nu)}{a^2} \sum_m \sum_n \frac{\xi_m^2 J_1^1(r\xi_m) P_n^1(z)}{[J_1(a\xi_m)]^2 \lambda_n} \Omega \right. \\
 & + \frac{2(2-\nu)}{a^2 r} \sum_m \sum_n \frac{\xi_m J_1(r\xi_m) P_n^1(z)}{[J_1(a\xi_m)]^2 \lambda_n} \Omega \\
 & + \frac{2(1-\nu)}{a^2} \sum_m \sum_n \frac{J_0(r\xi_m) P_n^{111}(z)}{[J_1(a\xi_m)]^2 \lambda_n} \Omega \\
 & - A \sum_m \sum_n \frac{\xi_m^2 J_1^1(r\xi_m)}{[J_1(a\xi_m)]^2} \left[\frac{P_n^1(z)}{\lambda_n} \Omega + B(t) \right] \\
 & \left. + \frac{2(2-\nu)}{a^2 r} \sum_m \sum_n \frac{\xi_m J_1(r\xi_m) P_n^1(z)}{[J_1(a\xi_m)]^2 \lambda_n} \Omega \right\} \tag{25}
 \end{aligned}$$

$$\begin{aligned}
 & \left. -\frac{A}{r} \sum \sum \frac{\xi_m J_1(r\xi_m)}{[J_1(a\xi_m)]^2} \left[\frac{P_n^1(z)}{\lambda_n} \Omega + B(t) \right] \right\} \\
 \sigma_{rz} = & 2G \left\{ \frac{2(1-\nu)}{a^2} \sum \sum \frac{\xi_m^3 J_1^{11}(r\xi_m)}{[J_1(a\xi_m)]^2} \frac{P_n^1(z)}{\lambda_n} \Omega \right. \\
 & + \frac{2(-\nu)}{a^2} \sum \sum \frac{\xi_m J_1(r\xi_m)}{[J_1(a\xi_m)]^2} \frac{P_n^{11}(z)}{\lambda_n} \Omega \\
 & + \frac{2(1-\nu)}{a^2} \sum \sum \frac{\xi_m}{[J_1(a\xi_m)]^2} \left\{ \frac{\xi_m r J_1^1(r\xi_m)}{r_2} \right\} \frac{P_n(z)}{\lambda_n} \Omega \\
 & \left. + A \sum \sum \frac{\xi_m J_1(r\xi_m)}{[J_1(a\xi_m)]^2} \left[\frac{P_n^4(z)}{\lambda_n} \Omega + B(t) \right] \right\}
 \end{aligned}$$

where

$$A = \left(\frac{1+\nu}{1-\nu} \right) \frac{2\alpha_t}{a^2},$$

$$\Omega_{(\overline{z})} e^{-kp^2 t} \left[\int_0^t \Psi e^{-kp^2 t^1} dt^1 + \overline{F}^*(m, n) \right],$$

$$B(t) = \int \Omega dt$$

5 SPECIAL CASE

Set $F(r, z) = z^2(1-r^2)$

Applying Marchi-Fasulo transform, are obtain

$$\overline{F}(r, n) = (1-r^2) \int_{-h}^h z^2 P_n(z) dz$$

$$\overline{F}(r, n) = (1-r^2) \Phi_n \left[\frac{2h^2 \sin(a_n h)}{a_n} + \frac{4h \cos(a_n h)}{a_n^2} - \frac{4 \sin(a_n h)}{a_n^3} \right]$$

(30)

where (28)

$$P_n(z) = Q_n \cos(a_n z) - W_n \sin(a_n z),$$

$$Q_n = a_n (\alpha_1 + \alpha_2) \cos(a_n h) + (\beta_1 - \beta_2) \sin(a_n h)$$

$$W_n = (\beta_1 - \beta_2) \cos(a_n h) + a_n (\alpha_1 - \alpha_2) \sin(a_n h)$$

Again on applying Hankel transform, we obtain

$$\bar{F}^*(m, n) = \Pi_n \left[\frac{a}{\xi_m} J_1(a\xi_m) - \frac{a(a^2 \xi_m^2 - 4)}{\xi_m^3} J_1(a\xi_m) - \frac{2a^2}{\xi_m^2} J_0(a\xi_m) \right] \times \left(\frac{a}{\xi_m} J_1(a\xi_m) - \frac{a(a^2 \xi_m^2 - 4)}{\xi_m^3} J_1(a\xi_m) - \frac{2a^2}{\xi_m^2} J_0(a\xi_m) \right) \quad (31)$$

where

$$\Pi_n = \Phi_n \left[\frac{2h^2 \sin(a_n h)}{a_n} + \frac{4h \cos(a_n h)}{a_n^2} - \frac{4 \sin(a_n h)}{a_n^3} \right]$$

and

$$\Phi_n = a_n (\alpha_1 + \alpha_2) \cos(a_n h) + (\beta_1 - \beta_2) \sin(a_n h).$$

Using equation (3.5.3) in equation (3.3.3), one obtains

$$T(r, z, t) = \frac{2}{a^2} \sum_m \sum_n \frac{J_0(r\xi_m)}{[J_1(a\xi_m)]^2} \frac{P_n(z)}{\lambda_n} e^{-kp^2 t} \times \left[\int_0^t \Psi e^{kp^2 t} dt^1 + \Pi_n \right]$$

6 NUMERICAL RESULTS

Set $a = 0.2m, k = 15.9 \times 10^{-6}, t = 1$ we get

$$T(r, z, t) = \frac{1}{2} \sum_m \sum_n \frac{J_0(r\xi_m)}{[J_1(a\xi_m)]^2} \frac{P_n(z)}{\lambda_n} e^{-(15.9 \times 10^6) P^2 t}$$

$$\times \left[\int_0^1 \Psi e^{(15.9 \times 10^6) P^2 t^1} dt^1 + \Pi_n \left(\frac{2}{\xi_m} J_1(2\xi_m) - \frac{2(4\xi_m^2 - 4)}{\xi_m^3} J_1(2\xi_m) - \frac{2}{\xi_m^2} J_0(2\xi_m) \right) \right]$$

As an illustration, we carried out numerical calculations for a thick circular plate made up of aluminum metal (refer Table 1 for parameter) and examine the thermoelastic behavior in the state for temperature distribution, displacement and thermal stresses in radial and axial direction.

Materials	Symbol	K	C_p	ρ	α	λ	E	ν
		Btu/ hr ft ⁰ F	Btu/ lb ° F	lb/ ft ³	ft ² / hr	1/F (X 10 ⁻⁶)	GP _a	
Aluminum	Al	117	0.208	169	3.33	12.84	70	0.35
Copper	Cu	224	0.091	558	4.42	9.3	117	0.36
Iron	Fe	36	0.104	491	0.70	6.7	193	0.21
Silver	Ag	242	0.056	655	6.60	10.7	83	0.37

Table 1: Thermal material properties

Figure 2 shows that the variation of temperature distribution along axial direction, it is clear that temperature decreases initially at time $t = 0.25, 0.50, 0.75, 1.00$ and slightly increasing at $z = 2.5$, the curve behaves like a sinusoidal type. But due to the axisymmetric internal heating at $t = 0.5$ temperature decreases upto zero at $z = 3$.

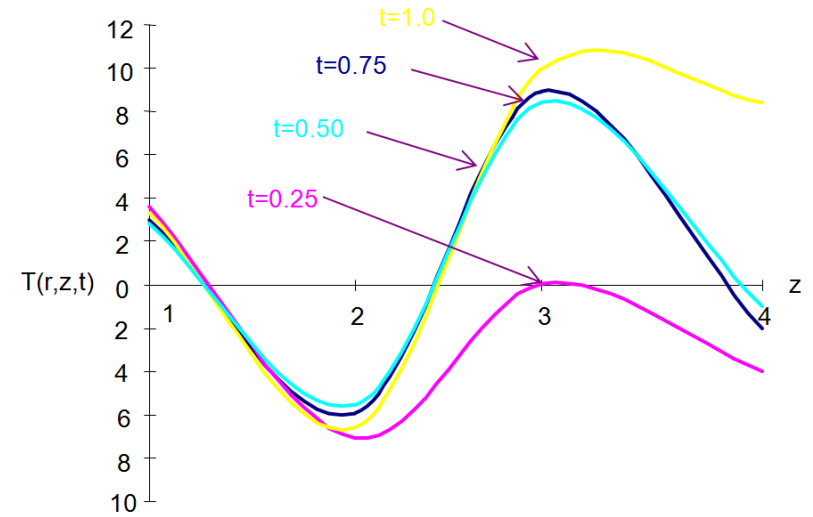


Figure 2. Temperature distribution along axial direction

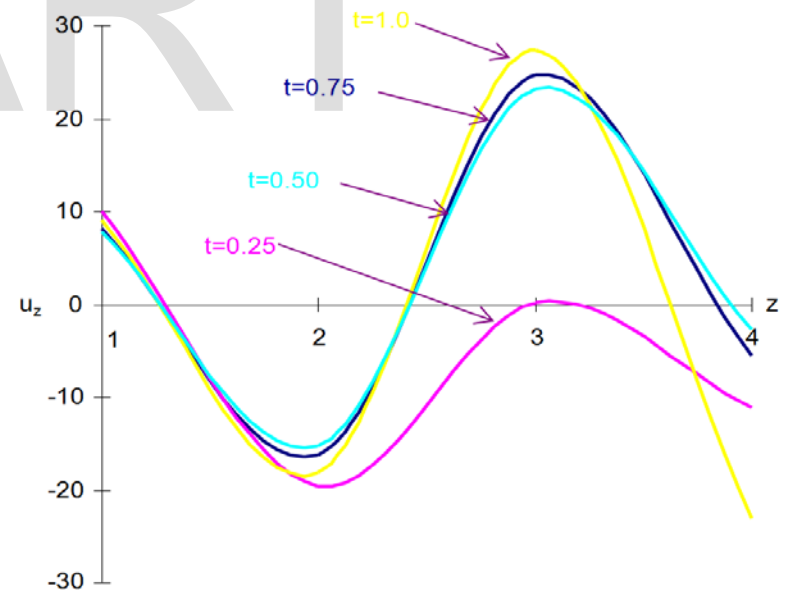


Figure 3. Displacement along axial direction

In figure 3 depicts the variation of displacement u_z along axial direction, it is clear that radial displacement decreases initially at time $t = 0.25, 0.50, 0.75, 1.00$ and slightly increasing at $z = 2.5$ and attain peak value for $z = 3$, again the curve behaves like a sinusoidal type. But due to the axis symmetric internal heating at $t = 0.5$ displacement decreases upto zero at $z = 3$.

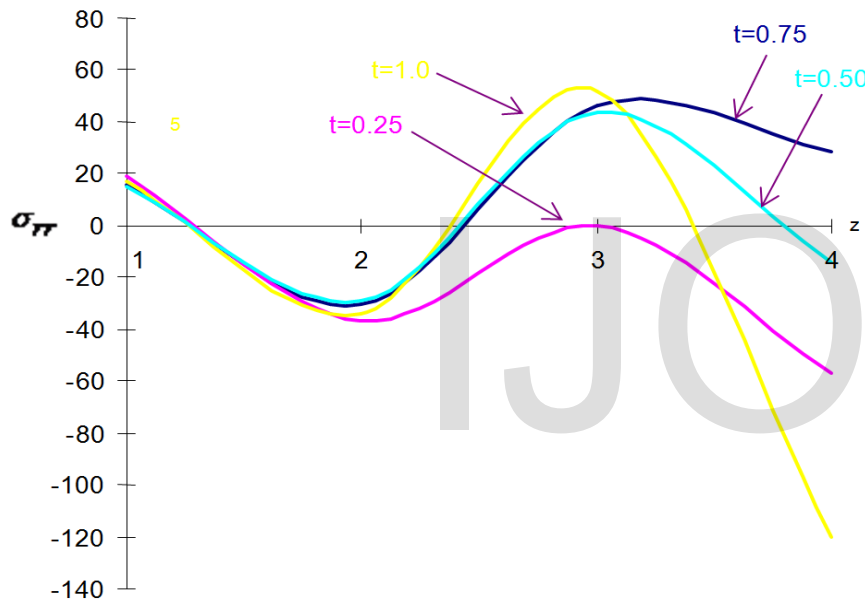


Figure 4. Radial stresses along axial direction

In figure 4 displays the variation of radial stresses along axial direction at different values of time, it is clear that radial stresses initially decreases at time $t = 0.25, 0.50, 0.75, 1.00$ and the start increasing at $z = 2.5$ and attain peak value for $z = 3$, again the curve behaves like a sinusoidal type. But due to the axis symmetric

internal heating at $t = 0.5$ temperature decreases upto zero at $z = 3$. It is also observed that the Axial stresses σ_{zz} and the Shear Stresses σ_{rz} along axial direction for different values of time were found similar to that of Radial stresses σ_{rr} and Tangential stresses $\sigma_{\theta\theta}$ along axial direction with only slight change in the magnitude.

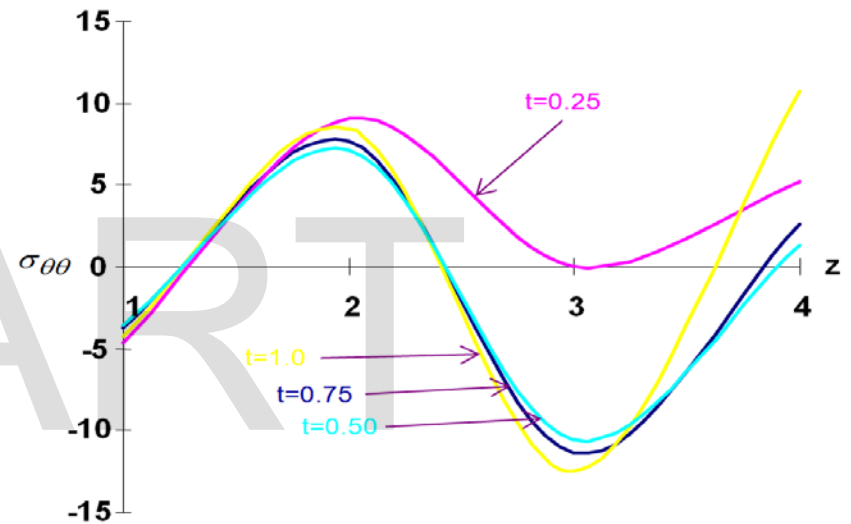


Figure 5. Tangential stresses along axial direction

7 CONCLUSION

The temperature distribution, displacement and thermal stresses of thick circular plate are investigated with known boundary conditions. Finite integral transform techniques are used to obtain numerical results. Any particular cases of special

interest can be assigned to the parameters and functions in expressions. The temperature, displacement and thermal stresses that are obtained can be useful to the design of structure or machines in engineering applications.

8 REFERENCES

- [1] archi E and Fasulo A: Heat conduction in sector of hollow cylinder with radiation, Atti, della Acc.sci. di.tori no, 1(1967), 373-382.
- [2] Nowacki W: the state of stress in thick circular plate due to temperature field. Ball. Sci. Acad. Palon
- [3] Noda N; Hetnarski, R.B. Tanigawa. Y: Thermal stresses, second edition Taylor and Francis, New York (2003). 260.
- [4] Ozisik M.N.: Boundary Value problem of heat conduction, International text book company, Scranton, Pennsylvania (1986), 135.
- [5] Wankhede P.C.: on the quasi-static thermal stresses in a circular plate. Indian J. Pure and Application Maths, 13, No. 11 (1982), 1273-1277.
- [6] Roy H.S.; Bagade S.H.; Khobragade N.W.: Thermal Stresses of a Semi Infinite Rectangular Beam. IJEIT vol.3 (2013) pp.442-445.
- [7] Khobragade N.W.; Khalsa L.H.; Gahane T.T. and Pathak A.C.: Transient Thermo elastic Problems of a Circular Plate with Heat Generation, IJEIT vol.3 (2013) pp. 361- 367.
- [8] Love, A.E.H: A treatise on the mathematical theory of elasticity (Dover publication, Inc, New York, 1964).

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