

# Design and Analysis of Piezoelectric Smart Beam for Active Vibration Control

Deepak Chhabra<sup>1</sup>, Kapil Narwal<sup>2\*</sup>, Pardeep Singh<sup>3</sup>

University Institute of Engineering and Technology, Maharshi Dayanand University, Rohtak, India  
Email: <sup>1</sup>deepaknit10@gmail.com, <sup>2\*</sup>mr.k.narwal@gmail.com, <sup>3</sup>pardeppapr@gmail.com

## ABSTRACT

This paper deals with the Active Vibration control of beam like structures with distributed piezoelectric actuator and sensor layers bonded on top and bottom surfaces of the beam. The patches are located at the different positions to determine the better control effect. The piezoelectric patches are placed on the free end, middle end and fixed end. The study is demonstrated through simulation in MATLAB for various controllers like Proportional Controller by Output Feedback, Proportional Integral Derivative controller (PID) and Pole Placement technique. A smart cantilever beam is modeled with SISO system. The entire structure is modeled using the concept of piezoelectric theory, Euler-Bernoulli beam theory, Finite Element Method (FEM) and the State Space techniques. The numerical simulation shows that the sufficient vibration control can be achieved by the proposed method.

**Keywords :** Smart Structure, Finite Element model, State Space model, Proportional Output feedback, PID, Pole Placement

## 1 INTRODUCTION

The development of high strength to weight ratio of mechanical structures are attracting engineers to build light weight aerospace structures as well as to build tall buildings and long bridges. The development of piezoelectric material has been used as sensors and actuators because if the forces are applied on that material it produces voltage and this voltage goes to active devices and controls the vibration. Their reliability, nearly linear response with applied voltage and their low cost make piezoelectric materials the most widely preferred one as collocated sensor and actuator pair. Active vibration control is the active application of force in an equal and opposite fashion to the forces imposed by external vibration. The finite element method is powerful tool for designing and analyzing smart structures. A design method is proposed by incorporating control laws such as Proportional Output Feedback (POF) and Proportional Integral Derivative (PID) and Pole Placement technique to suppress the vibration. Baz and Poh [2] investigated methods to optimize the location of piezoelectric actuators on beams to minimize the vibration amplitudes. Suleman [15] proposed the effectiveness of the piezo-ceramic sensor and actuators on the attenuation of vibrations on an experimental wing due to the gust loading. Brij N Agrawal and Kirk E Treanor [6] presented the analytical and experimental results on optimal placement of Piezoceramics actuators for shape control of beam structures. Manning, Plummer & Levesley [11] presented a smart structure vibration control scheme using system identification and pole placement technique. Kapil Narwal and Deepak Chhabra [8] presented a detailed analysis insight on the active vibration control of structures. Raja, S, prathap G & Sihna [14] studied active vibration control of a composite sandwich beam with two kinds of piezoelectric actuator or such as extension-bending and shear. Xu & Koko [18] proposed results by using

the commercial FE-package and ANSYS. Baillargeon & Vel [1] presented vibration suppression of adaptive sandwich cantilever beam using PZT shear actuators by experiments and numerical simulations. T. C. Manjunath & B. Bandyopadhyay [17] presented the modeling and design of a multiple output feedback based discrete sliding mode control scheme application for the vibration control of a smart cantilever beam of three four and five elements. N.S. Viliani<sup>1</sup>, S.M.R. Khalili [12] studied the active buckling control of smart functionally graded (FG) plates using piezoelectric sensor/actuator patches. M. Yaqoob Yasin, Nazeer Ahmad [10] presented the active vibration control of smart plate equipped with patched piezoelectric sensors and actuators. In most of present researches, FEM formulation of smart cantilever beam is usually done by ANSYS and by design of control laws are carried out in MATLAB toolbox. The objective of this work is to design and analysis of piezoelectric smart structures with control laws. Proportional Output Feedback (POF) Controller, Proportional Integral Derivative (PID) control law and Pole Placement Technique is used to suppress the vibrations. The eigenvalues of the closed loop system are also controlled with Pole Placement Technique. Numerical examples are presented to demonstrate the validity of the proposed design scheme. This paper has organized in to three parts, FEM formulation of piezoelectric smart structure with control laws, Numerical simulation and Conclusion.

## 2 MODELING OF SMART CANTILEVER BEAM

### 2.1 Finite Element Formulation of Beam Element

A beam element is considered with two nodes at its end. Each node is having two degree of freedom (DOF) i.e.

translation and rotation is considered. The shape functions of the element are derived by applying boundary conditions. The mass and stiffness matrix is derived using shape functions for the beam element. To obtain the mass and stiffness matrix of smart beam element which consists of two piezoelectric materials and a beam element, are added. The global mass and stiffness matrix is formed. The boundary conditions are applied on the global matrices for the cantilever beam. The first two rows and two columns should be deleted as one end of the cantilever beam is fixed. The actual response of the system i.e. the tip displacement is obtained for all the various models of the cantilever beam with and without the controllers.

$$\xi f_1(x) = \xi - 3x^2/l_b^2 + 2x^3/l_b^3 \quad (1)$$

$$[n] = \xi f_2(x) = [x - 2x^2/l_b + x^3/l_b^2] \quad (2)$$

$$\xi f_3(x) = [3x^2/l_b^2 - 2x^3/l_b^3] \quad (3)$$

$$\xi f_4(x) = [-x^2/l_b + x^3/l_b^2] \quad (4)$$

Where (n) gives the shape functions.

The equation of motion of the regular beam element is obtained by the lagrangian equation

$$\frac{d}{dt} \left[ \frac{\partial T}{\partial \dot{q}_i} \right] - \frac{\partial T}{\partial q_i} + \frac{\partial U}{\partial q_i} = [F_i] \quad (5)$$

$$\text{as } M^b \ddot{q} + K^b q = f^b(t) \quad (6)$$

Where  $M^b$ ,  $K^b$  and  $F^b$  are the mass, stiffness and force co-efficient vector matrices respectively of the regular beam element. The mass and stiffness matrices are obtained as

$$M^b = \frac{\rho A l_b}{420} \begin{bmatrix} 156 & 22l_b & 54 & -13l_b \\ 22l_b & 4l_b^2 & 13l_b & -3l_b^2 \\ 54 & 13l_b & 156 & -22l_b \\ 13l_b & -3l_b^2 & -22l_b & 4l_b^2 \end{bmatrix}$$

$$K_b = \frac{E_b I_b}{l_b} \begin{bmatrix} 12 & 6 & -12 & 6 \\ \frac{6}{l_b} & 4 & -\frac{6}{l_b} & 2 \\ \frac{12}{l_b} & -\frac{6}{l_b} & \frac{12}{l_b} & -\frac{6}{l_b} \\ \frac{6}{l_b} & 2 & -\frac{6}{l_b} & 4 \end{bmatrix}$$

The equation of motion of the smart structure is finally given by

$$M\ddot{q} + Kq = f_{ent} + f_{ctrl} = f_t \quad (7)$$

Where  $M, K, q, f_{ent}, f_{ctrl}, f^t$  is the global mass matrix, global stiffness matrix of the smart beam, the vector of displacements and slopes, and the external force applied to the beam, the controlling force from the actuator and the total force vector respectively.

## 2.2 Beam with Piezoelectric at Different Positions

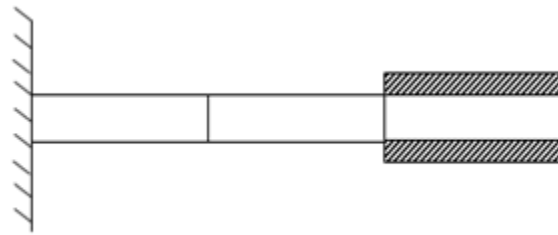


Fig1. Piezoelectric placed at the free end

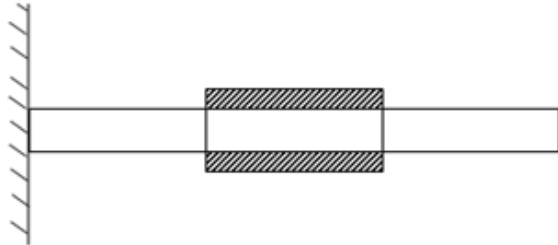


Fig2. Piezoelectric placed at the middle

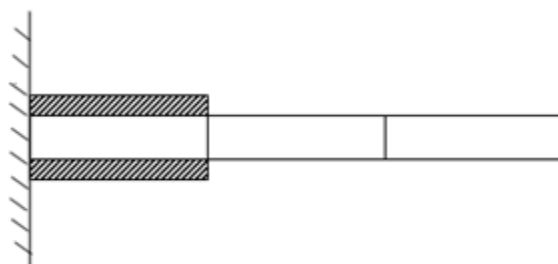


Fig3. Piezoelectric placed at the fixed end

## 2.3 Sensor Equation

The total charge  $Q(t)$  developed on the sensor surface is the spatial summation of all the point charges developed on the sensor layer. Thus, the expression for the current generated is obtained as

$$i(t) = \frac{dQ(t)}{dt} = \frac{d}{dt} \int_A e_{31} e_x dA = z e_{31} b \int_0^{l_p} n_1^T \dot{q} dx, \quad (8)$$

where  $z = \frac{t_b}{2} + t_a$

This current is converted into the open circuit sensor voltage  $V^s$  using a signal-conditioning device with the gain  $G_c$ . The sensor output voltage is obtained as

$$V^s(t) = G_c e_{31} z b \int_0^{l_p} n_1^T \dot{q} dx, \quad (9)$$

This Where  $d_{31}$  is the piezoelectric constant,  $e_{31}$  is the piezoelectric stress / charge constant,  $E_p$  is the young's modulus and  $\epsilon_x$  is the strain that is produced.

## 2.4 Actuator Equation

The actuator strain is derived from the converse piezoelectric equation. The strain developed  $\epsilon_a$  on the actuator layer is given by

$$e_a = d_{31}E_f \tag{10}$$

Where,  $d_{31}$  and  $E_f$  are the piezo strain constant and the electric field respectively. When the input to the piezoelectric actuator  $V^a(t)$  is applied in the thickness direction  $t_a$ , the electric field,  $E_f$  which is the voltage applied  $V^a(t)$  divided by the thickness of the actuator  $t_a$  and the stress,  $S_a$  which is the actuator strain multiplied by the young's modulus  $E_p$  of the piezo actuator layer are given by

$$E_f = \frac{V^a(t)}{t_a}$$

Finally, the control force applied by the actuator is obtained as

$$f_{ctrl} = E_p d_{31} b \bar{z} \int_0^{l_p} n_2 dx V^a(t) \tag{11}$$

Where  $\bar{z}$  is the distance between the neutral axis of the beam and the piezoelectric layer or can be expressed as a scalar vector product as

$$f_{ctrl} = hV^a(t) = hu(t) \tag{12}$$

where  $n_2^T$  is the first spatial derivative of the shape function of the flexible beam,  $h^T$  is a constant vector which depends on the type of actuator and its location on the beam, given by  $h = \begin{bmatrix} E_p d_{31} b \bar{z} & 0 & E_p d_{31} b \bar{z} & 0 \end{bmatrix}$  and  $u(t)$  is nothing but the control input to the actuator, i.e.  $V^a(t)$  from the controller. If any external forces are acting on the beam, then the total force vector becomes

$$f_t = f_{ext} + f_{ctrl} \tag{13}$$

### 2.5 State space model of the smart cantilever beam

The following equation can be written in state space from as follows:

$$M^* \ddot{g} + C^* \dot{g} + K^* g = f_{ent}^* + f_{ctrl}^* = f_t^*$$

Let the states of the system be defined as

$$\dot{g} = x = \begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \\ \dot{x}_3 \\ \dot{x}_4 \end{bmatrix} = \begin{bmatrix} \dot{u} \\ \ddot{u} \\ \dot{x}_3 \\ \dot{x}_4 \end{bmatrix} \text{ and } \ddot{g} = \begin{bmatrix} \ddot{x}_3 \\ \ddot{x}_4 \end{bmatrix}$$

Now equation becomes

$$M^* \begin{bmatrix} \dot{x}_3 \\ \dot{x}_4 \end{bmatrix} + c^* \begin{bmatrix} \dot{x}_3 \\ \dot{x}_4 \end{bmatrix} + K^* \begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \end{bmatrix} = r_{ent}^* + r_{ctrl}^*$$

$$A = \begin{bmatrix} \dot{e} & 0 & 1 & \dot{u} \\ \dot{e} & M^{*-1}K^* & -M^{*-1}C^* & \dot{u} \end{bmatrix}$$

$$B = \begin{bmatrix} \dot{e} & 0 \\ \dot{e} & M^{*-1}T^T h \end{bmatrix}$$

$$C^T = \begin{bmatrix} \dot{e} & P^T T \dot{u} \\ \dot{e} & \end{bmatrix}$$

D=null matrix

$$E = \begin{bmatrix} \dot{e} & 0 \\ \dot{e} & M^{*-1}T^T r \end{bmatrix} \dot{u}(t)$$

## 3 CONTROL LAWS

The various control laws such as one control law, which is based on Proportional Output Feedback by assuming arbitrary value and one classical control law Proportional Integral Derivative (PID) based on state feedback and one control law which is based on Pole Placement by state feedback has been explained as

### 3.1 Control with POF controller

In the first case, the responses are taken by giving impulse input. The Proportional Output Feedback controller is designed by taking the arbitrary value of gain. Output Feedback control provides a more consequential design. The responses are also plotted by changing the position of sensor and actuator on the beam i.e. free end, middle end and fixed end.

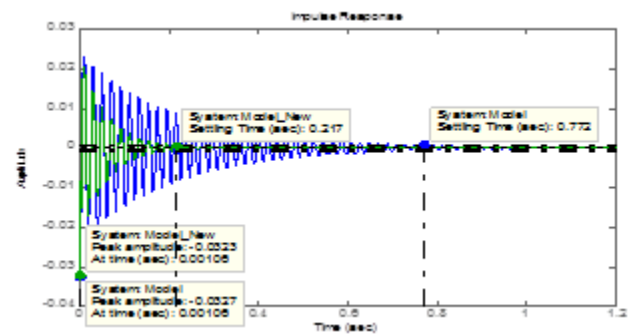


Fig. 4 Tip displacement of cantilever beam when piezoelectric patch placed on free end

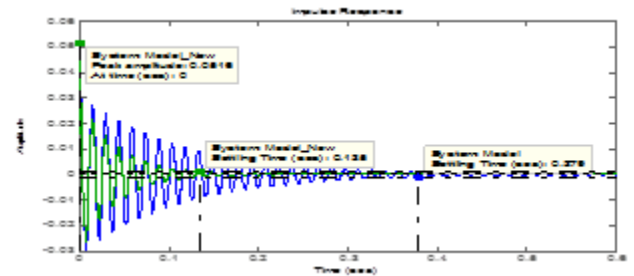


Fig. 5 Tip displacement of cantilever beam when piezoelectric patch placed on middle end

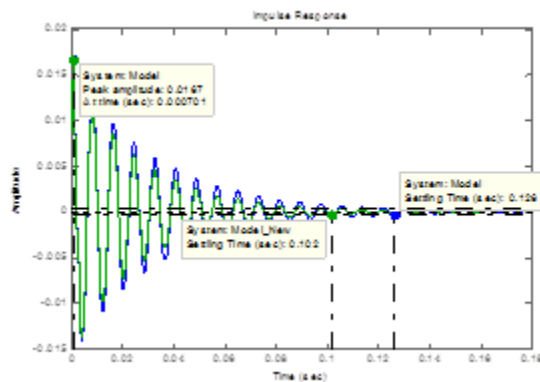


Fig. 6 Tip displacement of cantilever beam when piezoelectric patch placed on fixed end

### 3.2 Control with PID controller

In a PID controller the control action  $s$  generated as a sum of three terms. It is given by

$$G_1(s) = k_p + \frac{k_i}{s} + k_d s$$

$K_p$  = Proportional gain

$K_i$  = Integral gain

$K_d$  = Derivative gain

We use

$$k_d = 10, k_p = 100, k_i = 40$$

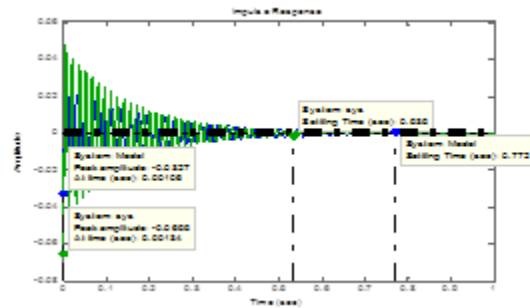


Fig. 7 Tip displacement of cantilever beam with and without PID controller when piezoelectric patch placed on free end

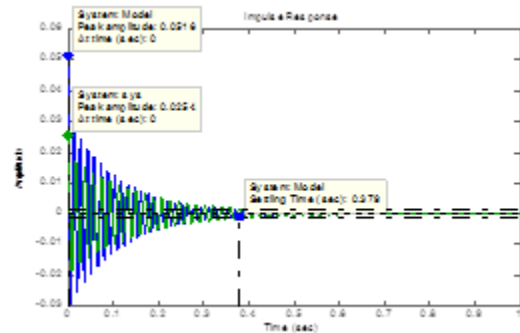


Fig. 8 Tip displacement of cantilever beam with and without PID controller when piezoelectric patch placed on middle end

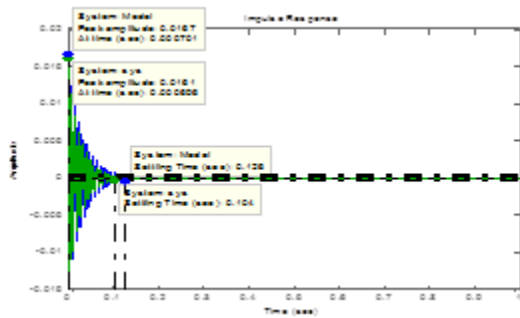


Fig. 9 Tip displacement of cantilever beam with and without PID controller when piezoelectric patch placed on fixed

end

### 3.3 Control with Pole placement technique

Pole Placement Technique is used to for the required control in which we can control according to the desired Eigen vectors and frequency. The present design technique begins with a determination of the desired closed-loop poles based on the transient-response and/or frequency-response requirements such as Eigen vectors, damping ratio.

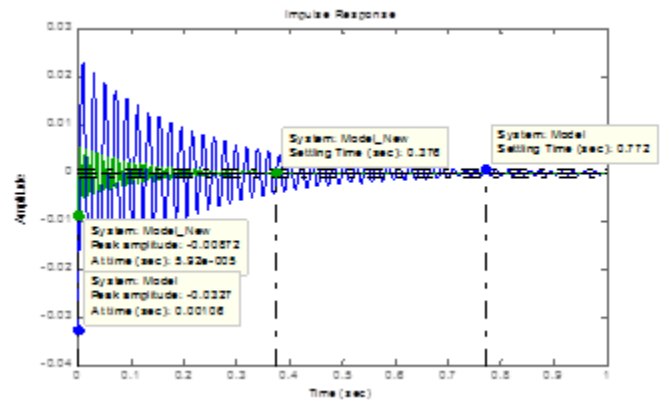


Fig. 10 Tip displacement of cantilever beam with and without Pole Placement technique when piezoelectric patch placed on free end

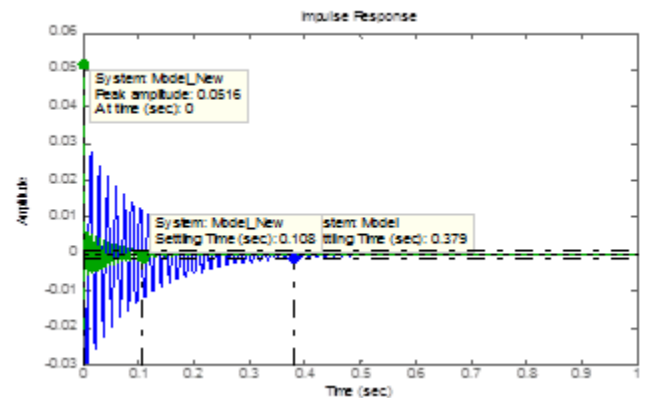


Fig. 11 Tip displacement of cantilever beam with and without Pole Placement technique when piezoelectric patch placed on middle end

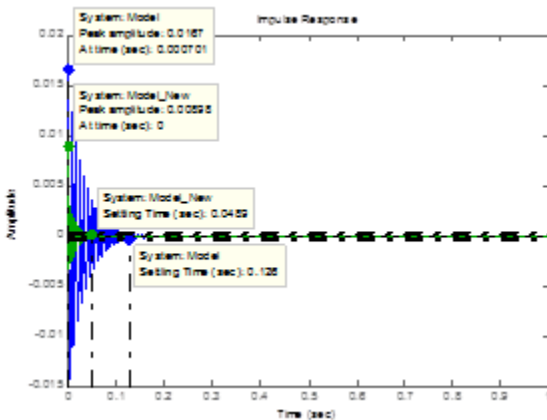


Fig. 12 Tip displacement of cantilever beam with and without Pole Placement technique when piezoelectric material on fixed end position

#### 4 RESULTS AND CONCLUSIONS

Present work deals with the mathematical formulation and the computational model for the active vibration control of a beam with piezoelectric smart structure. A general scheme of analysing and designing piezoelectric smart structures with control laws is successfully developed in this study. It has been observed that without control the transient response is predominant and with control laws, sufficient vibrations attenuation can be achieved. Numerical simulation showed that modeling a smart structure by including the sensor / actuator mass and stiffness and by varying its location on the beam from the free end to the fixed end introduced a considerable change in the system's structural vibration characteristics. From the responses of the various locations of sensor/actuator on beam, it has been observed that best performance of control is obtained, when the piezoelectric element is placed at fixed end position.

Different type of controller	Free end		Middle end		Fixed end	
	Settling time(in sec.)	peak response (in mm)	Settling time (in sec.)	Peak response	Settling time (in sec.)	Peak response
Uncontrolled loop	0.772	-	0.379	0.0516	0.126	0.0167
POF Controlled loop	0.217	0.0323	0.135	0.0516	0.102	0.0167
PID Controlled loop	0.535	0.0655	0.379	0.0254	0.104	0.0161
Pole Placement Controlled loop	0.378	0.0872	0.108	0.0516	0.0489	0.00898

Table 1 .Active Vibration control for different types of controller

#### REFERENCES

[1] Baillargeon, B. P., & Vel, S. S., "Active vibration suppression of sandwich beams using shear actuators: experiments and numerical simulations", *Journal of Intelligent Material Systems and Structures*, 16, 517-530, 2005.  
 [2] Baz and S. Poh , "Performance of an active control system with piezoelectric actuators", *Journal of Sound and Vibration*, 126:327-343, 1988.  
 [3] Benjeddou, A., Trindade, M.A., and Ohayon, R, "New shear actuated smart structure beam finite element", *AIAA Journal* 37(3): 378-383, 1999.  
 [4] Benjeddou, A. and Deii, J-F, "Piezoelectric transverse shear actuation and sensing of plates, Part 1: A three-dimensional mixed state space formulation",

*Journal of Intelligent Material Systems and Structures* 12 : 435-449, 2001a.  
 [5] Benjeddou, A., and Deii, J-F, "Piezoelectric transverse shear actuation and sensing of plates, Part 2: Application and analysis", *Journal of Intelligent Material* 2001b.  
 [6] Brij N Agrawal and Kirk E Treanor, "Shape control of a beam using piezoelectric actuators. *Smart Material. Structure*". 8, 729-740, 1999.  
 [7] Chori, S. B., Park, S. B., & Fukuda, T, "A proof of concept investigation on Active vibration control of hybrid structures". *Mechatronics*, 8, 673-689, (1998).  
 [8] Kapil Narwal and Deepak Chhabra, "Analysis of simple supported plate for active vibration control with piezoelectric sensors and actuators", *IOSR Journal of Mechanical and Civil Engineering*, Volume 1, Issue 1, 2278-1684, PP 26-39  
 [9] Kim V. V., Varadan, V. K. & Bao, X. Q., "Finite element modeling of a smart cantilever plate and comparison with experiments". *Smart Materials and Structures*, 5, 165-170, 1996.  
 [10] M. Yaqoob Yasin, Nazeer Ahmad, "Finite element analysis of actively controlled smart plate with patched actuators and sensors". *Latin American journal of solid and structure* 7, 227 - 247, 2010.  
 [11] Manning, W. J., Plummer, A. R., & Levesley, "M. C. Vibration control of a Flexible beam with integrated actuators and sensors", *Smart Materials and Structures*, 9, 932-939, 2000.  
 [12] N.S. Viliiani1, S.M.R. Khalili, "Buckling Analysis of FG Plate with Smart Sensor/ Actuator". *Journal of Solid Mechanics Vol. 1, No. 3, pp.201-212, 2009.*  
 [13] Raja, S., Prathap G., & Sihna, P. K. "Active vibration control of composite Sandwich beams with piezoelectric extension-bending and shear actuators", 2002.  
 [14] Singh, S. P., Pruthi, H. S., & Agarwal, V. P., "Efficient modal control strategies for active control of vibrations", *Journal of Sound and Vibration*, 262, 563-575, 2003.  
 [15] Suleman, "Wind Tunnel Aero elastic Response of Piezoelectric and Aileron Controlled 3-D Wing". *Can Smart Workshop Smart Materials and Structures*, Proceedings, Sep. 1998.  
 [16] Sun, C.T., and Zhang, X.D., "Use of thickness-shear mode in adaptive sandwich structures", *Smart Materials and Structures* 4: 202-206, 1995.  
 [17] T. C. Manjunath, B. Bandyopadhyay, "Control of vibration in smart structure using fast output sampling feedback technique", *World Academy of Science, Engineering and Technology* 34, 2007.  
 [18] Xu, S. X., & Koko, T. S, "Finite element analysis and design of actively Controlled piezoelectric smart structures", *Finite Elements in Analysis and Design*, 40, 241-262, 2004.