Comparasion of Energy Detection in Cognitive Radio over different fading channels.

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***Abstract*-With the advance of wireless communications, the problem of bandwidth scarcity has become more prominent. Cognitive radio technology has come out as a way to solve this problem by allowing the unlicensed users to use the licensed bands opportunistically. To sense the existence of licensed users, many spectrum sensing techniques have been devised. In this paper, energy detection and cyclic prefix is used for spectrum sensing.The comparison of ROC curves has been done for various wireless fading channels using squaring and cubing operation,the improvement has gone as high as up to 0.6 times for AWGN channel and 0.4 times for Rayleigh channel as we go from squaring to cubing operation in an energy detector. Closed form expressions for Probability of detection for AWGN and Rayleigh channels are described.Nakagami fading channel shows worst results .**

Keywords: Spectrum Sensing, Cognitive Radio, Probability of detection, Cooperative Detection.

I. INTRODUCTION

Today, by unprecedented growth of wireless applications, the problem of spectrum scarce is becoming more and more apparent. Most of the spectrum has been allocated to specific users, while other spectrum bands that haven't been assigned are overcrowded because of overuse. However, most of the allocated spectrum is idled in some times and locations.The Federal Communication Commission (FCC) research report [1] reveals that, seventy percent of the allocated spectrum is underutilized. So we need a technique to deal with the problem of spectrum underutilization, which makes the birth of cognitive radio. Cognitive radio [2][3]can sense external radio environment and learn from past experiences. It can access to unused spectrum band dynamically without affecting the primary users, in such a way to improve the spectrum efficiency. Sensing external radio environment quickly and accurately plays a key role in cognitive radio. Spectrum sensing includes the detection of primary users and secondary users in other cognitive networks in the same region, but most of papers on spectrum sensing only consider the detection of primary users. In this paper, we consider the cyclic prefix, a special feature embedded in the OFDM (Orthogonal Frequency Division Multiplexing) signals; is used to detect the presence of primary user's signal and is considered to be better than energy detection and matched filter detection as it performs well even in the fading channels. In addition, cooperative detection is used among the secondary users to improve the performance of spectrum sensing. Energy detector based approach, also known as radiometry or periodogram , is one of the popular methods for spectrum sensing as it is of non-coherent type and has low implementation complexity. In addition, it is more generic as receivers do not require any prior knowledge about the primary user's signal [4]. In this method, the received signal's energy is measured and compared against a pre-defined threshold to determine the presence or absence of primary user's signal. Moreover, energy detector is widely used in ultra wideband (UWB) communications to borrow an idle channel from licensed user. Detection probability (P_d), False alarm probability (P_f) and missed detection probability (P_m) are the key measurement metrics that are used to analyze the performance of an energy detector. The performance of an energy detector is illustrated by the receiver operating characteristics (ROC) curve which is a plot of Pd versus Pf or Pm versus Pf [5].
This paper is organized as follows: Section 2 describes the OFDM (Orthogonal Frequency Division Multiplexing) System Model. Section 3 and 4 describe the expressions for probability of detection for AWGN (Additive White Gaussian Noise) and Rayleigh channels respectively. Simulation Results for Cyclic Prefix and energy detection Based Spectrum Sensing over AWGN (Additive White Gaussian Noise) and Rayleigh channels and improvement using cooperative detection are presented in section 5 followed by conclusions in section 6.

II. OFDM SYSTEM MODEL

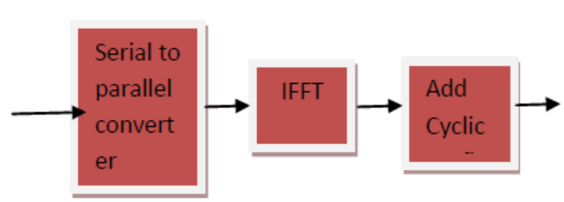

Fig. 1.Simplified Block Diagram of OFDM Transmitter

Consider a block of data symbols mapped on to the subcarriers is represented by:

$$\{s(0), s(1), s(2) \dots, s(T_d - 1)\}$$

The IFFT (Inverse Fast Fourier Transform) operation converts these frequency domain signals into timedomain signals and the time domain signals are represented by:

$$\{x(0), x(1), x(2) \dots, x(T_d - 1)\}$$

where IFFT block size is assumed to be Td .
Last Tc symbols of each block are added to the beginning of each block as cyclic prefix and the transmitted signal becomes:

$$\{x(-T_c), \dots, x(-1), x(0), x(1), \dots, x(T_d - T_c), \dots, x(T_d - 1)\}$$

where the block of symbols $\{x(-T_c), \dots, x(-1)\}$ is an exact copy of $\{x(T_d - T_c), \dots, x(T_d - 1)\}$ i.e. $x(t) = x(T_d + t)$ i.e. where $t \in [-T_c, -1]$.
Now, the relation between the signals before and after the IFFT block can be expressed by the following expression [10]:

$$x(t) = \frac{1}{\sqrt{T_d}} \sum_{n=0}^{T_d - 1} s(n) e^{\frac{j2\pi(t-T_c)n}{T_d}} \quad ,t = 0,1, \dots T_d - 1 \qquad (1)$$

A transmitted OFDM frame may contain several such blocks. Let denote the symbols of the transmitted OFDM frame. Detection is based on two hypotheses [5]:

$$H_0: \qquad r(t) = n(t) \qquad (2)$$

and

$$H_1: \qquad r(t) = y(t) + n(t) \qquad (3)$$

where r(t) is the received signal, n(t) is the additive white Gaussian noise [6].H_0 represents the hypothesis when the signal is absent and only noise is present. H_1 represents the hypothesis when both signal and noise are present. Let χ is a measure of correlation between two samples distance Td apart [10].

$$\chi = \sum_{t=1}^{W} \frac{r(t) r^*(t+T_d)}{E[|r(t)|^2]} \qquad (4)$$

For CP (Cyclic prefix) OFDM signal, the statistic χ under the above two hypothesis can be expressed as [10]:

$$H_0: \chi = \sum_{t=1}^{W} \frac{n(t) n^*(t+T_d)}{E[|n(t)|^2]} \qquad (5)$$

And

$$H_1: \chi = \sum_{t=1}^{W} \frac{(y(t)+n(t))(y^*(t+T_d)+n^*(t+T_d))}{E[|y(t)+n(t)|^2]} \qquad (6)$$

III. PROBABILITY OF DETECTION

A)Probability of Detection in Cyclic Prefix for AWGN Channel

Probability of Detection for cyclic prefix based spectrum sensing method over AWGN channel can be expressed as [10],[5]:

$$P_D = \frac{1}{2} erfc\left(\frac{T' - \mu_1}{\sqrt{W}}\right) \qquad (7)$$

where P_d is the probability of detection for AWGN Channel, μ_1 is the mean under Hypothesis H_1 , is the observation window size or number of samples, T' is the threshold and is given by :

$$T' = \sqrt{\frac{W}{2}} erfc^{-1}(2P_F) \qquad (8)$$

where P_f is the false alarm probability for AWGN channel. Under Hypothesis H_1 , Mean μ_1 can be calculated

$$\mu_1 = E[\ \chi | H_1] \qquad (9)$$

$$= E\left[\sum_{t=1}^{W} \frac{(y(t)+n(t))(y^*(t+T_d)+n^*(t+T_d))}{E[|y(t)+n(t)|^2]}\right] \qquad (10)$$

$$= \sum_{t=1}^{W} \frac{E[(y(t)(y^*(t+T_d))]}{E[|y(t)+n(t)|^2]} \qquad (11)$$

Since y(t)=$y^*(t+T_d)$ only when y(t) falls into the cyclic prefix period, equation (11) can be expressed as:

$$\mu_1 = \sum_{t=1}^{W} \frac{P(y(t) \in CP) E[|y(t)|^2]}{\sigma_y^2 + \sigma_n^2} \qquad (12)$$

$$= \sum_{t=1}^{W} \frac{T_c}{T_d + T_c} \frac{\sigma_y^2}{\sigma_y^2 + \sigma_n^2} \qquad (13)$$

$$= \sum_{t=1}^{W} K \frac{\sigma_y^2}{\sigma_y^2 + \sigma_n^2} \qquad (14)$$

$$= \frac{KW\gamma}{1+\gamma} \qquad (15)$$

where $K = \frac{T_c}{T_d + T_c}$ and

$$\gamma = \frac{\sigma_y^2}{\sigma_n^2}$$

is signal to noise ratio.

B) Probability of Detection in Cyclic Prefix for Rayleigh Channel

Probability of Detection for cyclic prefix based spectrum sensing method over rayleigh channel can be expressed as [11]:

$$P_{D,Ray} = \int_0^{\infty} P_D \, f(\gamma) d\gamma \qquad (16)$$

where Pd is the probability of detection for AWGN channel and f(γ) is the probability density function for Rayleigh channel [12, Eq.(4-44)].

$$f(\gamma) = \frac{1}{\bar{\gamma}} \exp\left(\frac{-\gamma}{\bar{\gamma}}\right) \qquad (17)$$

Using (7), (15) and (17), equation (16) can be rewritten as:

$$P_{D,Ray} = \frac{1}{2\bar{\gamma}} \int_0^{\infty} erfc\left(\frac{T' - \frac{KW\gamma}{1+\gamma}}{\sqrt{W}}\right) \exp\left(\frac{-\gamma}{\bar{\gamma}}\right) d\gamma \qquad (18)$$

Now considering the following notations:

$$\gamma = \frac{t}{1-t}, \; d\gamma = \frac{dt}{(1-t)^2}, \; \frac{T'}{\sqrt{W}} = a, \; K\sqrt{W} = b$$

Using these notations, equation (18) can be rewritten as:

$$P_{D,Ray} = \frac{1}{2\bar{\gamma}} \int_0^1 erfc(-bt) \exp\left(\frac{-t}{\bar{\gamma}(1-t)}\right) \frac{dt}{(1-t)^2} \qquad (19)$$

Taking the assumption t<<1 and applying Binomial approximation, we have:

$$P_{D,Ray} = \frac{1}{2\bar{\gamma}} \int_0^1 erfc(a-bt) \exp\left(\frac{-t}{\bar{\gamma}(1-t)}\right) (1 + 2t) dt \qquad (20)$$

or,

$$P_{D,Ray} = \frac{1}{2\bar{\gamma}} \int_0^1 erfc(a-bt) \exp\left(\frac{1}{\bar{\gamma}} - \frac{1}{\bar{\gamma}(1-t)}\right) (1 + 2t) dt \qquad (21)$$

or,

$$P_{D,Ray} = \frac{1}{2\bar{\gamma}} \int_0^1 erfc(a-bt) \exp\left(\frac{1}{\bar{\gamma}} - \frac{1}{\bar{\gamma}}(1+t)\right) (1 + 2t) dt \qquad (22)$$

or,

$$P_{D,Ray} = \frac{1}{2\bar{\gamma}} \int_0^1 erfc(a-bt) \exp\left(\frac{-t}{\bar{\gamma}}\right) (1 + 2t) \, dt \qquad (23)$$

Solving it using mathematica, we get the approximated expression for probability of detection for Rayleigh channel as:

$$P_{D,Ray} \cong \frac{1}{2\bar{\gamma}b^2} \left[e^{-\frac{a}{b\bar{\gamma}}} \left[\left(-1 + 2ab\bar{\gamma} + b^2\bar{\gamma}(1 + 2\bar{\gamma})\right) e^{\frac{1}{4b^2\bar{\gamma}^2}} erf\left(a - \frac{1}{2b\bar{\gamma}}\right) + \frac{1}{\sqrt{\pi}} \left[e^{-\frac{1+a^2\bar{\gamma}+b^2\bar{\gamma}}{\bar{\gamma}}} \left[-\left(-1 + 2ab\bar{\gamma} + b^2\bar{\gamma}(1 + 2\bar{\gamma})\right) e^{a^2+b^2+\frac{1}{4b^2\bar{\gamma}^2}+\frac{1}{\bar{\gamma}}} \sqrt{\pi} erf\left(a - b - \frac{1}{2b\bar{\gamma}}\right) + b\bar{\gamma} e^{\frac{a}{b\bar{\gamma}}} \left[-2e^{2ab} + 2e^{b^2+\frac{1}{\bar{\gamma}}} + b(1 + 2\bar{\gamma}) e^{a^2+b^2+\frac{1}{\bar{\gamma}}} \sqrt{\pi} erfc(a) - b(3 + 2\bar{\gamma}) e^{a^2+b^2} \sqrt{\pi} erfc(a - b) \right] \right] \right] \right] \right] \qquad (24)$$

IV.PROBABILITY OF DETECTION AND FALSE ALARM IN ENERGY DETECTION

A) In AWGN Channel

Probability of detection Pd and false alarm Pf can be evaluated respectively by [11]:

$$P_d = P(Y' > \Lambda | H_1)$$

$$P_f = P(Y' > \Lambda | H_0)$$

where λ is the decision threshold. Also, can be written in terms of probability density function as

$$P_f = \int_{\Lambda}^{\infty} f_{Y'}(y)\, dy$$

$$P_f = \frac{1}{2^d \Gamma(d)} \int_{\Lambda}^{\infty} y^{d-1} e^{-\left(\frac{y}{2}\right)} dy$$

Dividing and multiplying the R.H.S. of above equation by 2^{d-1}, we get

$$P_f = \frac{1}{2\Gamma(d)} \int_{\Lambda}^{\infty} \left(\frac{y}{2}\right)^{d-1} e^{-\left(\frac{y}{2}\right)} dy$$

Substituting $\frac{y}{2} = T, \frac{d_y}{2} = dt$ and changing the limits of integration to , we get

$$P_f = \frac{1}{\Gamma(d)} \int_{\Lambda/2}^{\infty} (t)^{d-1} e^{-(t)}\, dt$$

$$P_f = \frac{\Gamma(d, \Lambda/2)}{\Gamma(d)}$$

where $\Gamma(.)$ is the incomplete gamma function [13]. Now, Probability of detection can be written by making use of the cumulative distribution function

$$P_d = 1 - F_{Y'}(\Lambda)$$

The cumulative distribution function (CDF) of can be obtained (for an even number of degrees of freedom which is in our case) as

$$F_{Y'}(y) = 1 - Q_d(\sqrt{\lambda}, \sqrt{y})$$

$$P_d = Q_d(\sqrt{\lambda}, \sqrt{\Lambda})$$

$$P_d = Q_d(\sqrt{2\gamma}, \sqrt{\Lambda})$$

B) In Rayleigh Channel

Probability density function for Rayleigh channel is

$$f(\gamma) = \frac{1}{\bar{\gamma}} exp\left(\frac{-\gamma}{\bar{\gamma}}\right) \qquad \gamma \geq 0$$

The Probability of detection for Rayleigh Channels is obtained by averaging their probability density function over probability of detection for AWGN Channel

$$P_{d,R} = \int_0^{\infty} P_d\, f(\gamma) d\gamma$$

where Pd,r is the probability of detection for Rayleigh channel.

$$P_{d,R} = \frac{1}{\bar{\gamma}} \int_0^{\infty} Q_d\left(\sqrt{2\gamma}, \sqrt{\Lambda}\right) exp\left(\frac{-\gamma}{\bar{\gamma}}\right) d\gamma$$

Now, substituting $\sqrt{\gamma} = x\, , \gamma = x^2, d\gamma = 2xdx$

$$P_{d,R} = \frac{2}{\bar{\gamma}} \int_0^{\infty} x . Q_d\left(\sqrt{2}x, \sqrt{\Lambda}\right) exp\left(\frac{-x^2}{\bar{\gamma}}\right) dx$$

Probability of detection for Rayleigh channel can be expressed as

$$P_{d,R} = e^{(-\Lambda/2)} \sum_{n=0}^{d-2} \frac{1}{n!}\left(\frac{\Lambda}{2}\right)^n + \left(\frac{1+\gamma}{\gamma}\right)^{d-1} \left[\exp\left(-\frac{\Lambda}{2(1+\gamma)}\right) - \exp\left(-\frac{\Lambda}{2}\right) \sum_{n=0}^{d-2} \frac{1}{n!}\left(\frac{\Lambda\gamma}{2(1+\gamma)}\right)^n\right]$$

C) Probability of detection in Nakagami channel-m fading

$P_{d,1}$ for m $\geq$ 1/2 ; (η=1+$\frac{m}{\gamma}$)

$P_{d,1}$= 1-$e^{\frac{-\lambda}{2}}$ $(\frac{m}{\gamma\eta})^m \sum_u^{\infty}(\frac{\lambda}{2})$^$n$ 1/n! F_1 (m;n+1;$\frac{\lambda}{2\eta}$)

Average detection probability over Nakagami-*m* fading with i number of EGC branches ($P_{d,i}$) for m$\geq 1/2$

$$\overline{P}_{d,2} = 1-\sqrt{\pi}e^{-\frac{\lambda}{2}}\sum_{n=u}^{\infty}\sum_{k=0}^{\infty}\left(\frac{\lambda}{2}\right)^{n}\left(\frac{2m}{\overline{\gamma}+2m}\right)^{2m+k} \times \frac{4\,\psi_2(m,n,k)}{2^{4m+k}\,k!}\,{}_1F_1\left(2m+k;n+1;\frac{\lambda\overline{\gamma}}{2\left(\overline{\gamma}+2m\right)}\right)$$

$$\overline{P}_{d,3} = 1-\sqrt{\pi}e^{-\frac{\lambda}{2}}\sum_{n=u}^{\infty}\sum_{p=0}^{\infty}\sum_{k=0}^{\infty}\left(\frac{\lambda}{2}\right)^{n}\left(\frac{3m}{\overline{\gamma}+3m}\right)^{3m+p+k} \times \frac{8\,\psi_3(m,n,p,k)}{2^{4m+p+k}\,k!}\,{}_1F_1\left(3m+p+k;n+1;\frac{\lambda\overline{\gamma}}{2\left(\overline{\gamma}+3m\right)}\right)$$

V.SIMULATION RESULTS

The performance of energy detector is analysed using ROC (Receiver operating characteristics) curves for fading channels. Monte-Carlo method is used for simulation. It can be seen in the following figures that with increase in SNR (Signal to Noise Ratio), the performance of energy detection improves. FIGURE 2 and FIGURE 4 illustrates the ROC curves using squaring operation for AWGN and Rayleigh channel respectively. FIGURE 3 and FIGURE 5 depicts improvement in the performance of energy detector using cubing operation over AWGN and Rayleigh channel respectively. We assume time-bandwidth product=5.

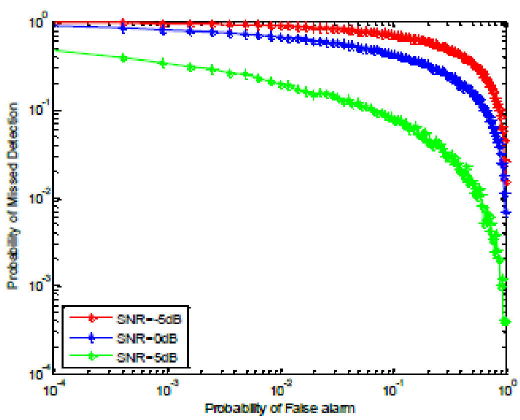

FIGURE 2: Complementary ROC Curves for AWGN using Squaring operation

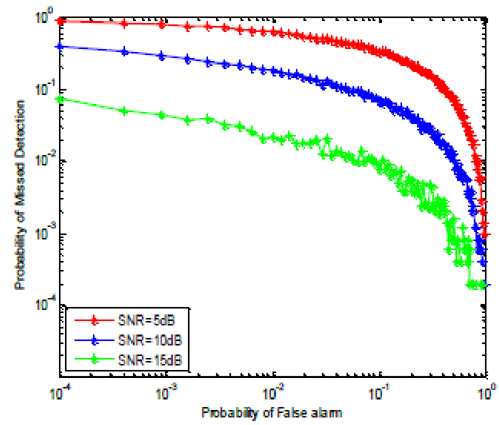

FIGURE 3 : Complementary ROC for Rayleigh using Squaring operation.

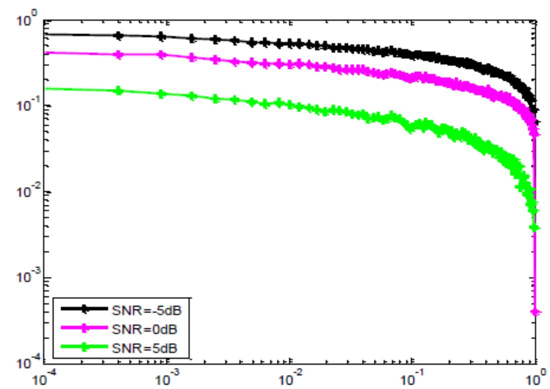

FIGURE 4:ROC curves using cubing operation in Energy Detection over AWGN channel

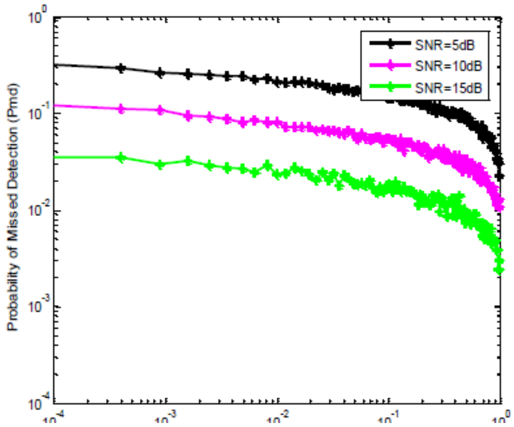

FIGURE 5:ROC curves for cubing operation of Energy Detection over Rayleigh channel

FIGURE. 5. Comparison of plots for Probability of detection versus signal to noise ratio (SNR) over AWGN and Rayleigh Channel.

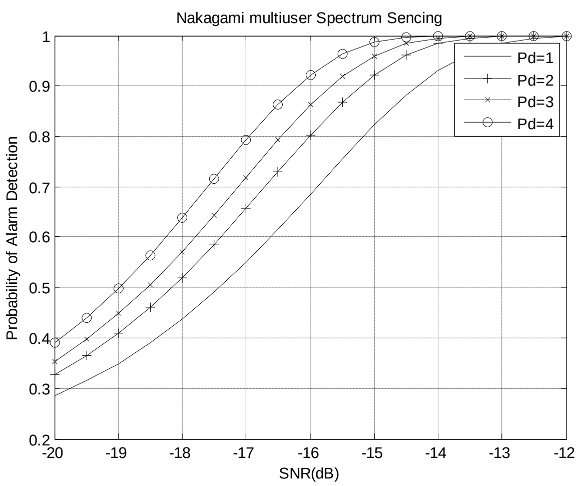

FIGURE 6:Energy detection over nakagami fading channel

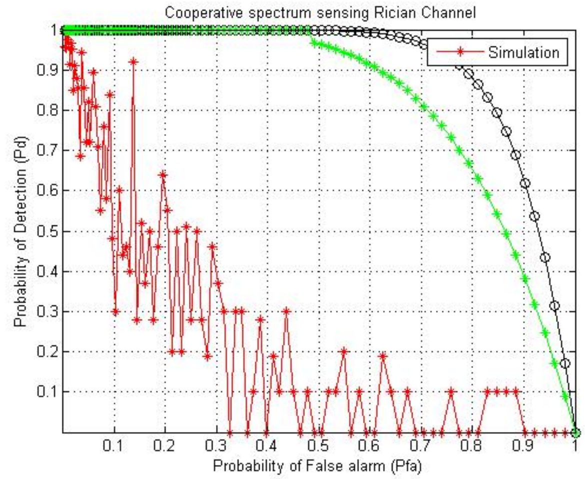

FIGURE7:ROC curves of Energuy Detection over rician channel

IV. CONCLUSION:

In the present work energy detection based spectrum sensing is analysed over different wireless fading channels.Closed form expressions for probability of detection for AWGN,nakagami and Rayleigh channels are described. Using ROC (Receiver Operating Characteristics) Curve, it has been shown that Nakagami shows worst results. The comparison of ROC curves has been done for various wireless fading channels using squaring and cubing operation,the improvement has gone as high as up to 0.6 times for AWGN channel and 0.4 times for Rayleigh channel as we go from squaring to cubing operation in an energy detector

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