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COMMON RANDOM FIXED POINT THEOREMS IN SYMMETRIC SPACES

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Abstract.

In this paper we obtain common random fixed point theorems for weakly compatible random operators under generalized contractive condition in symmetric space. In this paper we generalized the result of Beg and Abbas [4].

Keywords : Symmetric space, weakly compatible, random operators.

Mathematical subject classification (2000) : 47H10, 54H25.

1. Introduction :

In recent years, the study of random fixed points have attracted much attention, some of the recent literatures in random fixed point may be noted in [1, 2, 3, 4, 5, 7, 9]. In metric space some theorems can be proved without using some of the defining properties of metric. Hicks [6] established some common fixed point theorems in symmetric space. Recently Beg and Abbas [4] prove some random fixed point theorems for weakly compatible random operator under generalized contractive condition in symmetric space.

2 Preliminaries :

Throughout this paper, (Ω, Σ) denotes a measurable space (Σ - sigma algebra).

A symmetric on a set X is a non - negative real valued function d on $X \times X$ such that for all x, $y \in X$ we have

(a) d (x, y) = 0 if and only if x = y and

(b) d (x, y) = d(y, x).

Let d be a symmetric on a set X. For $\varepsilon > 0$ and $x \in X$, $B(x, \varepsilon)$ denotes the spherical ball centred at x with radius ε, defined as the set $\{y \in X : d(x, y) < \varepsilon.\}$

A topology t (d) on X is given by $U \in t(d)$ if and only if for each $x \in U$, $B(x, \varepsilon) \subset U$ for some $\varepsilon > 0$.

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Note that $\lim_{n\to\infty} d(x_n, x) = 0$ iff $x_n \to x$ in the topology t (d).

Let F be a subset of X. A mapping $\xi : \Omega \to X$ is measurable if $\xi^{-1}(U) \in \Sigma$ for each open subset U of X. The mapping $T : \Omega \times F \to F$ is a random map if and only if for each fixed $x \in F$, the mapping $T(., x): \Omega \to F$ is measurable.

The mapping T is continuous if for each $\omega \in \Omega$ the mapping $T(\omega, .) : F \to X$ is continuous. A measurable mapping $\xi: \Omega \to X$ is a random fixed point of random operator $T: \Omega \times F \to X$ if and only if $T(\omega, \xi(\omega)) = \xi(\omega)$ for each $\omega \in \Omega$. We denote the set of random fixed points of a random map T by RF (T) and the set of all measurable mapping from Ω into a symmetric space by $M(\Omega, X)$.

Let $\phi : R^+ \to R^+$ be a function satisfying the condition $0 < \phi(t) < t$, for each $t > 0$.

Definition 2.1. Random operators $S, T : \Omega \times X \to X$ are said to be commutative if $S(\omega, .)$ and $T(\omega, .)$ are commutative for each $\omega \in \Omega$.

Definition 2.2. [5] Let X be a Polish space, that is separable complete metric space. Mapping $f,g: X \to X$ are compatible if $\lim_{n\to\infty} d(fgx_n, gfx_n) = 0$, provided that $\lim_{n\to\infty} f(x_n)$ and $\lim_{n\to\infty} g(x_n)$ exists in X and $\lim_{n\to\infty} f(x_n) = \lim_{n\to\infty} g(x_n)$. Random operators $S, T : \Omega \times X \to X$ are compatible if $S(\omega, .)$ and $T(\omega, .)$ are compatible for each $\omega \in \Omega$.

Definition 2.3. Let X be a Polish space. Random operators $S,T : \Omega \times X \to X$ are weakly compatible if $T(\omega, \xi(\omega)) = S(\omega, \xi(\omega))$ for some $\xi \in M(\Omega, X)$ then $T(\omega, S(\omega, \xi(\omega))) = S(\omega, T(\omega, \xi(\omega)))$ for every $\omega \in \Omega$.

Definition 2.4.[8]. Let $\{x_n\}$ and $\{y_n\}$ be two sequences in a symmetric space (X,d) and x, $y \in X$. The space X is said to satisfy the following axioms:

(W.1) $\lim_{n\to\infty} d(x_n, x) = \lim_{n\to\infty} d(x_n, y) = 0$ implies that $x = y$.

(W.2) $\lim_{n\to\infty} d(x_n, x) = \lim_{n\to\infty} d(x_n, y_n) = 0$ implies that $d(y_n, x) = 0$.

Definition 2.5. Let $\{x_n\}$ and $\{y_n\}$ be two sequences in a symmetric space (X, d) and x ∈ X. The space X is said to satisfy axioms (H_E) ; if $\lim_{n\to\infty} d(x_n, x) = \lim_{n\to\infty} d(y_n, x) = 0$ implies that $\lim_{n\to\infty} d(x_n, y_n) = 0$.

Definition 2.6. Let d be a symmetric function on X. Two random mappings S and T from $\Omega \times X \to X$ are said to satisfy property (I) if there exists a sequence $\{\xi_n\}$ in $M(\Omega, X)$ such that for some $\xi \in M(\Omega, X)$,

$\lim_{n\to\infty} d(T(\omega, \xi_n(\omega)), \xi(\omega)) = \lim_{n\to\infty} d(S(\omega, \xi_n(\omega)), \xi(\omega)) = 0$ for every $\omega \in \Omega$.

Theorem 3.1. Let (X, d) be a separable symmetric space that satisfies (W.1) and (H_E). Let $T, S : \Omega \times X \to CB(X)$ be two weakly compatible random multivalued operators satisfying the property (I). Moreover, for all x,y∈ X we have.

$$d(T(\omega, x), T(\omega, y)) \le \phi(\max\{d(s(\omega, x), S(\omega, y)), d(s(\omega, x), T(\omega, y)), d(S(\omega, y), T(\omega, y)),$$

$$\frac{1}{2}[d(S(\omega, x), T(\omega, y)) + d(S(\omega, y), T(\omega, x))]\})$$

for every $\omega \in \Omega$. If $T(\omega, X) \subset S(\omega, X)$ and one of $T(\omega, X)$ or $S(\omega, X)$ is a complete subspace of X for every $\omega \in \Omega$, then T and S have unique common random fixed point.

Proof. Since random multivalued operators T and S satisfy the property (I), so there exists a sequence $\{\xi_n\}$ in $M(\Omega, X)$ such that :

$\lim_{n\to\infty} d(T(\omega, \xi_n(\omega)) = \lim_{n\to\infty} d(S(\omega, \xi_n(\omega)), \xi(\omega)) = 0$ for every $\omega \in \Omega$, for some $\xi \in M(\Omega, X)$.

Therefore by property (H_E), we have

$\lim_{n\to\infty} d(T(\omega_1 \xi_n(\omega)), S(\omega, \xi_n(\omega))) = 0$ for every $\omega \in \Omega$.

Suppose $S(\omega, X)$ is a complete subspace of X for every $\omega \in \Omega$.

Let $\xi_1 : \Omega \to X$ be the limit of the sequence of measurable mappings $\{S(\omega, \xi_n(\omega))\}$ and $\{S(\omega, \xi_n(\omega))\} \in S(\omega, X)$ for every $\omega \in \Omega$ and n∈N. Now since X is separable, therefore $\xi_1 \in M(\Omega, X)$. Moreover $\xi_1(\omega) \in S(\omega, X)$ for every $\omega \in \Omega$. This allows obtaining the measurable mappings $\overline{\xi} : \Omega \to X$ such that $\xi(\omega) = S(\omega, \overline{\xi}(\omega))$.

Now we show that $T(\omega, \overline{\xi}(\omega)) = S(\omega, \overline{\xi}(\omega))$ for every $\omega \in \Omega$.

If not then for some $\omega \in \Omega$. Consider

$$d\left(T(\omega, \overline{\xi}(\omega)), T(\omega, \xi_n(\omega))\right) \le \varphi\Big(\max\Big\{d\left(S(\omega, \overline{\xi}(\omega)), S(\omega, \xi_n(\omega))\right),$$

$$d\left(S(\omega, \overline{\xi}(\omega)), T(\omega, \xi_n(\omega))\right), d\left(S(\omega, \xi_n(\omega)), T(\omega\xi_n(\omega))\right),$$

$$\frac{1}{2}\left[d\left(S(\omega, \overline{\xi}(\omega)), T(\omega, \xi_n(\omega))\right) + d\left(S(\omega, \xi_n(\omega)), T\left(\omega, \overline{\xi}(\omega)\right)\right)\right]\Big\}\Big)$$

$$< \max\Big\{d(\xi(\omega), S(\omega, \xi_n(\omega))), d(\xi(\omega), T(\omega, \xi_n(\omega))),$$

$$d\left(S(\omega, \xi_n(\omega)), T(\omega, \xi_n(\omega))\right), \frac{1}{2}\left[d\left(\xi(\omega), T(\omega, \xi_n(\omega))\right) + d\left(S(\omega, \xi_n(\omega)), T\left(\omega, \overline{\xi}(\omega)\right)\right)\right]\Big)$$

Taking $n \to \infty$ we have

$$d\left(T(\omega, \overline{\xi}(\omega)), T(\omega, \xi_n(\omega))\right) < \max\left(0, 0, 0, \frac{1}{2}\left[0 + d(\xi(\omega), T(\omega, \overline{\xi}(\omega)))\right]\right)$$

or

$$d\left(T(\omega, \overline{\xi}(\omega)), \xi(\omega)\right) < \frac{1}{2} d\left(T(\omega, \overline{\xi}(\omega)), \xi(\omega)\right)$$

which is a contradiction, so $T(\omega, \overline{\xi}(\omega)) = S(\omega, \overline{\xi}(\omega))$ for every $\omega \in \Omega$.

The weak compatibility of random mappings T and S implies that

$$T\left(\omega, S(\omega, \overline{\xi}(\omega))\right) = S(\omega, T(\omega, \overline{\xi}(\omega))),$$

Then $T(\omega, T(\omega, \overline{\xi}(\omega))) = T(\omega, S(\omega, \overline{\xi}(\omega))) = S(\omega, T(\omega, \overline{\xi}(\omega))) = S(\omega, S(\omega, \overline{\xi}(\omega)))$ for every $\omega \in \Omega$.

Let us show that $T(\omega, T(\omega, \overline{\xi}(\omega))) = T(\omega, \overline{\xi}(\omega))$ for each $\omega \in \Omega$. If not, then for some $\omega \in \Omega$, consider

$$d(T(\omega, \overline{\xi}(\omega)), T(\omega, \xi(\omega))) \le \varphi\left(\text{Max}\left\{d(S(\omega, \overline{\xi}(\omega)), S(\omega, \xi(\omega)))\right.\right.$$

$$d\left(S(\omega, \overline{\xi}(\omega)), T(\omega, \xi(\omega))\right), d(s(\omega, \xi(\omega)), T(\omega, \xi(\omega))),$$

$$\frac{1}{2}[(d(S(\omega,\overline{\xi}(\omega)), T(\omega,\xi(\omega))) + d(S(\omega,\xi(\omega)), T(\omega,\overline{\xi}(\omega)))]\})$$

$$\leq \varphi\Big(\max\{d(T(\omega,\overline{\xi}(\omega)), T(\omega,\xi(\omega))), d(T(\omega,\overline{\xi}(\omega)), T(\omega,\xi(\omega)))$$

$$d(T(\omega,\xi(\omega)), T(\omega,\xi(\omega))), \frac{1}{2}[(d(T(\omega,\overline{\xi}(\omega)), T(\omega,\xi(\omega)))$$

$$+ d(T(\omega,\xi(\omega)), T(\omega,\overline{\xi}(\omega)))]\}\Big)$$

$$\leq \varphi\Big(\max\{d(T(\omega,\overline{\xi}(\omega)), T(\omega,\xi(\omega))), 0, d(T(\omega,\overline{\xi}(\omega)), T(\omega,\xi(\omega)))\}\Big)$$

$$\leq \varphi\, d(T(\omega,\overline{\xi}(\omega)), T(\omega,\xi(\omega))) .$$

$$< d(T(\omega,\overline{\xi}(\omega)), T(\omega,\xi(\omega))).$$

ie. $d(T(\omega,\overline{\xi}(\omega)), T(\omega,\xi(\omega))) < d(T(\omega,\overline{\xi}(\omega)), T(\omega,\xi(\omega)))$.

which is a contradiction, so $T(\omega,\overline{\xi}(\omega))$ is a random fixed point of T. Now

$$T(\omega,\overline{\xi}(\omega)) = T(\omega, T(\omega,\overline{\xi}(\omega))) = S(\omega, T(\omega,\overline{\xi}(\omega))) \text{ for every } \omega\in\Omega.$$

Therefore $T(\omega,\overline{\xi}(\omega))$ is a common random fixed point of T and S. The proof is similar when $T(\omega,X)$ is supposed to be a complete subspace of X for every $\omega\in\Omega$, as $T(\omega,X)\subset S(\omega,X)$ for each $\omega\in\Omega$.

To prove the uniqueness of common random fixed point, let $\eta,\overline{\eta}:\Omega\to X$ be two common random fixed points of random operators T and S such that $\eta(\omega)\neq\overline{\eta}(\omega)$ for some $\omega\in\Omega$, consider

$$d(\overline{\eta}(\omega), \eta(\omega)) = d(T(\omega,\overline{\eta}(\omega)), T(\omega,\eta(\omega))).$$

$$\leq \varphi\Big(\max\{d(S(\omega,\eta(\omega)), S(\omega,\eta(\omega))), d(S(\omega,\overline{\eta}(\omega)), T(\omega,\eta(\omega))),$$

$$d(S(\omega,\eta(\omega)), T(\omega,\eta(\omega))),$$

$$\frac{1}{2}[d(S(\omega,\overline{\eta}(\omega)), T(\omega,\eta(\omega))) + d(S(\omega,\eta(\omega)), T(\omega,\overline{\eta}(\omega)))]\}\Big)$$

$$\leq \varphi(\max\{d(\bar{\eta}(\omega), \eta(\omega)), d(\bar{\eta}(\omega), \eta(\omega)), d(\eta(\omega), \eta(\omega)),$$

$$\frac{1}{2}\left[d(\bar{\eta}(\omega), \eta(\omega)) + d(\eta(\omega), \bar{\eta}(\omega))\right] \})$$

$$\leq \phi\, d(\bar{\eta}(\omega), \eta(\omega))$$

$$< d(\bar{\eta}(\omega), \eta(\omega))$$

ie. $d(\bar{\eta}(\omega), \eta(\omega)) < d(\bar{\eta}(\omega), \eta(\omega))$.

This contradiction shows $\eta(\omega) = \bar{\eta}(\omega)$ for every $\omega \in \Omega$.

Theorem 3.2. Let (X, d) be a separable symmetric space that satisfies (W.1), (W.2) and (H_E). Let (A, S) and (B, T) be two pairs of weakly compatible random operators from $\Omega \times X \to X$ such that one of the pairs (A, S) or (B, T) satisfies the property (I). Moreover.

$$d(A(\omega,x), B(\omega,y)) \leq \varphi(\max\{d(A(\omega,x), S(\omega,x)), d(B(\omega,y)\, T(\omega,y)),$$

$$d(S(\omega,x), T(\omega,y)), \frac{1}{2}\left(d(A(\omega,x), T(\omega,y)) + d(B(\omega,y), S(\omega,x))\right\}\right)$$

for every $\omega \in \Omega$. If $A(\omega,X) \subset T(\omega,X)$ and $B(\omega,X) \subset S(\omega,X)$ and one of $T(\omega,X)$, $S(\omega,X)$, $B(\omega,X)$ or $A(\omega,X)$ is a complete subspace of X for every $\omega \in \Omega$, then A, B, T, and S have unique common random fixed point.

Proof :

Suppose the pair (B,T) of random mappings satisfies the property (I). So there exists a sequence $\{\xi_n\}$ in $M(\Omega, X)$ such that

$\lim_{n\to\infty} d(B(\omega, \xi_n(\omega)), \xi(\omega)) = \lim_{n\to\infty} d(T(\omega, \xi_n(\omega)), \xi(\omega)) = 0$ for every $\omega \in \Omega$ for some $\xi \in M(\Omega, X)$. As $\{B(\omega, \xi_n(\omega))\}$ is a sequence of measurable mappings and $B(\omega, \xi_n(\omega)) \in B(\omega, X)$ for every $\omega \in \Omega$ and $n \in N$, now the fact $B(\omega,X) \subset S(\omega,X)$ allows obtaining the sequence of measurable mappings $\eta_n : \Omega \to X$ such that $B(\omega, \xi_n(\omega)) = S(\omega, \eta_n(\omega))$ for every $\omega \in \Omega$. Hence $\lim_{n\to\infty} d(S(\omega, \eta_n(\omega)), \xi(\omega)) = 0$ for every $\omega \in \Omega$.

Now we show that $d(A(\omega, \eta_n(\omega)), \xi(\omega)) = 0$ for every $\omega \in \Omega$.

For this consider

$$d\left(A(\omega, \eta_n(\omega)),\ B(\omega, \xi_n(\omega))\right)$$
$$\leq \varphi\Big(\max\Big\{d\left(A(\omega, \eta_n(\omega)), S(\omega, \eta_n(\omega))\right),\ d(B(\omega, \xi_n(\omega)),\ T(\omega, \xi_n(\omega))),$$
$$d\left(S(\omega,\eta_n(\omega)),\ T(\omega,\xi_n(\omega))\right),$$
$$\frac{1}{2}\left(d\left(A(\omega, \eta_n(\omega)), T(\omega, \xi_n(\omega))\right) + d\left(B(\omega, \xi_n(\omega)),\ B(\omega, \xi_n(\omega))\right)\right\}$$

$$\leq \varphi\Big(\max\Big\{d\left(A(\omega,\eta_n(\omega)), B(\omega,\xi_n(\omega))\right),\ d(B(\omega, \xi_n(\omega)),\ T(\omega, \xi_n(\omega))),$$
$$\frac{1}{2}\left(d\left(A(\omega,\eta_n(\omega)), T(\omega, \xi_n(\omega))\right) + 0\right\}\ \Big)$$

$$\leq \phi\left(d\left(B(\omega, \xi_n(\omega)), T(\omega, \xi_n(\omega))\right)\right)$$
$$d\left(A(\omega, \eta_n(\omega)), B(\omega, \xi_n(\omega))\right) < d\left(B(\omega, \xi_n(\omega)), T(\omega, \xi_n(\omega))\right)$$

for every $\omega \in \Omega$.

Therefore by property (H_E), we have

$$\lim_{n\to\infty} d\left(B(\omega, \xi_n(\omega)),\ T(\omega, \xi_n(\omega))\right) = 0 \text{ for every } \omega \in \Omega.$$

Hence:

$$\lim_{n\to\infty} d\left(B(\omega, \xi_n(\omega)),\ A(\omega, \eta_n(\omega))\right) = 0 \quad \text{for every } \omega \in \Omega$$

By (W.2), we deduce that $\lim_{n\to\infty} d\left(A(\omega, \eta_n(\omega)),\ \xi(\omega)\right) = 0$ for every $\omega \in \Omega$. suppose for every $\omega \in \Omega$, $S(\omega, X)$ is a complete subspace of X. Now $S(\omega,\eta_n(\omega)) \in S(\omega, X)$ for every $\omega \in \Omega$. Let $\xi_1 : \Omega \to X$ be the limit of the sequence of measurable mappings $\{S(\omega, \eta_n(\omega))\}$. Since X is separable, therefore $\xi_1 \in M(\Omega, X)$. Moreover $\xi_1(\omega) \in S(\omega, X)$ for every $\omega \in \Omega$. This allows obtaining the measurable mapping $\overline{\xi} : \Omega \to X$ such that $\xi(\omega) = S(\omega, \overline{\xi}(\omega))$. Now consider

$$\lim_{n\to\infty} d\left(A(\omega, \eta_n(\omega)), S(\omega, \overline{\xi}(\omega))\right) = \lim_{n\to\infty} d\left(B(\omega, \xi_n(\omega)),\ S(\omega, \overline{\xi}(\omega))\right).$$
$$= \lim_{n\to\infty} d\left(T(\omega,\xi_n(\omega)), S(\omega, \overline{\xi}(\omega))\right)$$
$$= \lim_{n\to\infty} d\left(S(\omega, \eta_n(\omega)), S(\omega, \overline{\xi}(\omega))\right)$$

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$= 0 \qquad$ for every $\omega \in \Omega$.

Thus,

$$d\left(A(\omega, \bar{\xi}(\omega)), B(\omega, \xi_n(\omega))\right) \leq \varphi(\max\{d(A(\omega, \bar{\xi}(\omega)), S(\omega, \bar{\xi}(\omega))),$$

$$d(B(\omega, \xi_n(\omega)), T(\omega, \xi_n(\omega))), d(S(\omega, \bar{\xi}(\omega)), T(\omega, \xi_n(\omega)))$$

$$\frac{1}{2}\left(d(A(\omega, \bar{\xi}(\omega)), T(\omega, \xi_n(\omega))) + d(B(\omega, \xi_n(\omega)), S(\omega, \bar{\xi}(\omega)))\right)\})$$

for each $\omega \in \Omega$ This immediately gives :

$\lim_{n\to\infty} d\left(A(\omega, \bar{\xi}(\omega)), B(\omega, \xi_n(\omega))\right) = 0$ for every $\omega \in \Omega$.

By (W.1), we have $A(\omega, \bar{\xi}(\omega)) = S(\omega, \bar{\xi}(\omega))$ for every $\omega \in \Omega$.

The weak compatibility of random operators A and S implies that $S(\omega, A(\omega, \bar{\xi}(\omega))) = A(\omega, S(\omega, \bar{\xi}(\omega)))$ for every $\omega \in \Omega$. Now

$A(\omega, A(\omega, \bar{\xi}(\omega))) = A(\omega, S(\omega, \bar{\xi}(\omega))) = S(\omega, A(\omega, \bar{\xi}(\omega))) = S(\omega, S(\omega, \bar{\xi}(\omega)))$. for every $\omega \in \Omega$.

As $A(\omega, \bar{\xi}(\omega)) \in A(\omega, X)$ for every $\omega \in \Omega$ where $\bar{\xi} \in M(\Omega, X)$, the assumption $A(\omega, X) \subset T(\omega, X)$ for every $\omega \in \Omega$ allows obtaining $\bar{\xi}_1 \in M(\Omega, X)$ such that $A(\omega, \bar{\xi}(\omega)) = T(\omega, \bar{\xi}_1(\omega))$ for every $\omega \in \Omega$.

We now show that for every $\omega \in \Omega$, $B(\omega, \bar{\xi}_1(\omega)) = T(\omega, \bar{\xi}_1(\omega))$. If not, then for some $\omega \in \Omega$ consider

$$d\left(A(\omega, \bar{\xi}(\omega)), B(\omega, \bar{\xi}_1(\omega))\right)$$

$$\leq \varphi(\max\{d(A(\omega, \bar{\xi}(\omega)), S(\omega, \bar{\xi}(\omega))), d(B(\omega, \bar{\xi}_1(\omega)), T(\omega, \bar{\xi}_1(\omega))),$$

$$d(S(\omega, \bar{\xi}(\omega)), T(\omega, \bar{\xi}_1(\omega))),$$

$$\frac{1}{2}(d(A(\omega, \bar{\xi}(\omega)), T(\omega, \bar{\xi}_1(\omega))) + d(B(\omega, \bar{\xi}_1(\omega)), S(\omega, \bar{\xi}(\omega))))\}).$$

$$\leq \varphi(\max\{d(A(\omega, \bar{\xi}(\omega)), A(\omega, \bar{\xi}(\omega))), d(B(\omega, \bar{\xi}_1(\omega)), A(\omega, \bar{\xi}(\omega))),$$

$$d(A(\omega, \bar{\xi}(\omega)), T(\omega, \bar{\xi}_1(\omega))),$$

$$\frac{1}{2}\left(d(A(\omega,\bar{\xi}(\omega)), T(\omega,\bar{\xi}_1(\omega))) + d(B(\omega,\bar{\xi}_1(\omega)), A(\omega,\bar{\xi}(\omega)))\}\right).$$

$$\leq \varphi\left(\max\{0, d(B(\omega,\bar{\xi}_1(\omega)), A(\omega,\bar{\xi}(\omega))), 0,\right.$$

$$\left.\frac{1}{2}\left(0 + d(B(\omega,\bar{\xi}_1(\omega)), A(\omega,\bar{\xi}(\omega)))\right)\}\right)$$

$$\leq \varphi\left(\max\{d(A(\omega,\bar{\xi}(\omega)), B(\omega,\bar{\xi}_1(\omega))),\right.$$

$$\left.\frac{1}{2}\, d\left(A(\omega,\bar{\xi}(\omega)), B(\omega,\bar{\xi}_1(\omega))\right)\}\right)$$

$$\leq \varphi\, d\left(A(\omega,\bar{\xi}(\omega)), B(\omega,\bar{\xi}_1(\omega))\right)$$

$$< d\left(A(\omega,\bar{\xi}(\omega)), B(\omega,\bar{\xi}_1(\omega))\right)$$

i.e. $d\left(A(\omega,\bar{\xi}(\omega)), B(\omega,\bar{\xi}_1(\omega)\right) < d\left(A(\omega,\bar{\xi}(\omega)), B(\omega,\bar{\xi}_1(\omega))\right)$

which is a contradiction.

Hence

$$B\left(\omega,\bar{\xi}_1(\omega)\right) = T(\omega,\bar{\xi}_1(\omega)) = A(\omega,\bar{\xi}(\omega)) = S\left(\omega,\bar{\xi}(\omega)\right) \text{ for every } \omega\in\Omega$$

The weak compatibility of random operators B and T implies that

$$B\left(\omega, T(\omega,\bar{\xi}_1(\omega))\right) = T\left(\omega, B(\omega,\bar{\xi}_1(\omega))\right) \text{ for every } \omega\in\Omega,$$

$$T\left(\omega, T(\omega,\bar{\xi}_1(\omega))\right) = T\left(\omega, B(\omega,\bar{\xi}_1(\omega))\right) = B\left(\omega, T(\omega,\bar{\xi}_1(\omega))\right)$$

$$= B\left(\omega, B(\omega,\bar{\xi}_1(\omega))\right) \text{ for each } \omega\in\Omega$$

Let us show that $A\left(\omega, A(\omega,\bar{\xi}(\omega))\right) = A\left(\omega,\bar{\xi}(\omega)\right)$ for each $\omega\in\Omega$.

If not, then for some $\omega\in\Omega$, consider

$$d\left(A(\omega,\bar{\xi}(\omega)), A(\omega, A(\omega,\bar{\xi}(\omega)))\right) = d\left(A(\omega, A(\omega,\bar{\xi}(\omega))), B(\omega,\bar{\xi}_1(\omega))\right)$$

$$\leq \varphi\,(\max\{d(A(\omega, A(\omega,\bar{\xi}(\omega))), S(\omega, A(\omega,\bar{\xi}(\omega)))),$$

$$d(B(\omega,\bar{\xi}_1(\omega)), T(\omega,\bar{\xi}_1(\omega))), d(S(\omega, A(\omega,\bar{\xi}(\omega))), T(\omega,\bar{\xi}_1(\omega)))$$

$$\frac{1}{2}\left(d(A(\omega, A(\omega,\bar{\xi}(\omega))), T(\omega,\bar{\xi}_1(\omega))) + d(B(\omega,\bar{\xi}_1(\omega)), S\left(\omega, A(\omega,\bar{\xi}(\omega))\right)))\}\right)$$

$$\leq \varphi\,(\max\{d\,(A(\omega, A(\omega, \overline{\xi}(\omega))),\ A\,(\omega,\ A(\omega, \overline{\xi}(\omega)))),$$

$$d\,\left(A(\omega, \overline{\xi}(\omega)),\ A(\omega, \overline{\xi}(\omega))\right),\ d\left(A(\omega, A(\omega, \overline{\xi}(\omega))),\ A\,(\omega, \overline{\xi}(\omega))\right)$$

$$\frac{1}{2}\left(d\,(A(\omega, A(\omega,\ \overline{\xi}(\omega))),\ A\,(\omega, \overline{\xi}(\omega))\right) + d\,\left(A\,(\omega,\ \overline{\xi}(\omega)),\ A\,(\omega,\ A(\omega, \overline{\xi}\,(\omega))))\}\right)$$

$$\leq \phi(\max\{0,\ 0,\ d\,(A(\omega,\ A\,(\omega,\ \overline{\xi}(\omega))),\ A(\omega,\ \overline{\xi}(\omega))),$$

$$d\,\left(A(\omega, A(\omega, \overline{\xi}(\omega))),\ A\,(\omega, \overline{\xi}(\omega)))\}\right)$$

$$< \varphi d\,\left(A(\omega,\ A(\omega,\ \overline{\xi}(\omega))), A(\omega, \overline{\xi}(\omega)))\right)$$

$$< d\,\left(A\,(\omega,\ A\,(\omega, \overline{\xi}(\omega))),\ A\,(\omega, \overline{\xi}(\omega))\right)$$

which is a contradiction. Therefore

$A\left(\omega, A\,(\omega, \overline{\xi}(\omega))\right) = A\left(\omega,\ \overline{\xi}(\omega)\right) = S\left(\omega,\ A(\omega,\ \overline{\xi}(\omega))\right)$ for every $\omega \in \Omega$. So $A\left(\omega,\ \overline{\xi}\,(\omega)\right)$ is a common random fixed point of random operators A and S. Similarly, $B\left(\omega, \overline{\xi}_1(\omega)\right)$ is common random fixed point of random operators B and T. Since $A\left(\omega,\ \overline{\xi}(\omega)\right) = B\left(\omega,\ \overline{\xi}_1(\omega)\right)$ for every $\omega \in \Omega$, thus $A\left(\omega,\ \overline{\xi}(\omega)\right)$ is common random fixed point of random operators A, B, S and T. The proof is similar when for every $\omega \in \Omega$, $T(\omega,\ X)$ is complete subspace of X. The cases in which $A\left(\omega, X\right)$ or $B(\omega, X)$ is a complete subspace of X for every $\omega \in \Omega$ are similar to the cases in which $T\left(\omega_1 X\right)$ or $S(\omega_1 X)$, respectively, is a complete subspace of X, since $A\left(\omega, X\right) \subset T\left(\omega, X\right)$ and $B(\omega, X) \subset B(\omega, X)$ for every $\omega \in \Omega$.

UNIQUENESS:

To establish the uniqueness of common random fixed point of random operators, let ξ and η be two common random fixed points of the random operators such that $\xi(\omega) \neq \eta(\omega)$ for some $\omega \in \Omega$ consider.

$$d(\xi(\omega),\ \eta(\omega)) = d\,(A\,(\omega,\ \xi(\omega)),\ B\,(\omega,\ \eta(\omega))).$$

$$\leq \phi\,(\max\{d\,(A\,(\omega, \xi(\omega)),\ S\,(\omega,\ \xi(\omega))),\ d\,(B(\omega,\ \eta(\omega)),\ T(\omega, \eta(\omega))),$$

$$d\,(S(\omega,\ \xi(\omega)),\ T(\omega, \eta(\omega))),\ \frac{1}{2}(d\,(A(\omega,\ \xi(\omega)),\ T(\omega, \eta(\omega)))$$

$$+\ d\,(B\,(\omega, \eta(\omega)),\ S\,(\omega,\ \xi(\omega))))\})$$

$$\leq \phi\,(\max\{d\,(\xi(\omega), \xi(\omega)),\ d(\eta(\omega), \eta(\omega)), d(\xi(\omega), \eta(\omega)),$$

$$\frac{1}{2} d\left((\xi(\omega), \eta(\omega)) + d\,(\eta(\omega), \xi(\omega)))\}\right)$$

$$\leq \phi\left(\max\{0, 0, d(\xi(\omega), \eta(\omega))\}\right)$$

$$\leq \phi\ d\ (\xi(\omega), \eta(\omega))$$

$$d(\xi(\omega), \eta(\omega)) < d(\xi(\omega), \eta(\omega))$$

which is a contradiction. So the result follows.

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