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ACCEPTANCE SAMPLING PLAN FOR TRUNCATED LIFE TESTS BASED ON FRECHET DISTRIBUTION USING MEDIAN

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ABSTRACT

The design of acceptance sampling plans are proposed for the truncated life tests assuming that the lifetime of a product follows Frechet distribution using median. The minimum sample sizes of the zero-one double sampling plan and special double sampling plan are determined to ensure that the median life is longer than the given life at the specified consumer's confidence level. The operating characteristic values are analysed. The minimum median ratios are obtained so as to meet the producer's risk at the specified consumer's confidence level. Numerical illustrations are provided to explain the use of constructed tables. Efficiency of the proposed plan is studied by comparing the single sampling plan.

KEYWORDS

Zero-one double sampling plan, special double sampling plan, Truncated life tests, Operating characteristic function, Average sample number, Consumer's risk, Producer's risk.

INTRODUCTION

Acceptance sampling is the methodology that deals with procedures by which decision to accept or reject the lot is made on the sample results . If the quality characteristic is regarding the lifetime of the products, then the acceptance sampling plan is called a life test plan. In general the life testing experiments are time consuming hence it is usual to terminate the life test by a pre-fixed time. Such plans are called life test sampling plans. It is to be pointed out that the acceptance sampling plans are used to reduce the cost of inspection.

Sampling plans based on truncated life tests have been developed and investigated by many authors. Single Sampling plans for truncated life tests using exponential distribution was first discussed by Epstein[2].The results were extended by Goode and Kao[3] for Weibull distribution, Gupta and Groll [4] for gamma distribution, Kantam and Rosaiah [5] for half log-logistic , Kantam et al [6] for log-logistic distribution, Balki and El Masri [1] for Birnbaum-saunders distribution. Tsai and Wu[8] developed sampling plans for generalized Rayleigh distribution.

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Frechet distribution plays a vital role in modeling and analyzing the several extreme events ranging from accelerated life testing to earthquakes,floods,rainfall,sea currents and wind speeds. Balamurali et al [15] developed only single acceptance sampling plan for Frechet distribution using median.

This initiated the researcher to pursue with the designing of zero-one double sampling plan and special double sampling plan for truncated life test using Frechet distribution.Minimum sample sizes for the specified consumer's confidence level, the operating characteristic values and the minimum mean ratios of the life time for the specified producer's risk with illustration of tables are given. Finally the analysis of effectiveness is presented.

FRECHET DISTRIBUTION

Assume that the lifetime of a product follows Frechet distribution, whose probability density function and cumulative distribution function are given respectively as

$$f(t) = \left(\frac{\gamma}{s}\right)\left(\frac{t}{s}\right)^{-1-\gamma} e^{-\left(\frac{t}{s}\right)^{-\gamma}}, t > 0$$

$$and\ F(t) = e^{-\left(\frac{t}{s}\right)^{-\gamma}}, t > 0 \qquad (1)$$

where γ is the shape parameter, **s** is the scale parameter.

The median of the Frechet distribution is $m = \frac{s}{\sqrt[\gamma]{\log_e 2}}$

The probability of the failure of an item before experiment time using the median as the life of the product is given by:

$$p = e^{-\left(\frac{a}{bm/m_0}\right)^{-\gamma}} \qquad (2)$$

where,m/m_0 is the ratio of true median life to the specified median,**a** is the termination ratio and $b = \sqrt[\gamma]{\log_e 2}$

Assume that the life time of a product follows Frechet distribution and the quality of a product is represented by its median life time,m. The submitted lot will be accepted if the data supports the following null-hypothesis H_0: $m \geq m_0$ against the alternative hypothesis, H_1:$m < m_0$.The significance level for the test is 1-P* , where P* is the consumer's confidence level. Design of the sampling plans for the truncated life test consists of determination of (i) Sample sizes (ii) the ratio of true median life to the specified median life m/m_0 .

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DESIGN OF THE ZERO-ONE DOUBLE SAMPLING PLAN

The operating procedure of zero-one double sampling plan for the truncated life test consists of the following steps

Step1: Select a random sample of size n_1 from the submitted lot and put on test for preassigned experimental time t_0. Let d_1 be the number of failures. If $d_1=0$, accept the lot. If $d_1 \geq 2$, reject the lot.

Step2: If $d_1=1$ then select a second random sample of size n_2 and let d_2 be the number of failures. If $d_2=0$, accept the lot otherwise reject the lot.

For a given P*, the proposed zero-one double sampling plan may be characterized by the parameters (n_1, n_2, m/m_0, a).

The probability of accepting a lot under binomial model for zero-one double sampling plan is $(1-p)^{n_1}\left[1+n_1 p(1-p)^{n_2-1}\right]$

where p is given in equation (2)

The minimum sample sizes ensuring $m \geq m_0$ at the consumer's confidence level P*may be found as the solution of the inequality

$$(1-p)^{n_1}\left[1+n_1 p(1-p)^{n_2-1}\right] \leq 1-P^* \qquad (3)$$

(3) gives multiple solutions for the sample sizes n_1 and n_2 .In order to find the unique sample sizes the minimum ASN is incorporated along with specification (3).The determination of the minimum sample sizes for zero-one double sampling plan reduces to

Minimize $$ASN = n_1 + n_1 n_2 p(1-p)^{n_1-1}$$

subject to $$(1-p)^{n_1}\left[1+n_1 p(1-p)^{n_2-1}\right] \leq 1-P^* \qquad (4)$$

where n_1 and n_2 are integers with $n_2 \leq n_1$.

DESIGN OF THE SPECIAL DOUBLE SAMPLING PLAN

The operating procedure of special double sampling plan for the truncated life test has the following steps:

Step 1: Draw a sample of size n_1 from the lot and put on the test for pre-assigned experimental time t_0 and observe the number of defectives d_1. If $d_1 \geq 1$ reject the lot.

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Step 2: If $d_1 = 0$, draw a second random sample of size n_2 and put them on the test for time t_0 and observe the number of defectives d_2.If $d_2 \leq 1$ accept the lot, Otherwise reject the lot.

In a special double sampling plan the decision of acceptance is made only after inspecting the second sample. This aspect differs from usual double sampling plan in which decision of acceptance can be made after the inspection of either first or second sample.

The probability of accepting a lot under binomial model for special double acceptance sampling plan is obtained by $(1-p)^{n_1+n_2}\left(1+\frac{n_2 p}{1-p}\right)$

where p is the probability of a failure of an item before the experiment time using the median as the life of the product is given in (2).

The minimum sample sizes ensuring $m \geq m_0$ at the consumer's confidence level P*may be found as the solution of the inequality

$$(1-p)^{n_1+n_2}\left(1+\frac{n_2 p}{1-p}\right) \leq 1-P^* \qquad (5)$$

(5) provides multiple solutions for sample sizes n_1 and n_2 satisfying the specified consumer's confidence level. In order to find the unique sample sizes the minimum ASN is incorporated along with the probability of the acceptance of the lot which is less than or equal to 1-P* and $n_2 \leq n_1$.The determination of the minimum sample sizes for special double sampling plan reduces to

Minimize $$ASN = n_1 + n_2(1-p)^{n_1}$$

subject to $$(1-p)^{n_1+n_2}\left(1+\frac{n_2 p}{1-p}\right) \leq 1-P^* \qquad (6)$$

where n_1 and n_2 are integers.

The minimum sample sizes for the specified P*(=0.75,0.90,0.95,0.99),**a**(=0.6,0.8,1.0,1.2,1.4) shape parameter γ (=1,2,3) with Frechet distribution for zero-one double sampling plan and special double sampling plan using equation (4) and (6) respectively are presented in Tables 1 and 2.

Numerical values of Table 1 and 2 reveal that

(i) increase in P* increases the sample sizes irrespective of **a**

(ii) increase in **a** decreases the sample sizes irrespective of P*

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OPERATING CHARACTERISTIC VALUES

The operating characteristic function of the proposed life test acceptance sampling plan depict the performance of the sampling plan in discriminating the quality of the submitted product. The operating characteristic values for zero-one double sampling plan and special double sampling plan under Frechet distribution with γ=1 are given in Tables 3 and 4 respectively.

Numerical values of Table 3 and 4 reveal that

(i) increase in P* decreases the OC value for fixed **a** and m/m_0

(ii) increase in **a** decreases the OC value for fixed P* and m/m_0

(iii) increase in m/m_0 increases the OC value for fixed **a** and P*

Fig.1 and fig.2 show the trend of OC values according to the parameter γ of the Frechet distribution.

MINIMUM MEDIAN RATIO

The minimum median ratio, m/m_0 for zero-one double sampling plan and special double sampling plan to keep the producer's risk and consumer's confidence at the required level, may be obtained by solving $P_a \geq 1-\alpha$ using the sample sizes presented in Tables 1and 2 are presented in Tables 5 and 6 respectively.

Numerical values in Tables 5 and 6 indicate that

(i) increase in γ decreases the minimum median ratios for fixed P* and **a**

(ii) increase in **a** increases the minimum median ratios for fixed P* and γ

(iii) increase in P* increases the minimum median ratios for fixed **a** and γ

ILLUSTRATION FOR ZERO-ONE DOUBLE SAMPLING PLAN

Assume that the life time of product under consideration follows Frechet distribution with γ =1.Suppose that the experimenter would like to know whether the median life time is longer than or equal to 40 hours with a confidence of 95%.The experimenter wants to truncate the experiment at 24 hours.This leads to the experiment termination ratio a=0.6.Then from Table 1 the minimum sample sizes for the assumed specifications are n_1=9 and n_2=6.The selected zero-one double sampling plan for life test experiment is put into operations as follows:

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Select randomly 9 units as first sample from the lot and put on test for 24 hours. Based on first sample results accept the lot if no failures in the specified test and reject the lot if there are two or more failures. Otherwise take a second sample of 6 units randomly and put them on test for 24 hours.If no failure occurs accept the lot or else reject the lot.

ILLUSTRATION FOR SPECIAL DOUBLE SAMPLING PLAN

Assume that the life time of a product follows Frechet distribution with parameter $\gamma=1$. Suppose that the experiment wants to know whether the median life time of the product is longer than or equal to 730 days at 95% confidence level. The experimenter wants to run the experiment only for 438 days. This makes the termination time a=0.6. For this assessment, the sample sizes as $n_1=6$ and $n_2=6$ .The selected plan is applied as follows:

Select a random sample of 6 units from the lot and put them on test for 438 days. Reject the lot if the number of failures is greater than or equal to one. If the number of failures is zero then select a second random sample of 6 units and put them on test. If the number of failures is less than or equal to 1 accept the lot otherwise reject the lot.

COMPARITIVE STUDY WITH SINGLE SAMPLING

For a given quality level,amount of inspection is considered for comparing the effectiveness of designed plans.The ASN values of single sampling plan, zero-one double sampling plan and special double sampling plan when lifetime of units follow Frechet distribution with shape parameter $\gamma=1$ at consumer's confidence level P*=0.75 are presented in the following table

plan \ a		0.6	0.8	1.0	1.2	1.4
SSP	C=0	4	3	2	2	2
	C=1	7	5	4	4	3
DSP(0,1)		6.38	4.65	3.75	3.32	2.95
SDSP		3.96	3.19	2.25	2.19	2.15

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Table values reveal that

ASN value of zero-one double acceptance sampling plan is higher than the single sampling plan with c=0 and lesser than the single sampling plan with c=1.ASN of special double sampling plan is minimum when compared with the zero-one double acceptance sampling plan and single acceptance sampling plan. This explains the economy of special double sampling plan.

CONCLUSION

In this paper, truncated life test acceptance sampling plans are developed when the lifetime of the product follows the Frechet distribution.Minimum sample sizes required to guarantee a certain median lifetime of the test items is presented .The operating characteristic function values and the associated producer's risk are also discussed .The determination of optimal parameters by using two points on the operating characteristic curve approach is on progress.

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Table 1 Minimum sample sizes of zero-one double sampling plan under Frechet distribution

ϒ	P*	**a**				
		0.6	0.8	1.0	1.2	1.4
1	0.75	5,4	4,2	3,2	3,1	2,2
	0.9	7,6	5,4	4,3	3,2	3,1
	0.95	9,6	6,5	5,4	4,4	4,2
	0.99	13,8	9,6	7,5	6,4	5,5
2	0.75	11,10	5,3	3,2	2,2	2,1
	0.9	16,16	7,4	4,3	3,2	3,1
	0.95	20,20	8,6	5,4	4,2	3,2
	0.99	30,24	12,7	7,5	5,4	4,3
3	0.75	41,39	6,5	3,2	2,2	2,1
	0.9	61,58	9,7	4,3	3,2	2,2
	0.95	76,75	11,9	5,4	3,3	3,1
	0.99	76,75	16,12	7,5	5,2	4,2

Table 2 Minimum sample sizes of special double sampling plan under Frechet distribution

ϒ	P*	**a**				
		0.6	0.8	1.0	1.2	1.4
1	0.75	3,3	3,1	2,1	2,1	2,1
	0.9	5,4	4,2	3,2	3,1	2,2
	0.95	6,6	4,4	4,2	3,3	3,2
	0.99	9,7	6,6	5,4	4,4	4,3
2	0.75	7,7	3,3	2,1	2,1	2,1
	0.9	11,10	5,3	3,2	2,2	2,1
	0.95	14,13	6,4	4,2	3,2	2,2
	0.99	20,19	8,7	5,4	4,3	3,3
3	0.75	26,26	4,4	2,1	2,1	1,1
	0.9	40,40	6,6	3,2	2,2	2,1
	0.95	51,49	8,6	4,2	3,1	2,1
	0.99	73,73	11,10	5,4	3,3	3,2

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Table 3 Operating Characteristic values of zero-one double sampling plan under Frechet distribution when ϒ=1

P*	**a**	n_1	n_2	m/m_0					
				2	4	6	8	10	12
0.75	0.6	5	4	0.8081	0.9972	0.9999	1	1	1
	0.8	4	2	0.7266	0.9874	0.9996	0.9999	1	1
	1.0	3	2	0.6592	0.9688	0.9979	0.9999	0.9999	0.9999
	1.2	3	1	0.6252	0.9485	0.9944	0.9994	0.9999	0.9999
	1.4	2	2	0.5795	0.9198	0.9877	0.9982	0.9998	0.9999
0.9	0.6	7	6	0.6794	0.9943	0.9999	0.9999	1	1
	0.8	5	4	0.5645	0.9744	0.9991	0.9999	0.9999	1
	1.0	4	3	0.4944	0.9422	0.9958	0.9997	0.9999	0.9999
	1.2	3	2	0.4102	0.8938	0.9874	0.9987	0.9998	0.9999
	1.4	3	1	0.3576	0.8375	0.9722	0.9958	0.9994	0.9999
0.95	0.6	9	6	0.5973	0.9919	0.9999	0.9999	1	1
	0.8	6	5	0.4629	0.9630	0.9987	0.9999	0.9999	1
	1.0	5	4	0.3624	0.9107	1.0000	1	1	1
	1.2	4	4	0.3094	0.8494	0.9808	0.9979	0.9997	0.9999
	1.4	4	2	0.3018	0.8148	0.9677	0.9952	0.9993	0.9999
0.99	0.6	13	8	0.4167	0.9843	0.9998	0.9999	1	1
	0.8	9	6	0.2781	0.9318	0.9974	0.9999	0.9999	1
	1.0	7	5	0.2074	0.8516	0.9876	0.9992	0.9999	0.9999
	1.2	6	4	0.1661	0.7666	0.9674	0.9964	0.9996	0.9999
	1.4	5	5	0.1265	0.6572	0.9283	0.9884	0.9983	0.9997

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Table 4 Operating Characteristic values of special double sampling plan under Frechet distribution when ϒ=1

P*	**a**	n_1	n_2	m/m_0					
				2	4	6	8	10	12
0.75	0.6	3	3	0.7107	0.9705	0.9971	0.9997	0.9999	0.9999
	0.8	3	1	0.5579	0.9091	0.9835	0.9971	0.9995	0.9999
	1.0	2	1	0.5625	0.8789	0.9689	0.9922	0.998	0.9995
	1.2	2	1	0.4693	0.8114	0.9385	0.9804	0.9938	0.998
	1.4	2	1	0.3950	0.7430	0.9000	0.9623	0.9858	0.9947
0.9	0.6	5	4	0.5625	0.9512	0.9951	0.9995	0.9999	0.9999
	0.8	4	2	0.4449	0.8798	0.9781	0.9961	0.9993	0.9998
	1.0	3	2	0.3955	0.8207	0.9536	0.9883	0.9971	0.9993
	1.2	3	1	0.3214	0.7309	0.9091	0.9707	0.9907	0.9971
	1.4	2	2	0.3405	0.7289	0.8977	0.9619	0.9858	0.9947
0.95	0.6	6	6	0.4740	0.9460	0.9941	0.9994	0.9999	0.9999
	0.8	4	4	0.3921	0.8758	0.9779	0.9961	0.9993	0.9998
	1.0	4	2	0.2966	0.7695	0.9387	0.9845	0.9961	0.999
	1.2	3	3	0.2459	0.7107	0.9065	0.9704	0.9906	0.9971
	1.4	3	2	0.2140	0.6283	0.8517	0.9435	0.9788	0.9921
0.99	0.6	9	7	0.3328	0.9130	0.9912	0.9991	0.9999	0.9999
	0.8	6	6	0.2217	0.8154	0.9669	0.9941	0.9989	0.9998
	1.0	5	4	0.1752	0.7086	0.9229	0.9805	0.9951	0.9988
	1.2	4	4	0.1377	0.6245	0.8758	0.9606	0.9876	0.9961
	1.4	4	3	0.1074	0.5234	0.8039	0.9249	0.9719	0.9895

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Table 5 Minimum median ratios of zero-one double sampling plan under Frechet distribution

ϒ	P*	a				
		0.6	0.8	1.0	1.2	1.4
1	0.75	2.6841	3.0999	3.5841	3.9841	4.4999
	0.90	2.9841	3.5999	4.0399	4.2841	4.5999
	0.95	3.1899	3.7999	4.3999	5.0999	5.4999
	0.99	3.4999	4.1999	4.8999	5.5099	6.3999
2	0.75	1.4291	1.6691	1.8991	2.1791	2.3691
	0.90	1.5001	1.7699	2.0199	2.3091	2.5691
	0.95	1.5361	1.7999	2.0999	2.3691	2.6961
	0.99	1.6001	1.8999	2.1999	2.5591	2.8199
3	0.75	1.1791	1.3491	1.5391	1.7691	1.9798
	0.90	1.2091	1.3991	1.5991	1.8399	2.0591
	0.95	1.2261	1.4199	1.6499	1.8691	2.0961
	0.99	1.2501	1.4599	1.7001	1.9301	2.1999

Table 6 Minimum median ratios of special double sampling plan under Frechet distribution

ϒ	P*	a				
		0.6	0.8	1.0	1.2	1.4
1	0.75	3.5901	4.5999	5.2841	6.5441	7.4099
	0.90	3.8991	5.0009	5.9841	6.8941	7.5699
	0.95	4.0991	5.0099	6.2841	6.9441	8.3509
	0.99	4.4701	5.4299	6.4941	7.5441	8.7509
2	0.75	1.5901	1.9499	2.2841	2.7441	3.2079
	0.90	1.6701	2.0499	2.3991	2.7441	3.2089
	0.95	1.7107	2.1099	2.4991	2.9041	3.2099
	0.99	1.7501	2.1499	2.5681	2.9841	3.3999
3	0.75	1.2501	1.4899	1.7341	2.1031	2.2699
	0.90	1.2801	1.5299	1.8041	2.1041	2.4389
	0.95	1.2901	1.5599	1.8341	2.1801	2.4399
	0.99	1.3101	1.5799	1.8841	2.1841	2.5399

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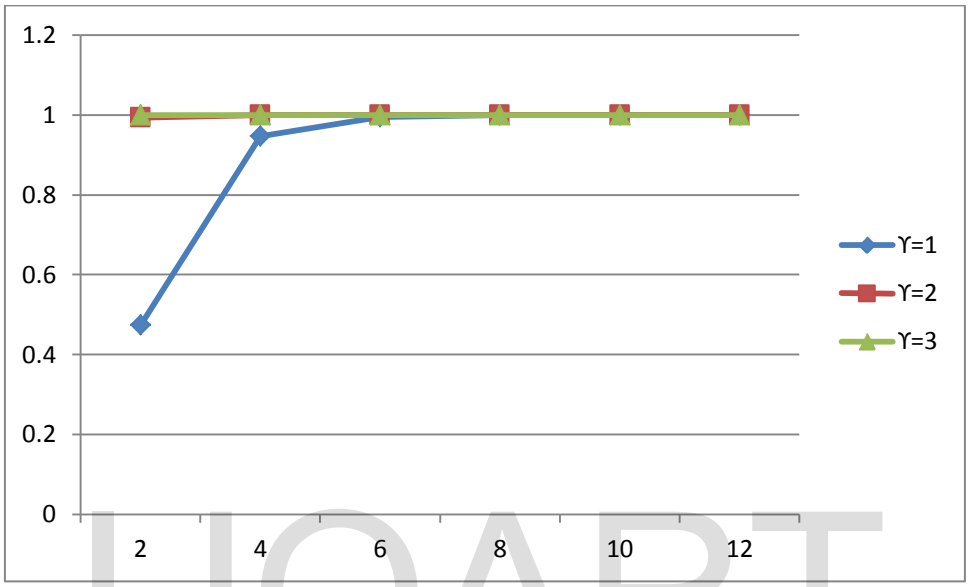

Fig 1 OC curves of zero-one double sampling plan under Frechet distribution with P*=0.95 and a = 0.6

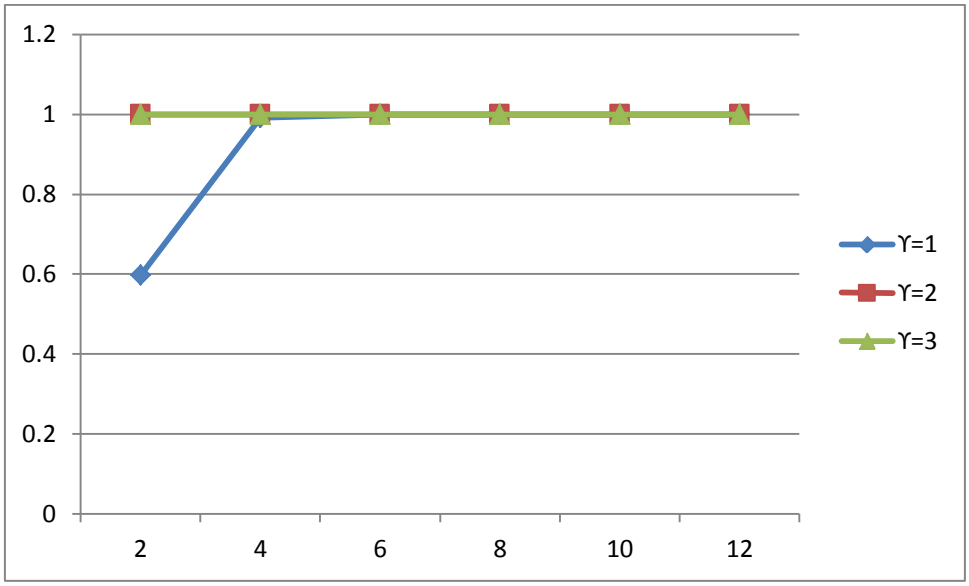

Fig 2 OC curves of special double sampling plan under Frechet distribution with P*=0.95 and a = 0.6

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