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A Hybrid Complex Method for Optimization of Rigidly Jointed Plane Frames

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ABSTRACT

An efficient and reliable procedure is presented for the minimum weight design of moment-resisting plane steel frames subjected to the stress constraints of the AISC-LRFD specifications. The cross-section of a typical member consists of a symmetrical I-section whose web and flange dimensions are considered as design variables. In order to achieve computational economy the problem is decomposed into two sets of variables. The more sensitive variables are found by modified Complex Method of Box and the less sensitive variables are determined by fully-stressed design procedure. To evaluate the performance of the developed algorithm two numerical problems are solved.

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1. Introduction

Generally, structures are designed by trial and error, that is; an initial design is assumed and then analyzed to evaluate its performance. The design is modified based upon the information provided by the analysis, subsequently the designer reanalyzes the design and this process continues. The design-analysis-redesigns steps are repeated until no further significant improvement is possible. This conventional design process is very time consuming. The designer usually terminates the process after several iterations and the design thus obtained is feasible but not necessarily optimal one. Design optimization process is an activity that can be fully automated.

It is theoretically possible to solve the entire structural optimization problem using mathematical programming techniques. However, in practice, difficulties emerge due to computational inefficiencies, when the dimension of the design space either becomes too large, or contains variables of different sensitivities, or both. In order to overcome these difficulties, the entire design space is decomposed into two subspaces. The important aspect of this decomposition, however, lies in the fact that two different strategies for finding the optimal solutions in the two subspaces are used. For instance, the variable cross-sectional dimensions of I-shaped member, web depth and flange width, playing more significant role in the optimal design of a member size are found by the application of a modified version of Complex Method of Box. The less sensitive variables i.e. web and flange thicknesses of the members are found by using the fully-stressed optimality criterion.

The Complex Method of Box [3] has been applied to small academic examples of structural optimization subjected to the stress constraints of the AISC-ASD specifications in the past (Fu [5], Haque [6, 7], Lai [9], and Lipson [10-14]), and performs well in this study. The present work considers the optimal weight design of two moment-resisting frames of fixed geometry under multiple load conditions subjected to stress design criteria stipulated in AISC-LRFD code. The results show favorable design improvements and rate of convergence.

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2. Optimum Design Problem and its Formulation

The optimum load and resistance factor design problem of a rigidly jointed plane frame with stress constraints can be stated as follows. Find the cross-sectional dimensions, flange width (b_m), flange thickness (t_m), web thickness (w_m), and web depth (d_m) of I-shaped members of a rigidly jointed plane frame so that the structure is able to carry safely a set of external loads and, at the same time, attain the minimum weight among all feasible designs. The geometry and topology of the structure are not changed during the optimization process.

The problem can be formulated as a mathematical programming problem as follows. Find a design vector

$$\mathbf{Z} = Z_i = (b_m, t_m, w_m, d_m) \quad (1)$$

$(i = 1, 2, \text{--------}, N)$, $(m = 1, 2, \text{--------}, M)$

so that the objective function

$$F = \sum_{i=1}^{M} \rho A_i L_i \quad (2)$$

attains a minimum value among all feasible designs that satisfy the explicit constraints.

$$b_m^L \le b_m \le b_m^U \quad (m = 1, 2, \text{--------}, M) \quad (3a)$$

$$t_m^L \le t_m \le t_m^U \quad (m = 1, 2, \text{--------}, M) \quad (3b)$$

$$w_m^L \le w_m \le w_m^U \quad (m = 1, 2, \text{--------}, M) \quad (3c)$$

$$d_m^L \le d_m \le d_m^U \quad (m = 1, 2, \text{--------}, M) \quad (3d)$$

and u implicit constraints.

$$g_i(\mathbf{Z}) < 0 \quad (i = 1, 2, \text{--------}, u) \quad (4)$$

in which Z_i represents the variables of the design vector, N is the dimension of the design space, M is the total number of members, ρ is the material density, and A_i is the cross-sectional area of member i. The superscripts L and U denote the lower and the upper bounds on the design variables. The implicit constraints impose restrictions on the stresses as governed by the AISC-LRFD specification.

Equation 1 is a general description of the design vector. In practice, not all elements of the design vector are independent variables of the design space. Some of the variables may be linked in order to satisfy symmetry. Thus, the dimension of the design vector in a particular case may be smaller than what has been suggested for the more general case.

3. Solution Procedure:

To solve the stated optimization problem, a computational methodology is developed consisting of three logically separable phases: the optimization phase, the structural analysis phase, and the design evaluation phase. During the optimization phase, attempts are made to improve by finding feasible points that are successively closer to an optimum. In the structural analysis phase, the structure, provisionally obtained in the optimization phase, is analyzed and, finally, the feasibility of the structure is checked in the design evaluation phase. An overview of the Complex Method of Box is given below.

3.1 The Complex Method of Box [3]

The Complex Method is a mathematical programming procedure for finding an optimal solution of non-linear, constrained optimization problems. This method derives its acronym COMPLEX from two words, Constrained and Simplex. The Complex Method was proposed originally by M. J. Box in 1965, where he demonstrated efficacy of the method in finding near optimal solution to non-linear, constrained optimization problems. It is a Zero-order method optimization method; that is, it does not require either the gradient of the objective function, or that of the constraints. The choice of Complex Method was made for its ability to span large portions of the design space, thereby providing a better chance of finding the global optimum, and for its ability to deal with constrained optimization problems.

The method attempts to find a design vector $\mathbf{x} \equiv \mathrm{x}_i \qquad i = 1, 2, ..., N$

where $\mathbf{x}$ denotes the design vector and x_i the coordinate of a point in the design space.

such that to minimize $f(\mathrm{x}_i) \qquad i = 1, 2, ..., N$

subject to N explicit constraints $\mathrm{x}_i^L \le \mathrm{x}_i \le \mathrm{x}_i^U \qquad i = 1, 2, ..., N$

and M implicit constraints $g_j(\mathrm{x}_i) \le 0 \qquad j = 1, 2, ..., M \qquad i = 1, 2, ..., N$

Where N is the number of design variables and M is the number of implicit constraints, x_i^L the lower and x_i^U the upper limit for design variables.

The Complex Method optimizes a provisional design by reflecting the worst point (design) through the centroid to find the best point (design). The optimization process is divided into two phases. In the first phase a set of feasible points (satisfying all constraints) are generated randomly. After generating the initial complex, the algorithm moves to the reflection phase. In this phase, the method calls for the improvement of the worst point in the complex. To improve a point, the algorithm reflects it through the centroid of the remaining points (vertices of the Complex). If the reflected point is worst, or it violates an implicit constraint, it is moved back half the distance to the centroid. The method continues in this manner until convergence criteria are met, or the maximum number of iterations is reached. Details of the method and its successful application to structural design problems can be found in references [3, 5-7, 9, 10-14].

3.2 Proposed Modifications and Implementation of the Complex Method

The modifications to the Complex Method as used in this study are summarized as follows.

i) ***Separation of design space***: Theoretically, it is possible to apply the mathematical programming techniques to the variables of the entire design space. In general, such a design problem involves significant number of design variables and a significant number of linear and nonlinear constraints for structure to ensure that there are sufficient margins between design load effects and their load bearing capabilities. In addition, since the variables are of different nature, numerical problems may arise during the solution process. Thus, from the computational point of view, it is highly desirable to decompose the design space into a number of subspaces; with each subspace having its own optimization strategy.

In the Hybrid Complex Method proposed in this study, the entire design space is decomposed into two subspaces. During the Optimization process, the more sensitive variables i.e. web depths and flange widths of the members are strictly obtained by the modified Complex Method and the less sensitive variables i.e. web and flange thicknesses of the members are found by using the fully-stressed optimality criterion.

ii). *Feasibility of the initial design*: An initial point in an n-dimensional design space is chosen. In the original procedure this point was required to be feasible, but the present algorithm has been written in such a way that if the initial chosen point is not feasible it is made feasible by adjusting one or more of the coordinates of the design vector.

iii). *Satisfying the implicit constraints*: In the original procedure proposed by Box the points in the initial complex which violated the implicit constraints were moved halfway back towards the centroid of the remaining, already accepted points. The process of moving halfway in towards the centroid is repeated until the point becomes feasible. In the present method, an attempt is made to satisfy all the implicit constraints for each randomly chosen point during the sizing of web and flange thicknesses of the members using the fully-stressed optimality criterion. If it is impossible to satisfy all the implicit constraints by this method, then, the original procedure is used.

iv). *The improvement procedure*: The improvement procedure has been modified in that at every iteration the worst design is reflected through the centroid of the remaining designs in the design space to a new point. Then, when this new point has been optimally sized, its objective function is evaluated and compared with that of worst design in the complex. If the new point is less, it is accepted as a design improvement and termination criteria are checked; if greater, instead of continuously halving α , it is halved only thrice and then centroid is considered as a candidate for improvement. If centroid is still greater than the worst, then a new point is located at the mid point of a line joining centroid to the best point in the complex. If the objective function is still greater than the worst, then the worst point is replaced by the best design in the complex.

v). *Termination criteria*: The procedure in 3 is repeated until a preset termination criterion is reached. The first termination criterion used in this study is based on the objective function values of all k points in the complex. This convergence criterion is met if the ratio of the difference between the maximum objective function value and the minimum objective function value to maximum objective function value of the points in the complex is less than or equal to the value of $\in$ (a user defined variable) i.e.

$$(f_{\max} - f_{\min}) / f_{\max} \leq \in \qquad (5)$$

The second criterion that is checked for the convergence of the solution is a measure of the design space spanned by the vertices of complex,

$$\frac{\sum_{i=1}^{n}\sum_{j=1}^{k}\left(\mathrm{x}_{ij} - c_i\right)^2}{2\sum_{i=1}^{n}\left(\mathrm{x}_i^U - \mathrm{x}_i^L\right)^2} \leq \delta \qquad \text{\_\_\_\_\_\_\_ (6)} \qquad \text{where } c_i = \frac{1}{k}\sum_{j=1}^{k}\mathrm{x}_{ij}$$

Finally, a constraint is placed on the maximum number of iterations that may occur before terminating the optimization. The optimization process is terminated as soon as any of the termination criteria is satisfied.

3.3 Sizing of Members

The design procedure used is an iterative technique, and commences with the smallest web and flange thicknesses of the members whose web depths and flange thicknesses have been established by the complex method. The program takes each load condition and calculates the deflections and forces. It determines the maximum axial force, shear force and bending moment for each member of the structure. After completing the final loading condition, it summarizes the condition of forces and deflections, by selecting their highest value from the loading conditions for each member. The cross-sectional design of the members is undertaken in the member design space. During this phase of calculations, the web depths and flange widths of the members are regarded as fixed, while their web and flange thicknesses are determined based on fully-stressed design. The design of the member is carried out in two phases. In the first phase the minimum thickness of webs for all members are determined in order to carry safely the induced shear forces under all load cases. At this stage, any symmetry of member design (if specified) is also taken into account before proceeding with the flange design.

In the second phase, after the web dimensions have been completely determined for the entire structure, the flange thicknesses are calculated to resist the already determined bending moments and axial forces. It then considers each group of members and selects the maximum value for that group. The program substitutes the modified cross-sectional dimensions for the members, making the necessary alterations to the stiffness matrix, and modifies the loading vectors for the change in self weight. The analysis and design cycle is repeated until the program is unable to modify any group of members. This constitutes the final design for the structure whose web depths and flange thicknesses have been established by the modified complex method.

4. Examples

Two numerical examples are solved to demonstrate the versatility of the proposed procedure. The results of the examples were generated with a FORTRAN 77 computer program executed on a Pentium IV, 2.66 GHz laptop computer with 512 MB of RAM. The program is in three interacting modules, performs search for optimum, structural analysis and structural design. In the development of the optimization routine guidance is taken from Kuester and Mize [8], and Belegundu and Chandrupatla [2] in order to produce a hybrid version of the modified Complex method. In the development of the structural analysis routine, extensive modifications to the frame analysis programs given in Crawley and Dillon [4] are made to meet the special needs of this study. The structural design routine is developed by using the Third Edition of Load and Resistance Factor Design (LRFD) procedure from American Institute of Steel Construction (AISC). The decision-making and the computational assignments are carried out by a separate subroutine in each phase. It is only during the last phase that the provisions of the AISC-LRFD specification are invoked; thus letting the other two subroutines function independently. This feature is highly desirable, especially when the need for modification to the design code may arise in the future. However, different modules representing the provision of other commonly used design codes can be appended to the existing design routine with relative ease.

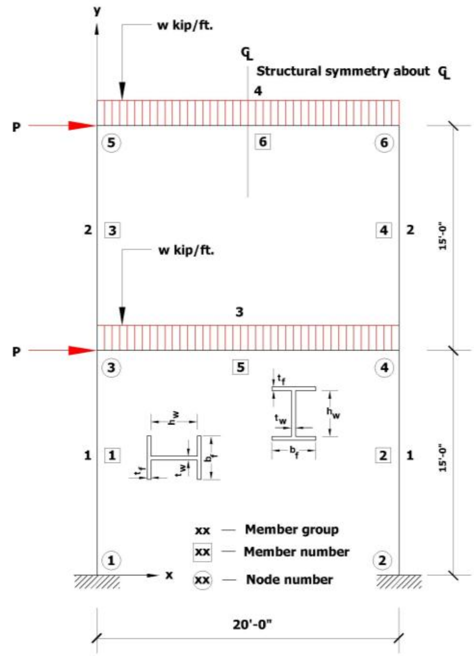
The material specified for all members is steel with yield stress of 36 ksi, modulus of elasticity 29,000 ksi and unit weight of 490 lb/ft^3. The implicit constraints are to meet the relevant provisions of AISC-LRFD stress specifications.

4.1 Example 1: One-Bay Two-Storey Frame

Consider the one-bay two-storey frame shown in figure 1. The aim of this design exercise is to minimize the weight of the frame by selecting a set of cross-sectional dimensions for the members so that the structure can carry the externally applied loads while the member stress requirements as per AISC-LRFD specifications are satisfied. The figure 1 shows the geometry, typical cross-sections, member, node and member group numbering for the frame. The design variables b_f, t_f, h_w and t_w define the thickness and width of the member's flange and web, respectively. This six-member rigid joint plane frame has been used as an example by previous investigators using sizing variables only. The frame is subjected to two loading conditions shown in figure 1. Member 2 is linked with member 1, while member 4 is linked with member 3, in order to impose symmetry. The unbraced length of 15 ft and 10 ft are specified for the design of beams and columns, respectively. The results of the axial forces in the members, and the axial force capacity of the members are shown in figure 2. The bending moment curves for load cases, the bending moment envelope and the moment capacity curves are shown for comparison in figure 3. At the optimum design, the most critical constraints are formed by the interaction of the combined effects of the axial forces and bending moments. The interaction equation checks, in members 2, 4 and 5 for load case 2 reached their limiting value unity and are shown in figure 4. Furthermore, there is no violation of constraints that can be seen from figures 2, 3 and 4, in other members.

The initial complex, randomly generated, consisted of sixteen designs, with frame weight ranging from 10398.12 lb to 14661.47 lb. The minimum design in the initial complex is 28.08% heavier than that of the final design. Figure 5 depicts the maximum and minimum objective function values in the complex with successive iteration. It can be noted that the objective function is steadily decreasing, as one would expect from a minimization problem. The frame becomes 12.44%, 8.62%, 5.17% and .86% heavier than the final design at iterations 4, 26, 43 and 125 respectively. The objective function and design space convergence parameters for every iteration are plotted in figure 6.

After 296 iterations, the complex improvement procedure was stopped by the termination criteria. The optimal weight obtained based on AISC-LRFD specifications for this problem is 8117.77 lb. Results of the final design are compared to the design reported by Reinschmid and coworkers [16], Arora and coworkers [1], and Haque [7]. They reported optimal weights of 8810 lb, 8292 lb and 8656 lb, respectively. All of these three studies are based on the AISC-ASD specifications. It is, therefore not possible to strictly compare the results of the present study with either of them. However, the proximity of the optimal weights in the four studies is worth noticing.



Load case	P (kip)	w (kip/ft.)
1	0.00	9.00
2	76.5	0.00

Figure 1 Skeletal geometry and load cases for 1-bay 2-storey frame

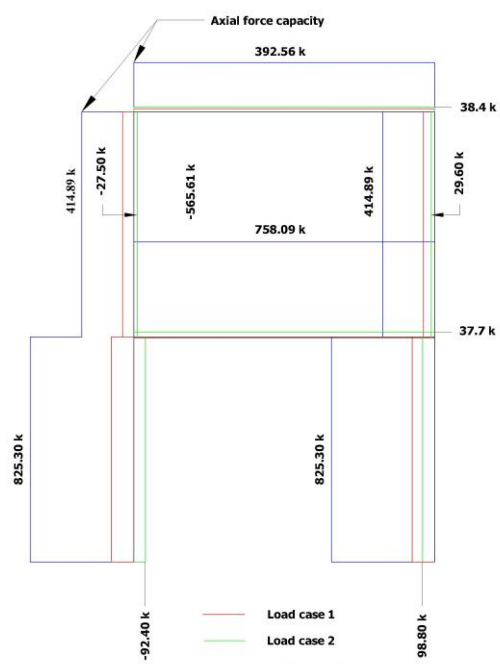


Figure 2 Axial force and Axial force capacity diagrams for 1-bay 2-storey frame

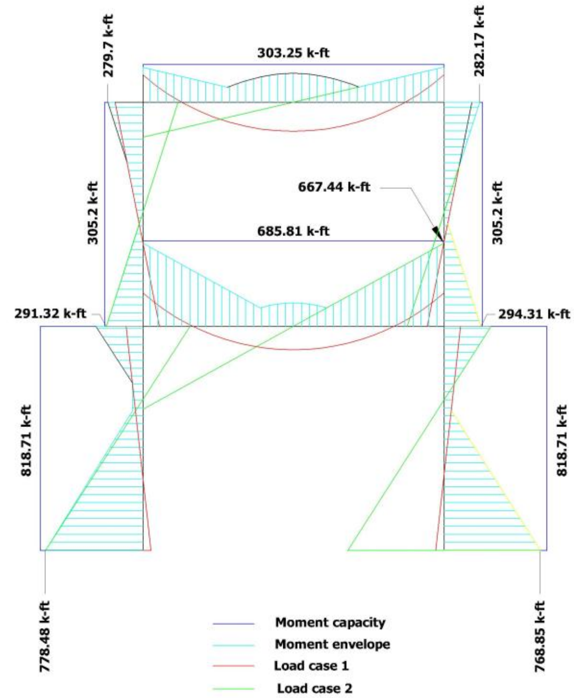


Figure 3 Moment envelop and moment capacity diagrams for 1-bay 2-storey frame

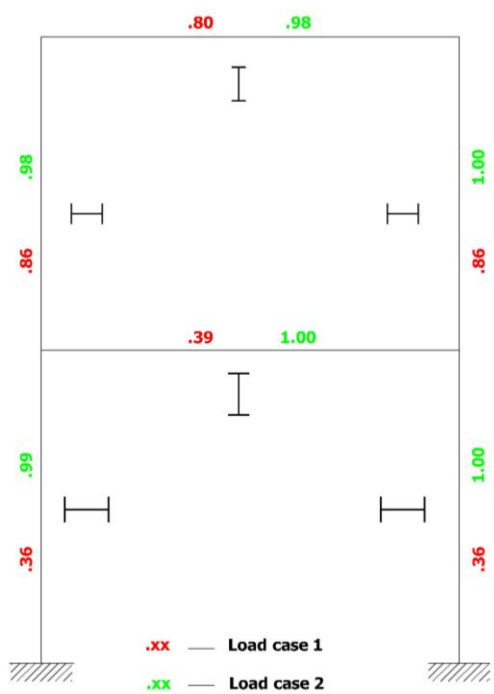


Figure 4 Final design and interaction eq. checks for 1-bay 2-storey frame

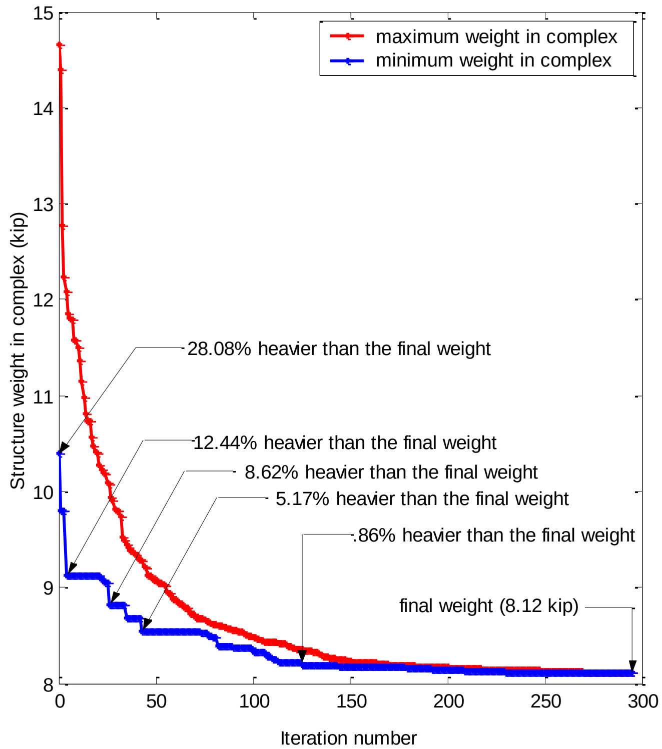


Figure 5 History of max/min weight of structure in the complex

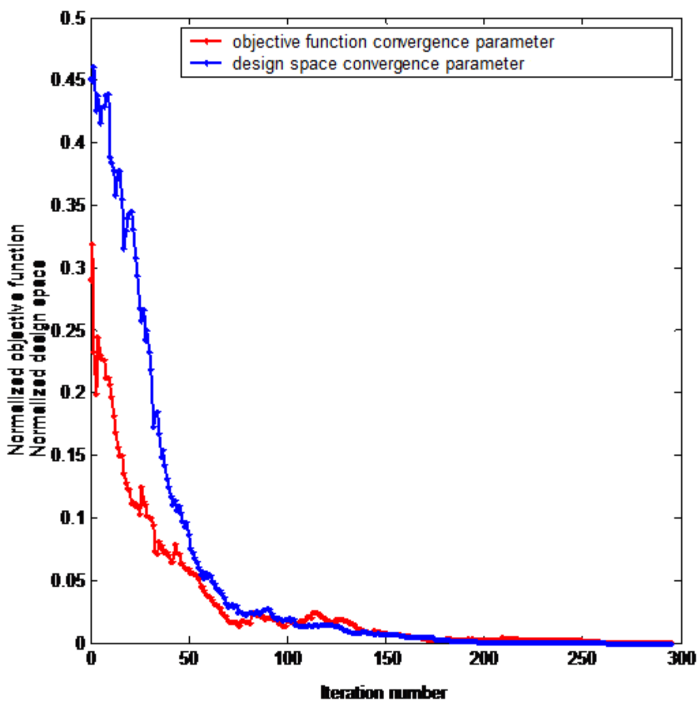
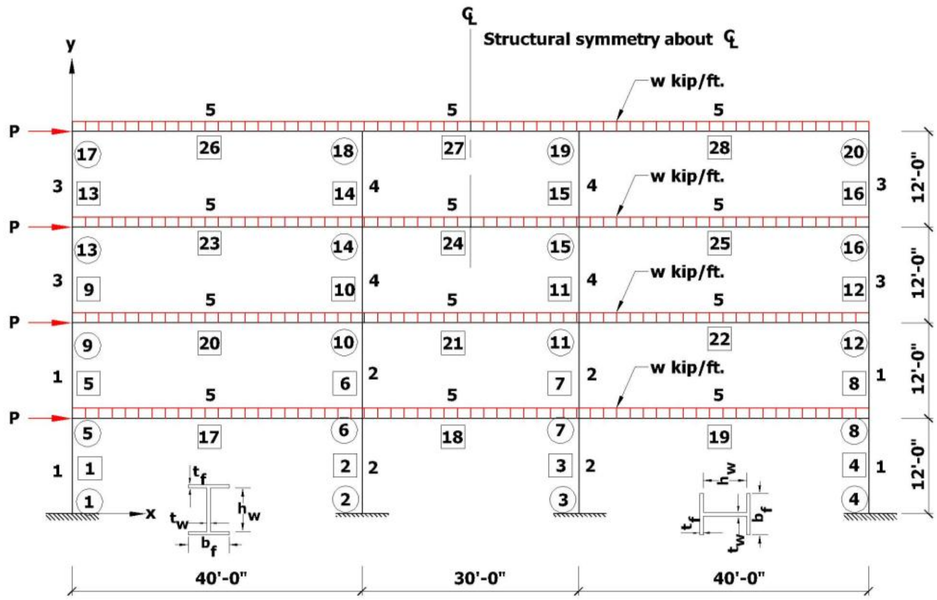


Figure 6 History of convergence criteria

4.2 Example 2: Three-Bay Four-Storey Rigid Joint Frame

A three-bay, four-storey steel rigid joint frame shown in figure 7 is used to test the performance of the developed algorithm. The design optimization problem is to find the cross-sectional dimensions of all the members such as to minimize the weight of the steel frame structure. In addition to the implicit constraints, the flange width and web depths of the members are required to lie within the intervals 4 in. – 18 in., and 4 in. – 30 in., respectively. The unbraced length of 12 ft is specified for the design of beams and columns. Conforming to the usual design practice, all beams and columns are specified to have the same flange width, b_f and the structure has to be symmetrical about the vertical centerline. The design involves five member fabrication groups i.e., four column groups and one beam group. The skeletal geometry of the structure and the load cases for this problem are shown in figure 7. This figure 7 also shows the member and node numbers, and also the member grouping numbers to which they belong to. For simplicity, a uniform member web depth of 28 in. and flange width of 16 in. is selected for all members in the initial design.

The initial design has a total weight of 58.91 kip. The structure weight ranges from 46.66 kip to 81.71 kip in the initial complex. It is interesting to note that the weight of the best design in the initial complex is about 7% heavier than the final design. It takes 117 iterations to obtain the optimum weight of 43.75 kip for this problem. In figure 8, the axial forces for applied load cases and axial force capacity of the members are shown. The bending moment for the two load cases, as well as the moment envelope and the moment capacity curves are shown in figure 9. The interaction equation checks i.e. (the interaction of the combined effects of axial and bending stresses) form the most critical constraints and are shown in figure 10. It is evident from figure 10 that the stresses in members 2, 4, 10, 13 and 17 are at their limiting values and thus govern the designs of their respective groups. The design history of the maximum and minimum weight of the structure in complex versus iteration is given in figure 11, which shows that the structure design converges to its optimum smoothly. The histories of the design variables and normalized objective function/design space are shown in figures 12 and figure 13 respectively.



Load case	P (kip)	w (kip/ft.)
1	0.00	3.00
2	30.00	2.00

xx — Member group
[xx] — Member number
(xx) — Node number

Figure 7 Geometry and load cases for 3-bay 4-storey frame

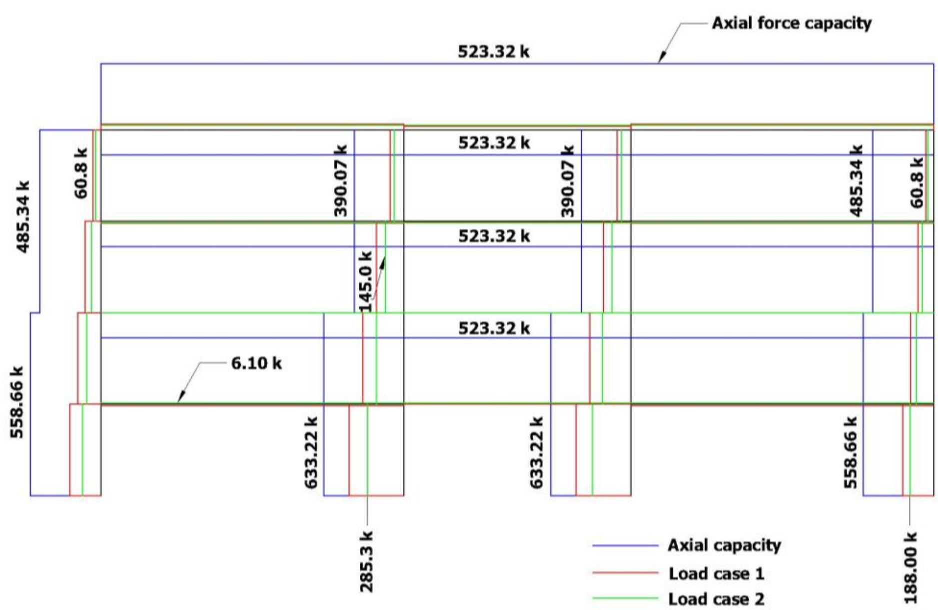


Figure 8 Axial force and Axial force capacity diagrams for 3-bay 4-storey frame

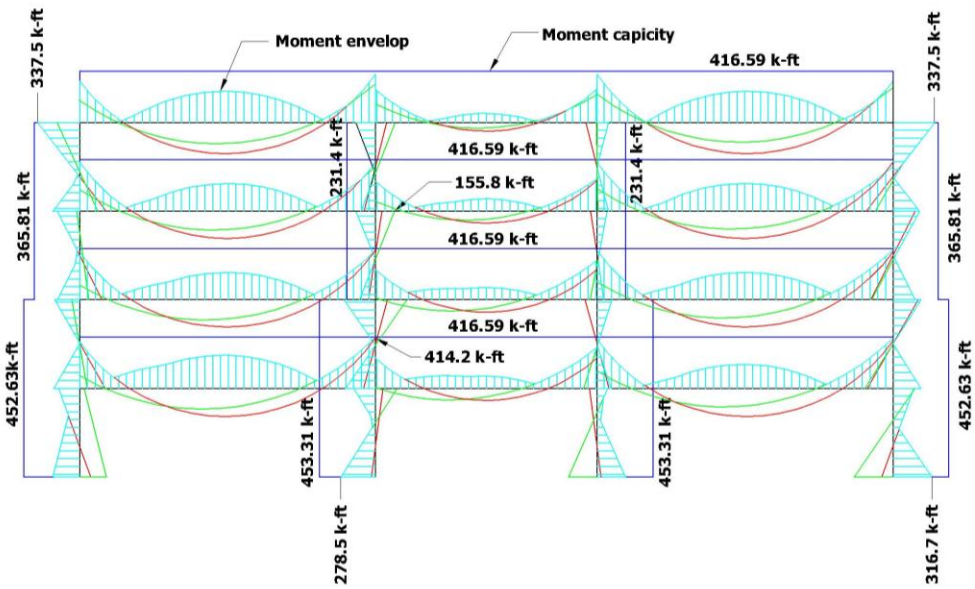


Figure 9 Moment envelop and moment capacity diagrams of 3-bay 4- storey frame

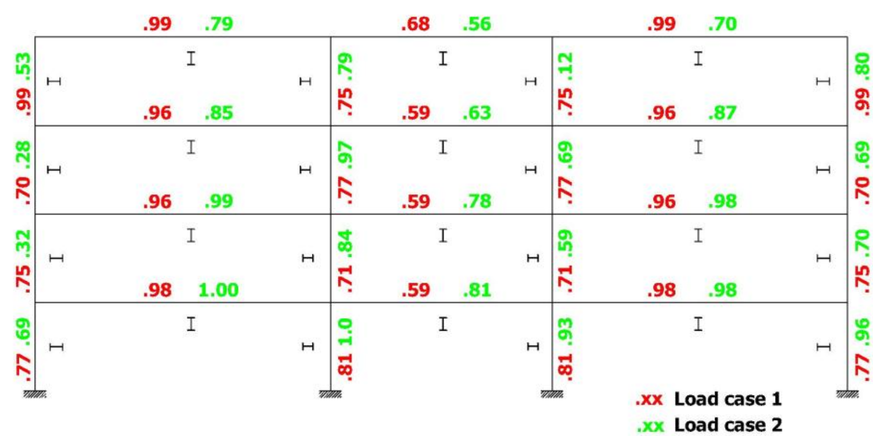


Figure 10 Final design and interaction eq. checks for 3-bay 4-storey frame

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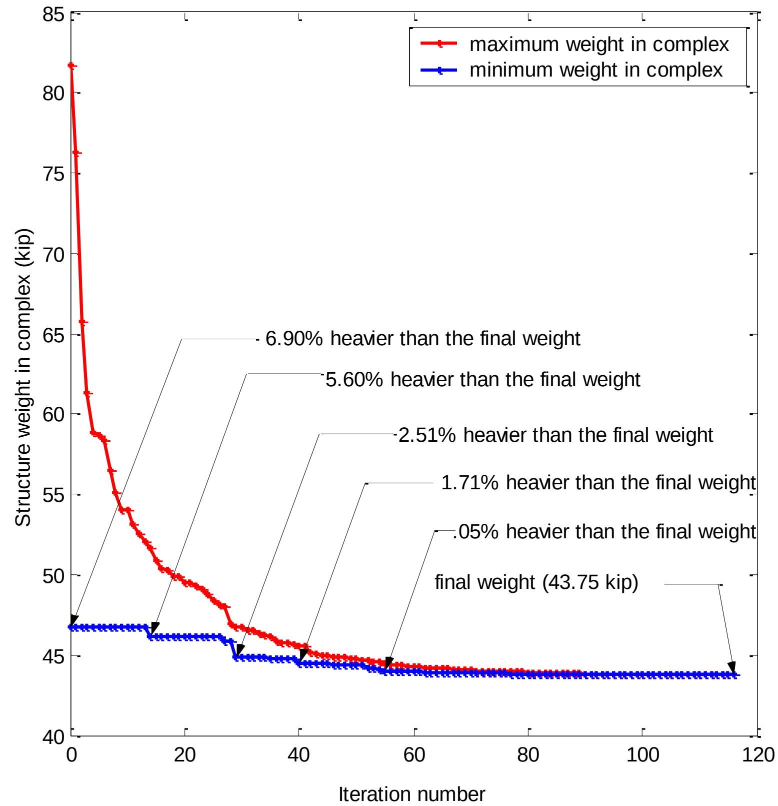

Figure 11 History of max/min weight of structure in the complex

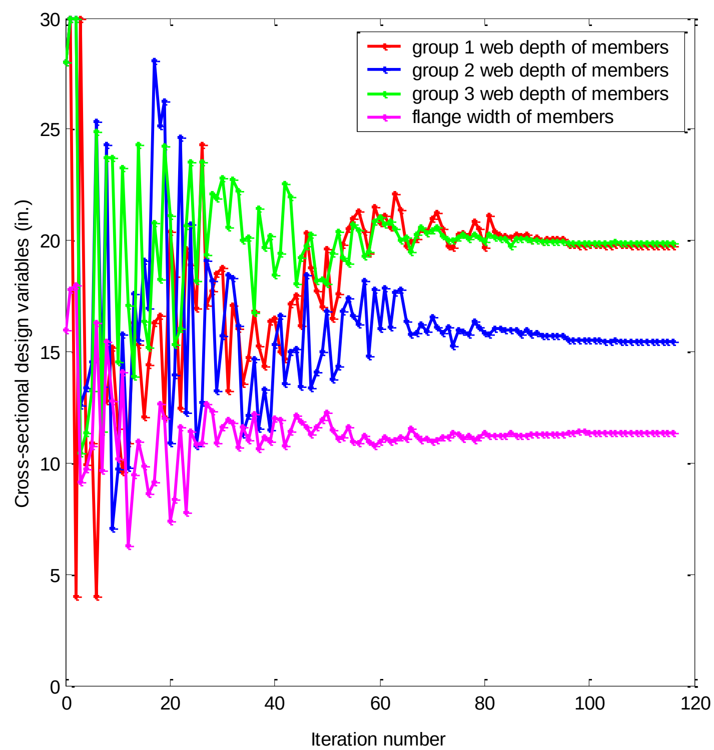

Figure 12 History of cross-sectional design variables

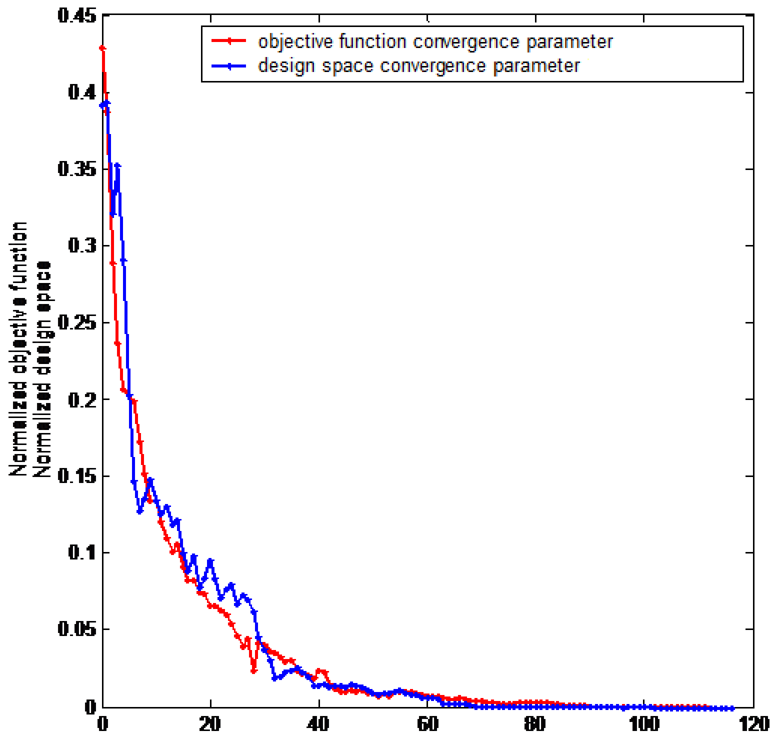

Figure 13 History of convergence criteria

5. Summary and Conclusion

In this study, a direct-search optimization algorithm based on the Complex Method with appropriate modifications is used in conjunction with the fully-stressed design for moment-resisting plane frame problems utilizing member sizing design parameters subject to strength considerations. A sequence of increasingly better designs can be achieved quite effectively by decomposing the design space into two subspaces. An important feature of this resolution is that the design requirements can be easily accommodated during the latter phase of sub optimization.

The modified Complex Method is able to search the design space of a given model and is an effective tool in locating the optimal design of the structure. It does require the generation of an initial complex of feasible points that are spread out throughout the design space and tends to move in the direction of improvements in the various parts of the design space. This feature provides the potential for locating global optimum. The improvement in function value is very rapid in the initial 10-20 iterations after the initial feasible complex is established. It is simple to formulate the problem generally in a manner suitable for Box Complex method. It becomes very easy to apply the method when there are more than a few constraints in the problem. Multiple loading conditions can be accommodated easily by this method without significantly increasingly the computational effort.

This procedure offers the designer the ability to search more of the design space with a minimal amount of effort, and the ability to optimize complicated models, for which the gradients would be time consuming, if not impossible to calculate. The designer can examine the many different optimum design states and can set some additional goals or constraints. If for some reason it is not possible to use optimum design, one of the slightly heavier designs found could then be selected.

Additional work can further improve the efficiency and effectiveness of the approach, by attempting to evaluate the effects of changing the number of points in the complex and reflection factor (α). For the test problems presented in this dissertation, there were 2N points in the complex and α was 1.3 (suggested by Box [3]). The user can specify any value of the reflection factor and specify the number of points to be used (except that there can not be less than 2 points). The amount of time to run and the amount of the design space searched is dependent on these variables.

This work can be extended with some effort to include three-dimensional space frames by simply modifying the analysis algorithm. Similarly, other design codes may be incorporated by appropriate changes to the program. Although the criterion for optimality in the present work is weight of the structures, the method can be applied to a cost criterion very easily by an appropriate change in the objective function. Such a change would not affect the already nonlinear nature of the problem nor the basic procedure. It is simple to incorporate deflection constraints as well since the deflection of all joints is available at each analysis. This adds to only the number of total constraints but does not need any change in the procedure. It may, however, increase the computations to arrive at the solution.

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