0-1 Integer Interval Number Programming Approach for the Multilevel Generalized Assignment Problem

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ABSTRACT

In this paper, an approach is suggested to solve the multilevel generalized assignment problem with 0-1 integer interval number programming. In the multilevel generalized assignment problem (MGAP) agents can perform tasks at more than one efficiency level. MGAP is the complicated form of the classical generalized assignment problem (GAP). A profit is associated with each assignment and the objective of the problem is profit maximization. In mathematical programming problem, the coefficients in the objective function and the constraint functions are always determined as crisp values. In practice, however, there are many decision situations where the objective functions and/or the constraints are uncertain to some degree. Over the last two decades, interval programming based on the interval analysis has been developed as a useful and simple method to deal with this type of uncertainty. The interval numbers will be in the constraints. A parametric study is carried out for the problem of concern.

Keywords: Multilevel generalized assignment; Interval numbers; Parametric study;

1 INTRODUCTION

MGAP is concerned with assigning n tasks to *m* agents with a maximum of l efficiency levels. Each task j must be assigned to exactly one agent i at a level k. The resource of each agent i has an upper limit of bi, which must not be exceeded. More than one task may be assigned to one agent.

Generally, the data of real-world problems are imprecise or uncertain. Then, the input data can be only estimated as with some kind of uncertainty, this uncertainty may be represented by an interval number. For MGAP, we apply interval numbers for the resource used if task j is assigned to agent i.

MGAP was first described by Glover et al [7]. The same problem was addressed later by laguna et al [9], who tackled the problem with a tabu search procedure.

In traditional mathematical programming, the coefficients of the problems are always treated as deterministic values. However uncertainty always exists in practical engineering problems. For uncertain optimization problems, fuzzy and stochastic approaches are commonly used to describe the imprecise characteristics.

The studies of random instances of assignment problems date back to as early as Donath [6]. The probabilistic analysis of assignment problems are covered only briefly as a number of comprehensive surveys on solution methods for various classes of assignment problems [13]. This probabilistic analysis of assignment problems are available in the literature (see Burkard and Cela [3], Burkard [4], Anstreicher [2] and Loiola et al [11]). Linzhong [10] introduced a fuzzy approach for the quadratic assignment problem. Deng Kui Huang et al [5] developed a fuzzy multi-criteria decision making approach for solving a bi-objective personnel assignment problem.

The rest of this paper is organized as follows. The MGAP with 0-1 integer interval number programming is described in section 2. Section 3 presents an optimization approach to solve the MGAP which is described in section 2. A parametric study is carried out for the problem of concern in section 4. A numerical example is provided in Section 5 to clarify the proposed approach. Finally section 6 contains the conclusions.

2 STATEMENT OF THE PROBLEM

Now, we define the MGAP by using 0-1 integer interval number programming as follows:

$$\text{Max}\, z = \sum_{i=1}^{m}\sum_{j=1}^{n}\sum_{k=1}^{l} p_{ijk}\, x_{ijk} \quad (1)$$

subject to

$$\sum_{i=1}^{m}\sum_{k=1}^{l} x_{ijk} = 1, \qquad j \in N = \{1,2,...,n\}, \quad (2)$$

$$\sum_{j=1}^{n}\sum_{k=1}^{l} \left[a_{ijk}^{L}, a_{ijk}^{R}\right] x_{ijk} \le b_i, \quad i \in M = \{1,2,...,m\}, \quad (3)$$

$$x_{ijk} = 0 \text{ or } 1, \quad i \in M, \ j \in N, \ k \in L = \{1,2,...,l\}. \ (4)$$

In the above formulation $\left[a_{ijk}^{L}, a_{ijk}^{R}\right]$ is an interval number represent the resource used if task j is assigned to agent i at the kth level. b_i is the resource available from agent i. The superscripts L and R denote lower and upper bounds of an interval number. p_{ijk} is the benefit of assigning task j to agent i at the kth level. The binary variable x_{ijk} is defined to be 1 if task i is assigning to agent j at the kth level. The objective function is given by (1). Constraint (2) ensures that each task is completely assigned to some agent at some level. Constraint (3) ensures that the total resources required from an agent do not exceed capacity. Without loss of generality, it will be assumed that $a_{ijk} \ge 0$, that $a_{ijk} = 0 \Rightarrow x_{ijk} = 0$, and that $a_{ijk} < b_i$, $i \in M$, $j \in N$, $k \in L$ [1].

3 The Optimization approach

Based on the proposed approach of Jiang [8] for treating interval number, we well treat the uncertainty in the left hand side of constraints (3).

3.1 Treatment of the Uncertain Inequality Constraints

The possibility degree of interval number represents is a certain degree that one interval number is larger or smaller than another [14]. The set of constraints (3) can be written as

$$-\sum_{j=1}^{n}\sum_{k=1}^{l} \left[a_{ijk}^{L}, a_{ijk}^{R}\right] x_{ijk} \ge -b_i, \ i = 1,2,...,m.$$

As in stochastic programming, this inequality satisfied with a possibility degree and formulate a deterministic inequality by the possibility degre $P_{E \ge b}$:

$$p_{E \ge b} = \begin{cases} 0, & b > E^R, \\ \dfrac{E^R - b}{E^R - E^L}, & E^L < b \le E^R, \\ 1, & b \le E^L. \end{cases}$$

where $g_i(x) = -\left[a_{ijk}^{L}, a_{ijk}^{R}\right] x_{ijk} + b_i$ $E = \left[g_i^L(x), g_i^R(x)\right]$ is the interval of the constraint function at x and its bounds can be obtained through the following two deterministic equations:

$$g_i^L(x,a) = \min_{a \in \Gamma} g_i(x,a), \quad g_i^R(x,a) = \max_{a \in \Gamma} g_i(x,a).$$

Where Γ is an uncertain vector and its components are all interval numbers. $P_{E \ge b} \ge \lambda_i$ is the possibility degree of the ith constraint. $0 \le \lambda_i \le 1$ is a predetermined possibility degree level.

3.2 The Deterministic form of MGAP

Through the above treatments for the MGAP which is described in the form (1) – (4), it can be transformed into the following deterministic form:

$$\text{Max } z = \sum_{i=1}^{m}\sum_{j=1}^{n}\sum_{k=1}^{l} p_{ijk} x_{ijk} \quad (5)$$

subject to

$$\sum_{i=1}^{m}\sum_{j=1}^{n} x_{ijk} = 1, \qquad j \in N = \{1,2,...,n\}, \quad (6)$$

$$p_{E \ge b} \ge \lambda_i, \qquad i = 1, 2, \ldots, m \quad (7)$$

$$x_{ijk} = 0 \text{ or } 1, \quad i \in M, \ j \in N, \ k \in L = \{1,2,...,l\}. \quad (8)$$

$$\lambda_i \in [0,1] \qquad i = 1, 2, \ldots, m \quad (9)$$

4 Parametric study for the problem (5) – (9)

Let λ_i, i = 1,2,…,m are assumed to be parameters rather than constants. The decision space of the problem (5) – (9) can be defined as follows:

$$X(\lambda) = \left\{ \begin{array}{l} x_{ijk} \in \{0,1\}, i = 1,2,...,m, \ j = 1,2,...,n, \\ k = 1,2,...,l \ \left| \text{satisfies set of constraints (6)} - (9) \right. \end{array} \right\}$$

In what follows, we give the definitions of some basic notions for problem (5) – (9).

The set of feasible parameters [12]

The set of feasible parameters of problem (5) - (9) which is denoted by U, is defined by

$$U = \left\{ \lambda^* \in R^m \left| X(\lambda) \ne \phi \right. \right\}$$

The solvability set

The solvability set of problem (5) - (9) which is denoted by V, is defined by

$$V = \left\{ \lambda^* \in U \left| \text{problem (5) - (9) has optimal solution} \right. \right\}$$

The stability set of the first kind

The stability set of the first kind of problem (5) – (9) that is denoted by S(x*) is defined by

$$S(x^*) = \left\{ \lambda^* \in V \left| x^* \text{ is optimal solution of problem (5) - (9)} \right. \right\}.$$

4.1 Determination of the Stability Set of the First Kind S(x*)

Going back to problem (5) – (9), the Kuhn-Tucker necessary conditions corresponding to this problem will take the following form

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$$\sum_{i=1}^{m}\sum_{j=1}^{n}\sum_{k=1}^{l}\frac{\partial p_{ijk}x_{ijk}}{\partial x_{ijk}}+\mu_j\sum_{i=1}^{m}\sum_{k=1}^{l}\frac{\partial x_{ijk}-1}{\partial x_{ijk}}+\rho_{ijk}\frac{\partial x_{ijk}-1}{\partial x_{ijk}}+\sigma_{ijk}\frac{\partial -x_{ijk}}{\partial x_{ijk}}+$$

$$\beta_i\frac{\partial}{\partial x_{ijk}}\left[\lambda_i\left(\sum_{j=1}^{n}\sum_{k=1}^{l}a^R_{ijk}x_{ijk}-\sum_{j=1}^{n}\sum_{k=1}^{l}a^L_{ijk}x_{ijk}\right)-\sum_{j=1}^{n}\sum_{k=1}^{l}a^R_{ijk}x_{ijk}+b_i\right]=0,$$

$$\mu_j\sum_{i=1}^{m}\sum_{k=1}^{l}x_{ijk}-1=0,\qquad j=1,2,\dots,n$$

$$\left.\begin{aligned}&\rho_{ijk}\left(x_{ijk}-1\right)=0,\\&\sigma_{ijk}\left(-x_{ijk}\right)=0\end{aligned}\right\}\ i=1,2,\dots,m,\ \ j=1,2,\dots,n,\ \ k=1,2,\dots,l \qquad (10)$$

$$\beta_i\left[\lambda_i\left(\sum_{j=1}^{n}\sum_{k=1}^{l}a^R_{ijk}x_{ijk}-\sum_{j=1}^{n}\sum_{k=1}^{l}a^L_{ijk}x_{ijk}\right)+\sum_{j=1}^{n}\sum_{k=1}^{l}a^R_{ijk}x_{ijk}-b_i\right]=0,$$

$$i=1,2,\dots,m$$

$$\sum_{i=1}^{m}\sum_{k=1}^{l}x_{ijk}-1=0,\qquad j=1,2,\dots,n$$

$$\left.\begin{aligned}&x_{ijk}-1\le 0,\\&x_{ijk}\le 0\end{aligned}\right\}\ i=1,2,\dots,m,\ \ j=1,2,\dots,n,\ \ k=1,2,\dots,l$$

$$\lambda_i\left(\sum_{j=1}^{n}\sum_{k=1}^{l}a^R_{ijk}x_{ijk}-\sum_{j=1}^{n}\sum_{k=1}^{l}a^L_{ijk}x_{ijk}\right)+b^L_i+\sum_{j=1}^{n}\sum_{k=1}^{l}a^R_{ijk}x_{ijk}\le 0,$$

$$i=1,2,\dots,m$$

$$\mu_j,\rho_{ijk},\sigma_{ijk},\beta_i\le 0,$$

$$i=1,2,\dots,m,\quad j=1,2,\dots,n,\quad k=1,2,\dots,k$$

where $\mu_j,\rho_{ijk},\sigma_{ijk}$ and β_i are called Lagrange multipliers. All the relation of system (10) are evaluated at the optimal solution x^*. According to whether any of the Lagrange multipliers is zero or negative, then the stability set of the first kind can be determined. Also by treating the relations of system (10), we can express the set of feasible parameters and the solvability set.

5 NUMERICAL EXAMPLE

Consider the instance of MGAP with 0-1 integer interval number programming given by m = 3, n = 8 and l = 2

$$p=\begin{Bmatrix}27,34 & 12,20 & 12,34 & 16,19 & 24,9 & 31,\ 3 & 41,19 & 13,34\\ 14,27 & 5,12 & 27,12 & 9,24 & 36,16 & 25,41 & 1,31 & 34,13\\ 34,14 & 34,37 & 20,15 & 9,36 & 19,\ 9 & 19,\ 1 & 3,25 & 34,34\end{Bmatrix}$$

$$,\ (b)=\begin{Bmatrix}22\\24\\30\end{Bmatrix}$$

$$(a^L,a^R)=\begin{Bmatrix}(10,12)(13,16) & (13,16)(17,21) & (8,11)(12,15) & (6,9)(8,12)\\ (6,9)(10,14) & (10,15)(9,13) & (4,8)(6,12) & (12,15)(10,16)\\ (8,11)(11,13) & (5,9)(10,14) & (11,15)(8,12) & (10,14)(6,8)\\ (6,9)(4,8) & (3,6)(10,15) & (8,11)(6,9) & (4,6)(12,15)\\ (10,12)(8,11) & (12,16)(4,8) & (11,13)(12,15) & (10,14)(9,13)\\ (10,12)(4,7) & (10,13)(4,7) & (10,13)(8,11) & (10,13)(6,8)\end{Bmatrix}$$

Let $\lambda_1=0.2,\ \lambda_2=0.4,\ \lambda_3=0.5$.

The deterministic MGAP with 0-1 integer interval number programming can be written in the following form:

$$\max\ z=p_{111}x_{111}+p_{112}x_{112}+p_{121}x_{121}+p_{122}x_{122}+p_{131}x_{131}+p_{132}x_{132}+$$
$$p_{141}x_{141}+p_{142}x_{142}+p_{151}x_{151}+P_{152}x_{152}+p_{161}x_{161}+p_{162}x_{162}+p_{171}x_{171}+$$
$$p_{172}x_{172}+p_{181}x_{181}+p_{182}x_{182}+p_{211}x_{211}+p_{212}x_{112}+p_{221}x_{221}+p_{222}x_{222}+$$
$$p_{231}x_{231}+p_{232}x_{232}+p_{241}x_{241}+p_{242}x_{242}+p_{251}x_{251}+p_{252}x_{252}+p_{261}x_{261}+$$
$$p_{262}x_{262}+p_{271}x_{271}+p_{272}x_{272}+p_{281}x_{281}+p_{282}x_{282}+p_{311}x_{311}+p_{312}x_{312}+$$
$$p_{321}x_{321}+p_{322}x_{322}+p_{331}x_{331}+p_{332}x_{332}+p_{341}x_{341}+p_{342}x_{342}+p_{351}x_{351}+$$
$$p_{352}x_{352}+p_{361}x_{361}+p_{362}x_{362}+p_{371}x_{371}+p_{372}x_{372}+p_{381}x_{381}+p_{382}x_{382}$$

subject to

$$x_{111}+x_{112}+x_{211}+x_{212}+x_{311}+x_{312}=1,$$
$$x_{121}+x_{122}+x_{221}+x_{222}+x_{321}+x_{322}=1,$$
$$x_{131}+x_{132}+x_{231}+x_{232}+x_{331}+x_{332}=1,$$
$$x_{141}+x_{142}+x_{241}+x_{242}+x_{341}+x_{342}=1,$$
$$x_{151}+x_{152}+x_{251}+x_{252}+x_{351}+x_{352}=1,$$
$$x_{161}+x_{162}+x_{261}+x_{262}+x_{361}+x_{362}=1,$$
$$x_{171}+x_{172}+x_{271}+x_{272}+x_{371}+x_{372}=1,$$
$$x_{181}+x_{182}+x_{281}+x_{282}+x_{381}+x_{382}=1,$$
$$x_{ijk}=0,1\qquad i=1,2,3,\quad j=1,2,\dots,8,\quad k=1,2$$

$$11.6x_{111}+11.4x_{112}+15.4x_{121}+20.2x_{122}+10.4x_{131}+$$
$$14.4x_{132}+8.4x_{41}+10.2x_{142}+8.4x_{151}+13.2x_{152}+14x_{161}+$$
$$12.2x_{162}+7.2x_{171}+10.8x_{172}+14.4x_{181}+14.8x_{182}\ge 22$$

$$9.8x_{211}+12.2x_{212}+7.4x_{221}+12.4x_{222}+13.4x_{231}+10.4x_{232}$$
$$+12.4x_{241}+7.2x_{242}+7.8x_{251}+6.4x_{252}+4.8x_{261}+11x_{262}+$$
$$9.8x_{271}+7.8x_{272}+5.2x_{281}+13.8x_{282}\ge 24$$

$$11x_{311}+9.5x_{312}+14x_{321}+6x_{322}+12x_{331}+13.5x_{332}+12x_{341}$$
$$+11x_{342}+11x_{351}+5.5x_{352}+11.5x_{361}+5.5x_{362}+11.5x_{371}$$
$$+9.5x_{372}+11.5x_{381}+7x_{382}\ge 30$$

Then the above problem can be solved by using any package of ILP and its optimal solution is found:

$$x^*_{171} = x^*_{182} = x^*_{231} = x^*_{251} = x^*_{262} = x^*_{311} = x^*_{321} = x^*_{342} = 1$$

and all other variables = 0. The objective function z = 283. By solving the system (10) at the above optimal solution x^*, we get:

set of feasible parameters: $U = \{\lambda_i \,|0 \le \lambda_i \le 1,\ i = 1,2,3\}$,

solvability set: $V = \{\lambda_i \,|0 \le \lambda_i \le 1,\ i = 1,2,3\}$,

stability set of the first kind:

$$S(x^*) = \left\{\lambda \,|0 \le \lambda_1 \le 1,\ 0 \le \lambda_2 \le \frac{2}{3}, 0 \le \lambda_3 \le 0.5\right\}.$$

6 Conclusions

An approach based on 0-1 integer interval number programming have been developed for MGAP. The proposed approach gave a better solution than stated in reference [1] especially for the objective function value. A parametric study has been carried out for the same problem. A numerical example is given to clarify the proposed approach. However, An exact approach is needed for solving MGAP in case of interval numbers are found in both of objective function and both sides of constraints.

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